Type-Checking

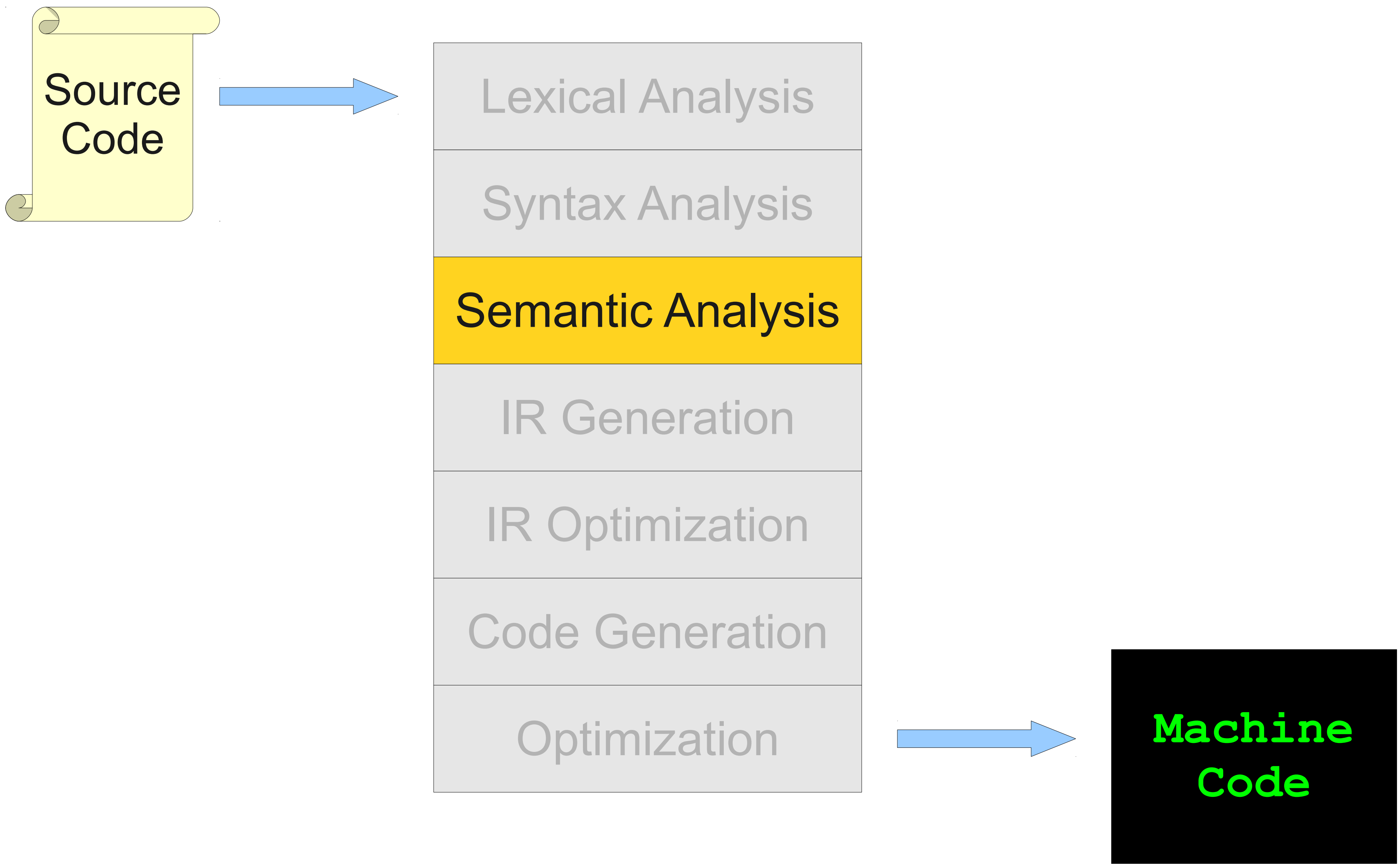
Announcements

- Programming Project 2 due **tonight** at 11:59PM.
  - Office hours from 1:00PM – 3:00PM in Gates 160.
- Programming Project 3 out.
  - Scope checkpoint due **Saturday, July 23 at 11:59PM**.
  - This is a **hard deadline**, no late days allowed.
  - Final submission due **Wednesday, July 27 at 11:59PM**.
  - **Start early**; this assignment is significantly larger than the previous two assignments.

More Announcements

- Programming Assignment 1 graded and returned on paperless.stanford.edu.
  - Mean: 52.9 / 60
  - Stdev: 8
- Written Assignment 1 graded. Hard copies returned after class, electronic copies will be emailed later today.
  - Mean: 20.2 / 24
  - Stdev: 3
- **Let us know ASAP if you haven't heard back from us by tomorrow morning.**

Where We Are


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

Review from Last Time

```
class MyClass implements MyInterface {
    string myInteger;

    void doSomething() {
        int[] x;
        x = new string;

        x[5] = myInteger * y;
    }
    void doSomething() {

    }
    int fibonacci(int n) {
        return doSomething() + fibonacci(n - 1);
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Can't multiply strings

Variable not declared

Can't redefine functions

Can't add void

No main function

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What Remains to Check?

- **Type errors**.
- Today:
  - What are types?
  - What is type-checking?
  - A type system for Decaf.

What is a Type?

- This is the subject of some debate.
- To quote Alex Aiken:
  - **“The notion varies from language to language**.
  - The consensus:
    - A set of values.
    - A set of operations on those values”
- **Type errors** arise when operations are performed on values that do not support that operation.

Types of Type-Checking

- **Static type checking.**
  - Analyze the program during compile-time to prove the absence of type errors.
  - Never let bad things happen at runtime.
- **Dynamic type checking.**
  - Check operations at runtime before performing them.
  - More precise than static type checking, but usually less efficient.
  - (Why?)
- **No type checking.**
  - Throw caution to the wind!

Type Systems

- The rules governing permissible operations on types forms a **type system**.
- **Strong type systems** are systems that never allow for a type error.
  - Java, Python, JavaScript, LISP, Haskell, etc.
- **Weak type systems** can allow type errors at runtime.
  - C, C++

Type Wars

- **Endless** debate about what the “right” system is.
- Dynamic type systems make it easier to prototype; static type systems have fewer bugs.
- Strongly-typed languages are more robust, weakly-typed systems are often faster.

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- Strongly-typed languages are more robust, weakly-typed systems are often faster.
- **I'm staying out of this!**

Our Focus

- Decaf is typed **statically** and **weakly**:
  - Type-checking occurs at compile-time.
  - Runtime errors like dereferencing **`null`** or an invalid object are disallowed.
- Decaf uses **class-based inheritance**.
- Decaf distinguishes primitive types and classes.

Typing in Decaf

Static Typing in Decaf

- Static type checking in Decaf consists of two separate processes:
  - Inferring the type of each expression from the types of its components.
  - Confirming that the types of expressions in certain contexts matches what is expected.
- **Logically** two steps, but you will probably combine into one pass.

An Example

```
while (numBitsSet(x + 5) <= 10) {

    if (1.0 + 4.0) {
        /* … */
    }

    while (5 == null) {
        /* … */
    }

}
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Well-typed expression with wrong type.

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Expression with type error

Inferring Expression Types

- How do we determine the type of an expression?
- Think of process as **logical inference**.

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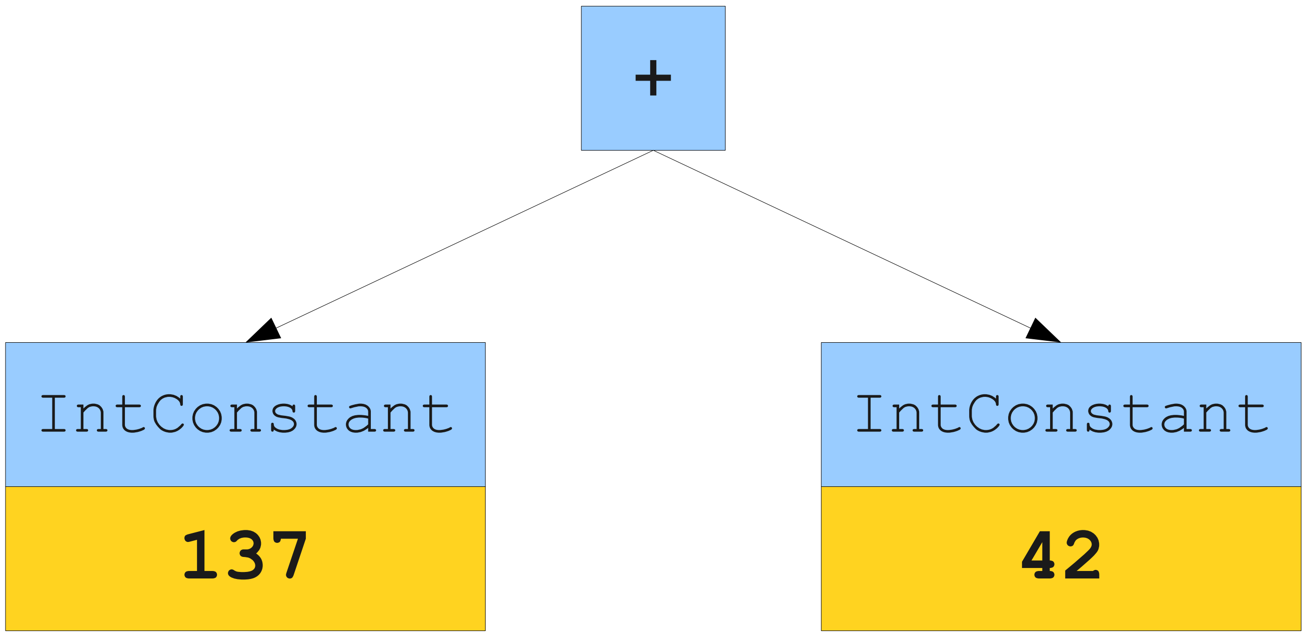

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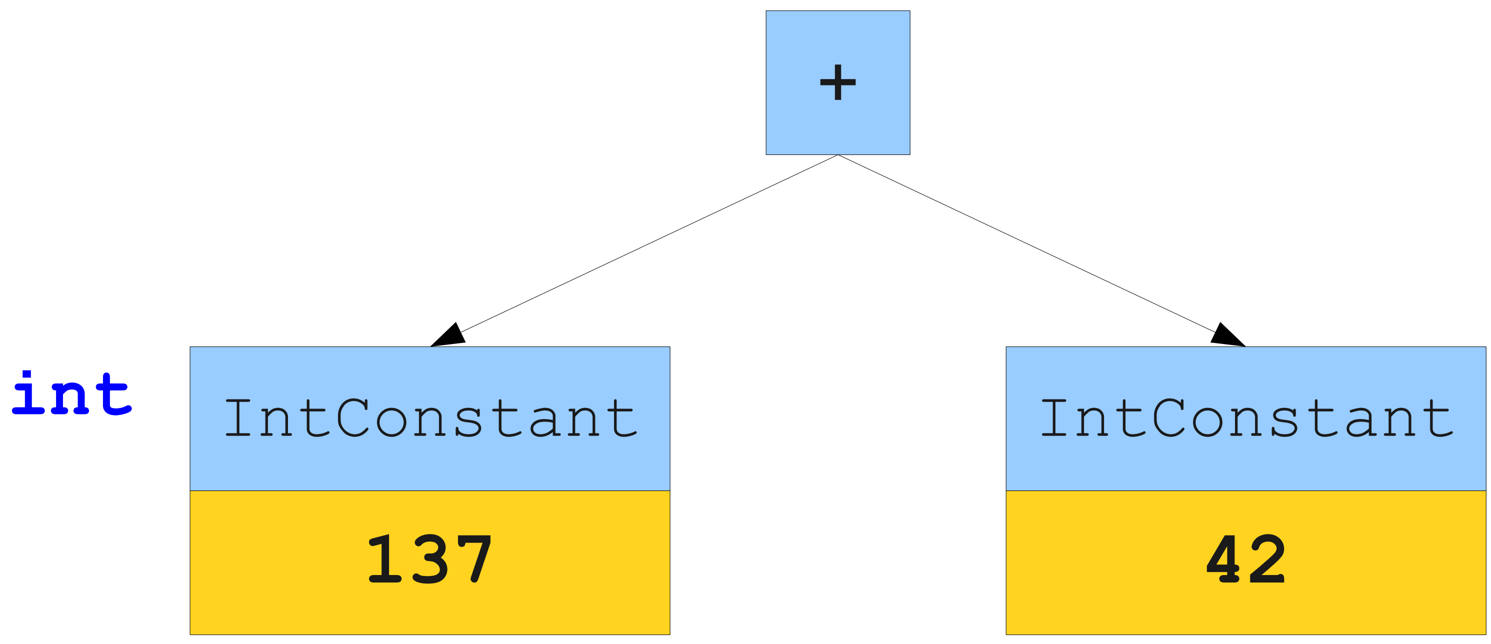

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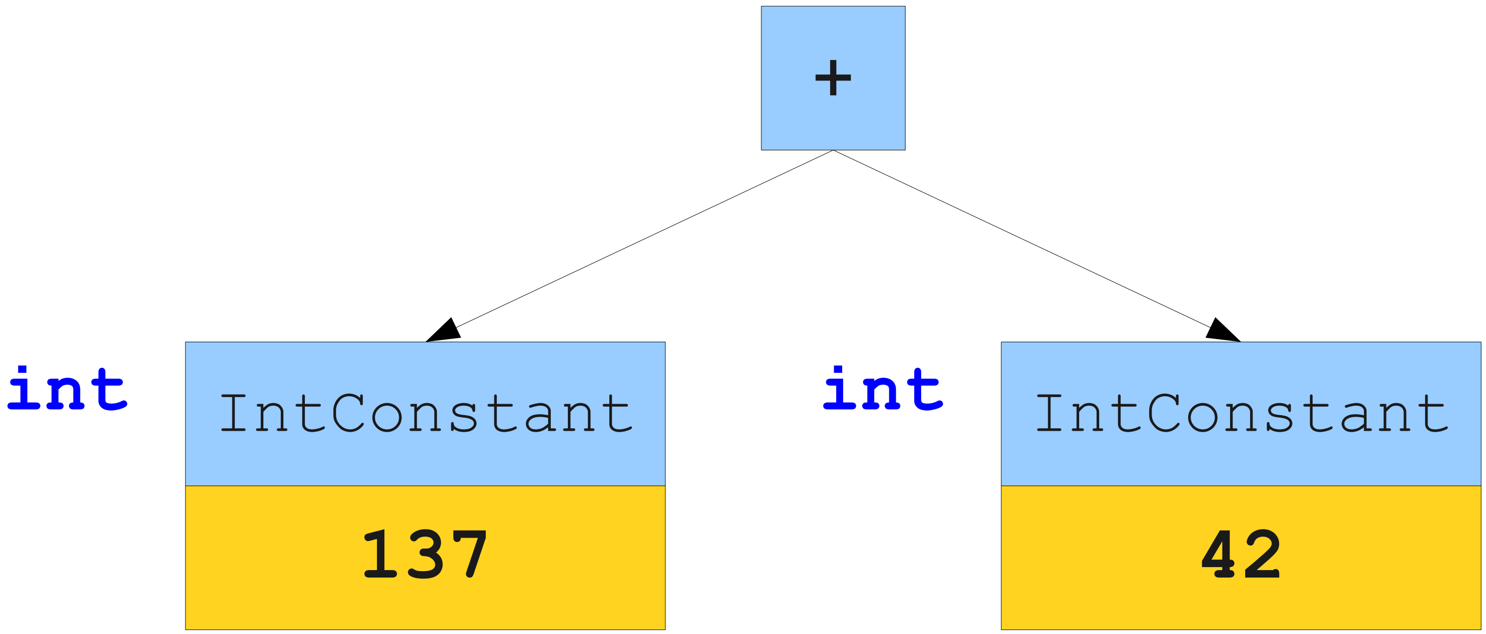

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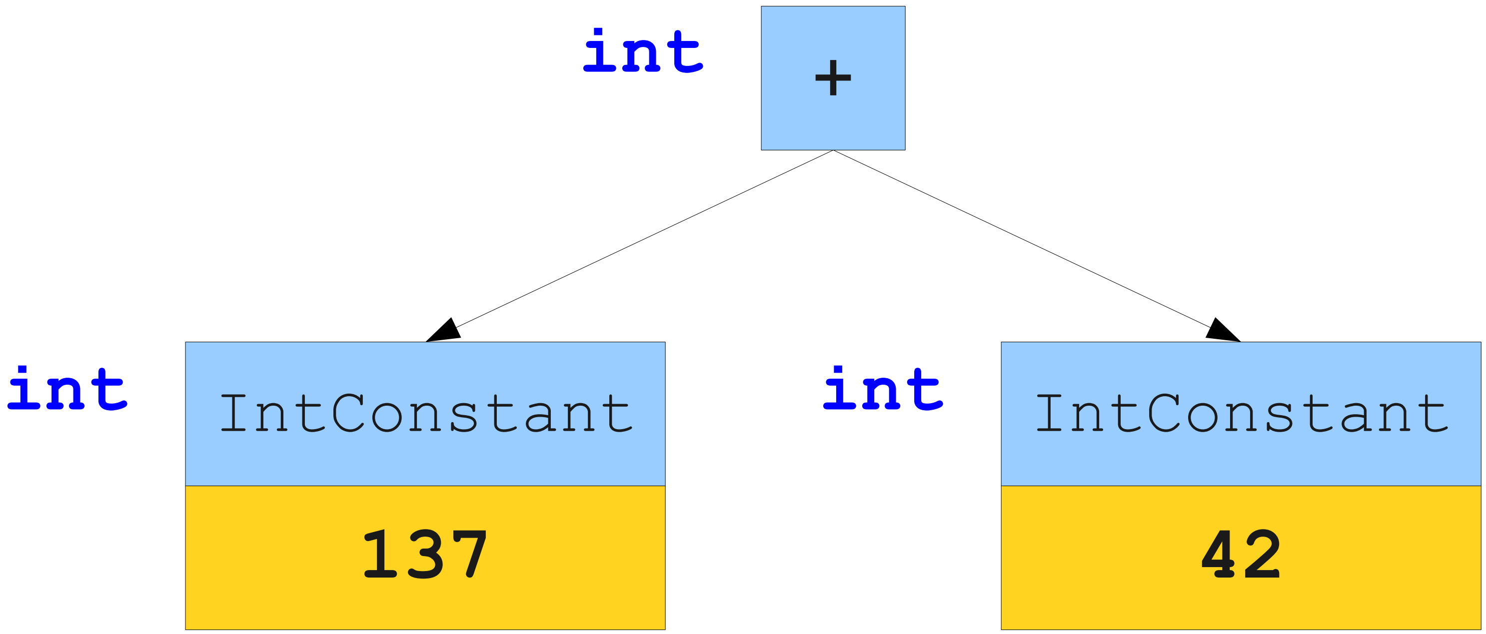

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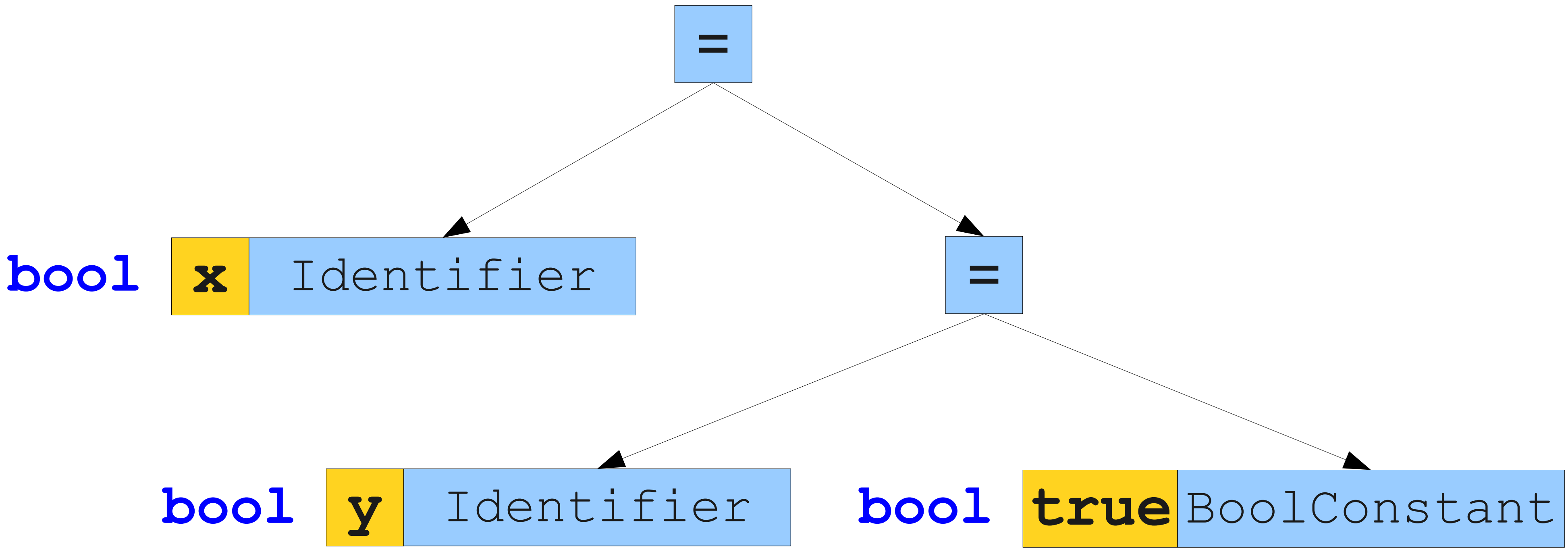

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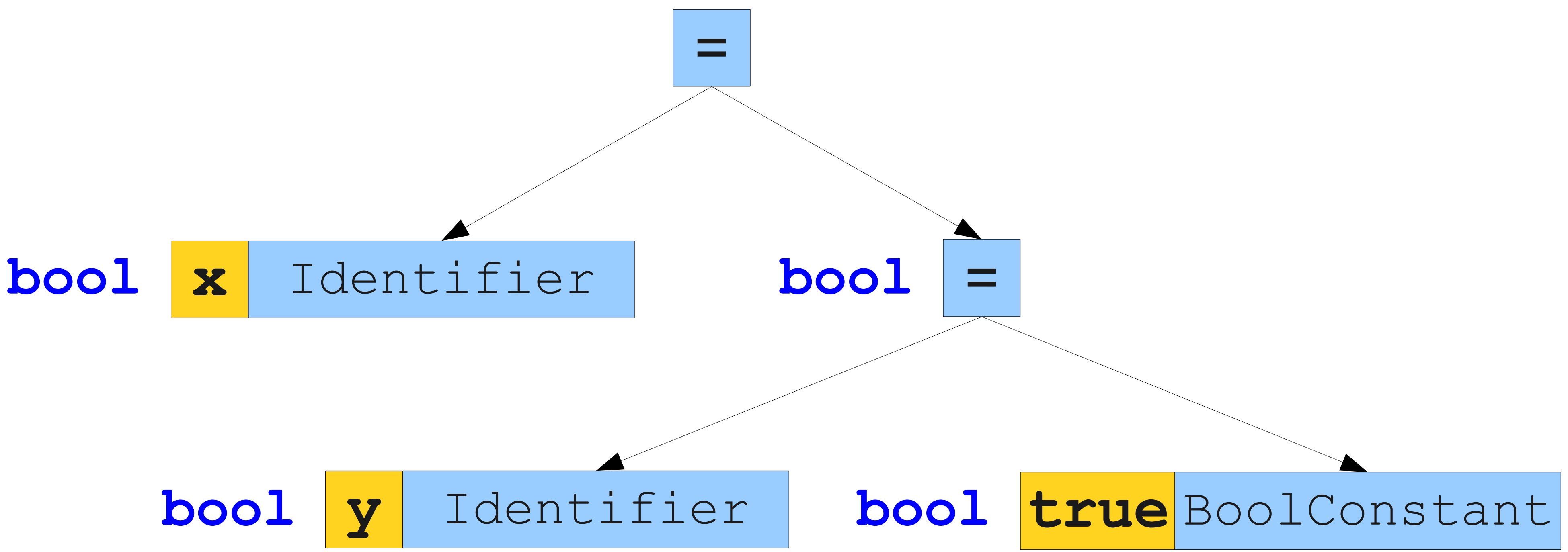

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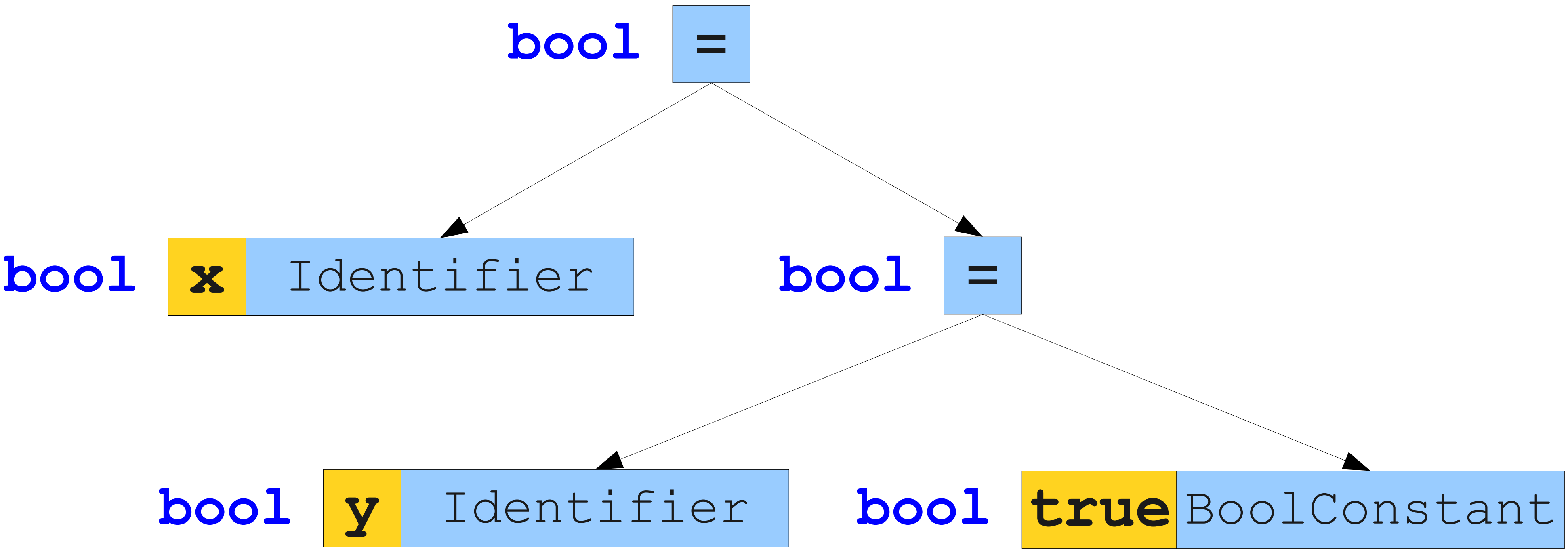

Sample Inference Rules

- “If **x** is an identifier that refers to an object of type **t**, the expression **x** has type **t**.”
- “If **e** is an integer constant, **e** has type **`int`**.”
- “If the operands e_1 and e_2 of $\mathbf{e_1 + e_2}$ are known to have types **`int`** and **`int`**, then $\mathbf{e_1 + e_2}$ has type **`int`**.”

Type Checking as Proofs

- We can think of syntax analysis as **proving** claims about the types of expressions.
- We begin with a set of **axioms**, then apply our **inference rules** to determine the types of expressions.
- **Many type systems can be thought of as proof systems**.

Formalizing our Notation

- We will encode our axioms and inference rules using this syntax:

$$\frac{\text{Preconditions}}{\text{Postconditions}}$$

- This is read "if **preconditions** are true, we can infer **postconditions**."

Examples of Formal Notation

$$\frac{A \to Bv \text{ is a production.} \quad t \in \text{FIRST}(B)}{t \in \text{FIRST}(A)}$$

$$\frac{A \to \varepsilon \text{ is a production.}}{\varepsilon \in \text{FIRST}(A)}$$

$$\frac{A \to B_1B_2...B_ntv \text{ is a production.} \quad \varepsilon \in \text{FIRST}(B_i) \text{ for } 1 \leq i \leq n}{t \in \text{FIRST}(A)}$$

$$\frac{A \to B_1 \ldots B_n \text{ is a production.} \quad \varepsilon \in \text{FIRST}(B_i) \text{ for } 1 \leq i \leq n}{\varepsilon \in \text{FIRST}(A)}$$

Formal Notation for Type Systems

- We write

$$\vdash \mathbf{e} : \mathbf{T}$$

  if the expression **e** has type **T**.
- The symbol $\vdash$ means “we can infer...”

Our Starting Axioms

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$$\frac{}{\vdash \texttt{true} : \texttt{bool}} \qquad \frac{}{\vdash \texttt{false} : \texttt{bool}}$$

Some Simple Inference Rules

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$$\frac{i \text{ is an integer constant}}{\vdash i : \texttt{int}}$$

$$\frac{s \text{ is a string constant}}{\vdash s : \texttt{string}}$$

$$\frac{d \text{ is a double constant}}{\vdash d : \texttt{double}}$$

More Complex Inference Rules

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$$\frac{\vdash e_1 : \textbf{int} \qquad \vdash e_2 : \textbf{int}}{\vdash e_1 + e_2 : \textbf{int}}$$

$$\frac{\vdash e_1 : \textbf{double} \qquad \vdash e_2 : \textbf{double}}{\vdash e_1 + e_2 : \textbf{double}}$$

More Complex Inference Rules

If we can show that e_1 and e_2 have type int...

$$\frac{\vdash e_1 : \mathbf{\texttt{int}} \qquad \vdash e_2 : \mathbf{\texttt{int}}}{\vdash e_1 + e_2 : \mathbf{\texttt{int}}}$$

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More Complex Inference Rules

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$$\frac{\vdash e_1 : \texttt{double} \qquad \vdash e_2 : \texttt{double}}{\vdash e_1 + e_2 : \texttt{double}}$$

… then we can show that $e_1 + e_2$ has type int as well

Even More Complex Inference Rules

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$$\frac{\vdash e_1 : T \qquad \vdash e_2 : T \qquad T \text{ is a primitive type}}{\vdash e_1 \;\texttt{==}\; e_2 : \textbf{bool}}$$

$$\frac{\vdash e_1 : T \qquad \vdash e_2 : T \qquad T \text{ is a primitive type}}{\vdash e_1 \;\texttt{!=}\; e_2 : \textbf{bool}}$$

Why Specify Types this Way?

- Gives a **rigorous definition of types** independent of any particular implementation.
  - No need to say “you should have the same type rules as my reference compiler.”
- Gives **maximum flexibility in implementation**.
  - Can implement type-checking however you want, as long as you obey the rules.
- Allows **formal verification of program properties**.
  - Can do inductive proofs on the structure of the program.
- **This is what's used in the literature**.
  - Good practice if you want to study types.

A Problem

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$$\frac{x \text{ is an identifier.}}{\vdash x : \mathbf{??}}$$

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How do we know the type of x if we don't know what it refers to?

An Incorrect Solution

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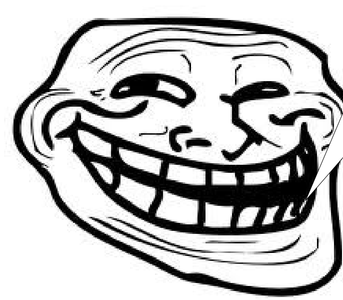

Strengthening our Inference Rules

- The facts we're proving have no **context**.
- We need to strengthen our inference rules to remember **under what circumstances the results are valid**.

Adding Scope

- We write

$$S \vdash \mathbf{e} : \mathbf{T}$$

  if **in scope S**, the expression **e** has type **T**.
- Types are now proven relative to the scope they are in.

Old Rules Revisited

$$\frac{}{S \vdash \texttt{true} : \texttt{bool}}$$

$$\frac{}{S \vdash \texttt{false} : \texttt{bool}}$$

$$\frac{i \text{ is an integer constant}}{S \vdash i : \texttt{int}}$$

$$\frac{s \text{ is a string constant}}{S \vdash s : \texttt{string}}$$

$$\frac{d \text{ is a double constant}}{S \vdash d : \texttt{double}}$$

$$\frac{S \vdash e_1 : \texttt{double} \quad S \vdash e_2 : \texttt{double}}{S \vdash e_1 + e_2 : \texttt{double}}$$

$$\frac{S \vdash e_1 : \texttt{int} \quad S \vdash e_2 : \texttt{int}}{S \vdash e_1 + e_2 : \texttt{int}}$$

A Correct Rule

$$\frac{x \text{ is an identifier.} \quad x \text{ is a variable in scope S with type T.}}{S \vdash x : T}$$

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$$\frac{\begin{array}{c} x \text{ is an identifier.} \\ \textbf{\textit{x} is a variable in scope S} \text{ with type T.} \end{array}}{S \vdash x : T}$$

Rules for Functions

$$\frac{}{S \vdash f(e_1, \ldots, e_n) \ : ??}$$

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f is an identifier.
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Read rules
like this

Rules for Arrays

$$\frac{S \vdash e_1 : T\texttt{[]} \qquad S \vdash e_2 : \texttt{int}}{S \vdash e_1[e_2] : T}$$

Rule for Assignment

$$\frac{S \vdash e_1 : T \qquad S \vdash e_2 : T}{S \vdash e_1 = e_2 : T}$$

Rule for Assignment

$$\frac{S \vdash e_1 : T \qquad S \vdash e_2 : T}{S \vdash e_1 = e_2 : T}$$

Why isn't this rule a problem for this statement?

```
5 = x;
```

Rule for Assignment

$$\frac{\begin{array}{c} S \vdash e_1 : T \\ S \vdash e_2 : T \end{array}}{S \vdash e_1 = e_2 : T}$$

If **`Derived`** extends **`Base`**, will this rule work for this code?

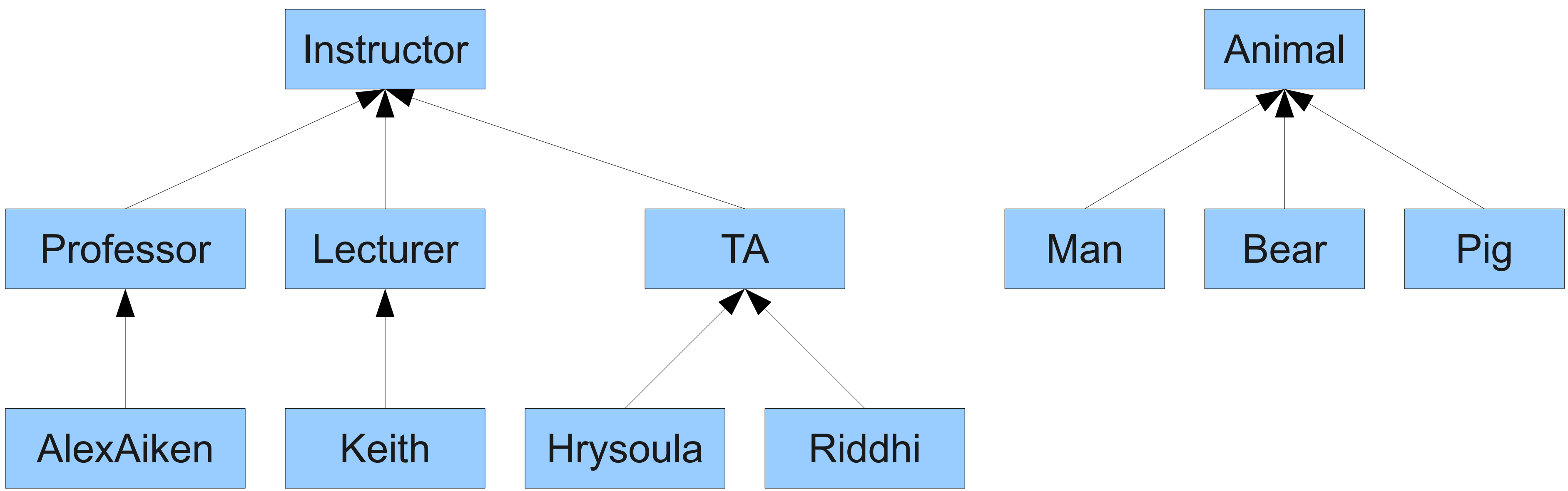
```
Base    myBase;
Derived myDerived;

myBase = myDerived;
```

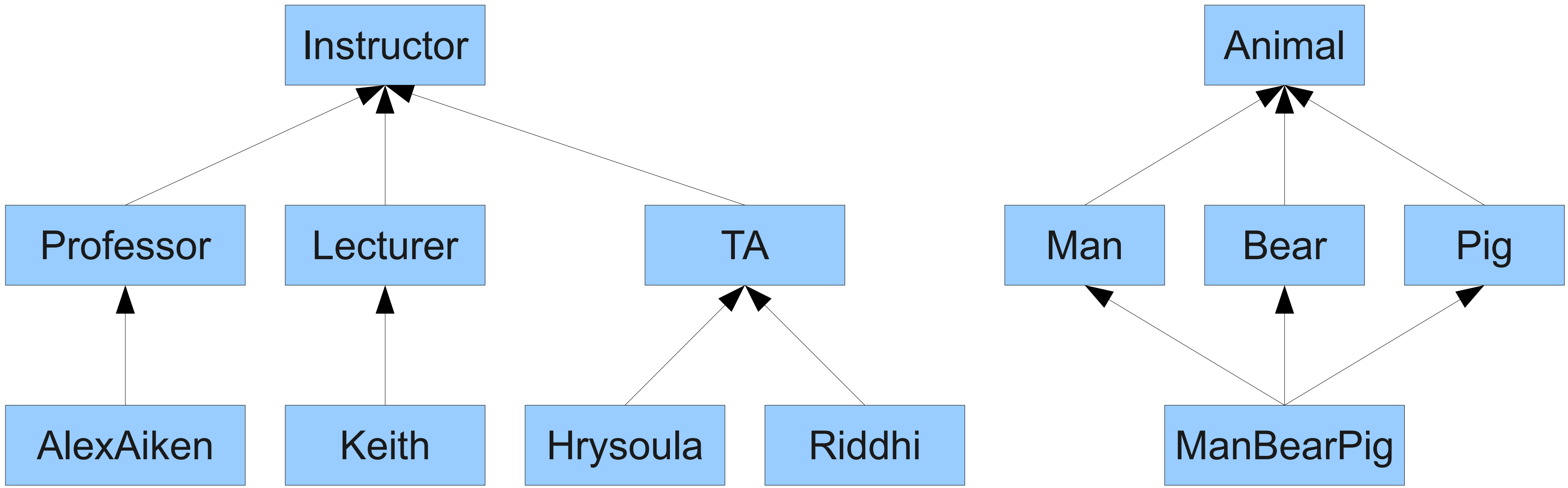
Typing with Classes

- How do we factor inheritance into our inference rules?
- We need to consider the **shape** of class hierarchies.

Single Inheritance



Multiple Inheritance



Properties of Inheritance Structures

- Any class is convertible to itself. (**Reflexivity**)
- If A is convertible to B and B is convertible to C, then A is convertible to C. (**Transitivity**)
- If A is convertible to B and B is convertible to A, then A and B are the same type. (**Antisymmetry**)
- This defines a **partial order** over types.

Types and Partial Orders

- We say that $A \leq B$ if A is convertible to B.
- We have that
  - $A \leq A$
  - $A \leq B$ and $B \leq C$ implies $A \leq C$
  - $A \leq B$ and $B \leq A$ implies $A = B$

Updated Rule for Assignment

$$\frac{}{S \vdash e_1 = e_2 : ??}$$

Updated Rule for Assignment

$$\frac{S \vdash e_1 : T_1 \qquad S \vdash e_2 : T_2}{S \vdash e_1 = e_2 : ??}$$

Updated Rule for Assignment

$$\frac{S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad T_2 \leq T_1}{S \vdash e_1 = e_2 : ??}$$

Updated Rule for Assignment

$$\frac{\begin{array}{c} S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_2 \leq T_1 \end{array}}{S \vdash e_1 = e_2 : T_1}$$

Updated Rule for Assignment

$$\frac{S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad T_2 \leq T_1}{S \vdash e_1 = e_2 : \boxed{T_1}}$$

Can we do better than this?

Updated Rule for Assignment

$$\frac{S \vdash e_1 : T_1 \qquad S \vdash e_2 : T_2 \qquad T_2 \leq T_1}{S \vdash e_1 = e_2 : \mathbf{T_2}}$$

Updated Rule for Assignment

$$\frac{\begin{array}{c} S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_2 \leq T_1 \end{array}}{S \vdash e_1 = e_2 : \boxed{T_2}}$$

Not required in your semantic analyzer, but easy extra credit!

Updated Rule for Comparisons

Updated Rule for Comparisons

$$\frac{S \vdash e_1 : T \qquad S \vdash e_2 : T \qquad T \text{ is a primitive type}}{S \vdash e_1 \,\texttt{==}\, e_2 : \texttt{bool}}$$

Updated Rule for Comparisons

$$\frac{S \vdash e_1 : T \quad S \vdash e_2 : T \quad T \text{ is a primitive type}}{S \vdash e_1 == e_2 : \mathtt{bool}}$$

$$\frac{S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad T_1 \text{ and } T_2 \text{ are of class type.} \quad T_1 \leq T_2 \text{ or } T_2 \leq T_1}{S \vdash e_1 == e_2 : \mathtt{bool}}$$

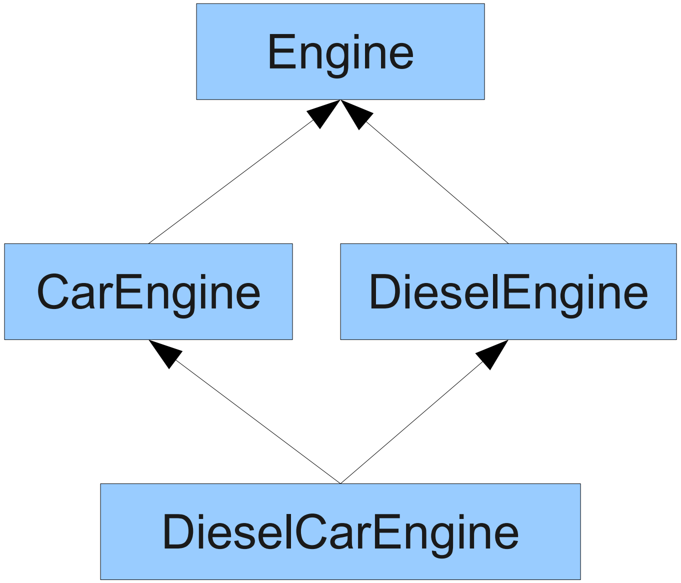
Updated Rule for Comparisons

Can we unify these rules?

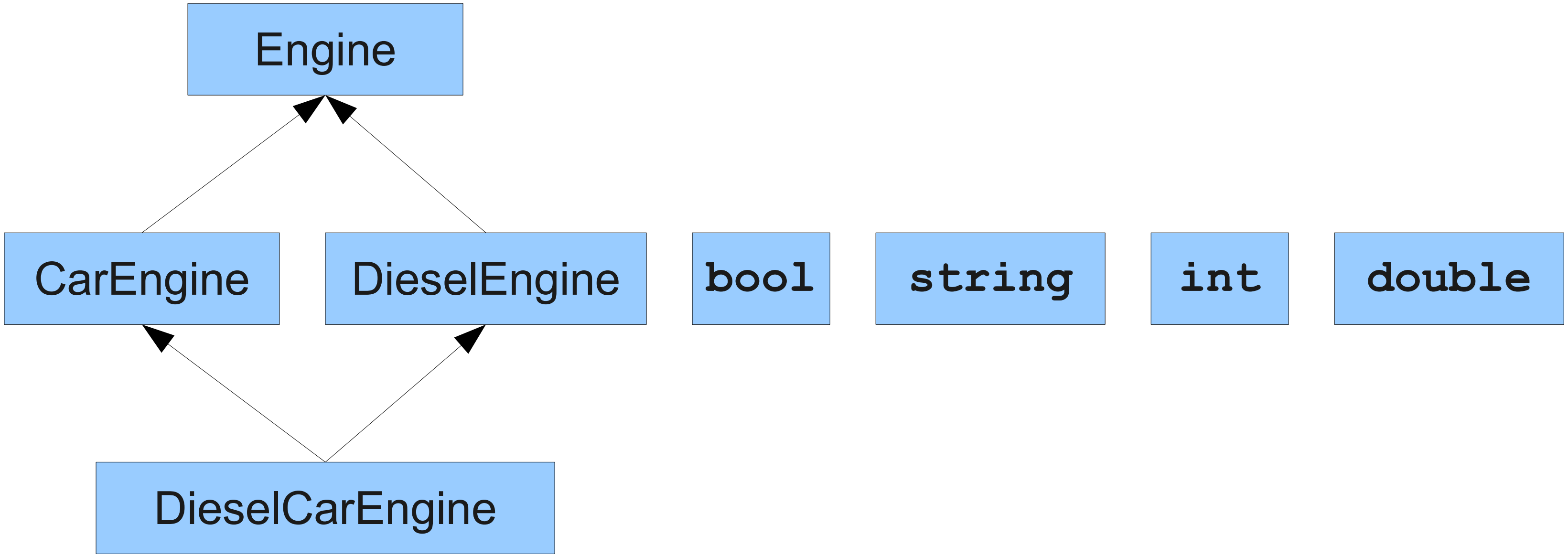
$$\frac{S \vdash e_1 : T \qquad S \vdash e_2 : T \qquad T \text{ is a primitive type}}{S \vdash e_1 \texttt{ == } e_2 : \textbf{bool}}$$

$$\frac{S \vdash e_1 : T_1 \qquad S \vdash e_2 : T_2 \qquad T_1 \text{ and } T_2 \text{ are of class type.} \qquad T_1 \leq T_2 \text{ or } T_2 \leq T_1}{S \vdash e_1 \texttt{ == } e_2 : \textbf{bool}}$$

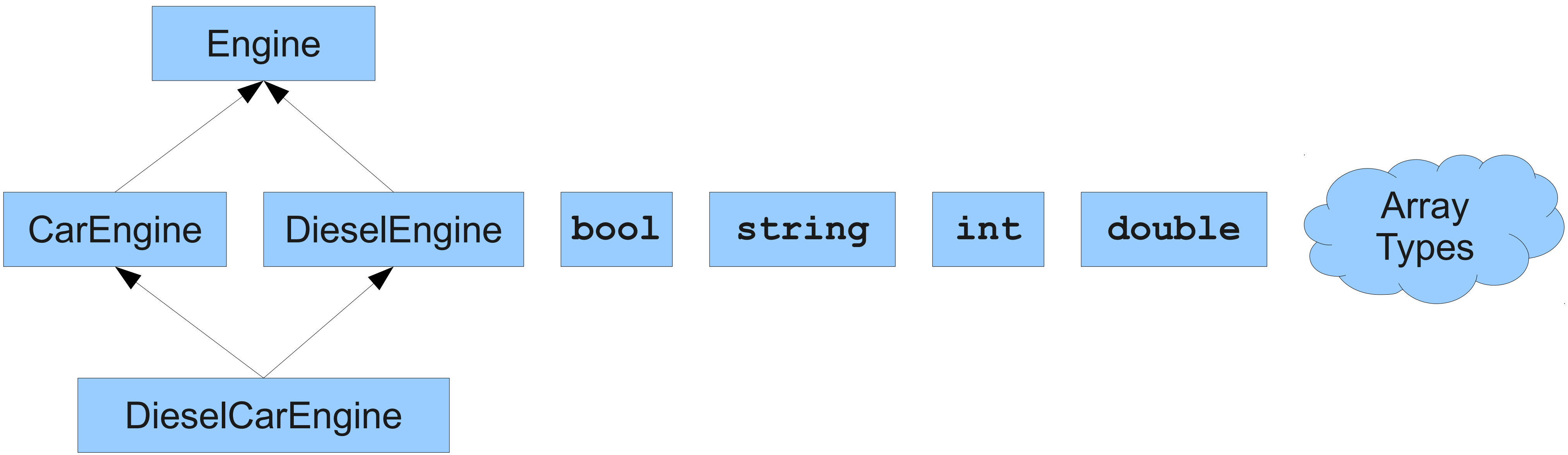
The Shape of Types



The Shape of Types



The Shape of Types



Extending Convertibility

- If A is a primitive or array type, A is only convertible to itself.
- More formally, if A and B are types and A is a primitive or array type:
  - $A \leq B$ implies $A = B$
  - $B \leq A$ implies $A = B$

Updated Rule for Comparisons

$$\frac{S \vdash e_1 : T \qquad S \vdash e_2 : T \qquad T \text{ is a primitive type}}{S \vdash e_1 == e_2 : \texttt{bool}}$$

$$\frac{S \vdash e_1 : T_1 \qquad S \vdash e_2 : T_2 \qquad T_1 \text{ and } T_2 \text{ are of class type.} \qquad T_1 \leq T_2 \text{ or } T_2 \leq T_1}{S \vdash e_1 == e_2 : \texttt{bool}}$$

Updated Rule for Comparisons

$$\frac{\begin{array}{c} S \vdash e_1 : T \\ S \vdash e_2 : T \\ T \text{ is a primitive type} \end{array}}{S \vdash e_1 \mathbin{\texttt{==}} e_2 : \texttt{bool}}$$

$$\frac{\begin{array}{c} S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \text{ and } T_2 \text{ are of class type.} \\ T_1 \leq T_2 \text{ or } T_2 \leq T_1 \end{array}}{S \vdash e_1 \mathbin{\texttt{==}} e_2 : \texttt{bool}}$$

$$\frac{\begin{array}{c} S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \leq T_2 \text{ or } T_2 \leq T_1 \end{array}}{S \vdash e_1 \mathbin{\texttt{==}} e_2 : \texttt{bool}}$$

Updated Rule for Comparisons

$$\frac{S \vdash e_1 : T \quad S \vdash e_2 : T \quad T \text{ is a primitive type}}{S \vdash e_1 == e_2 : \texttt{bool}}$$

$$\frac{S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad T_1 \text{ and } T_2 \text{ are of class type.} \quad T_1 \leq T_2 \text{ or } T_2 \leq T_1}{S \vdash e_1 == e_2 : \texttt{bool}}$$

$$\frac{S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad T_1 \leq T_2 \text{ or } T_2 \leq T_1}{S \vdash e_1 == e_2 : \texttt{bool}}$$

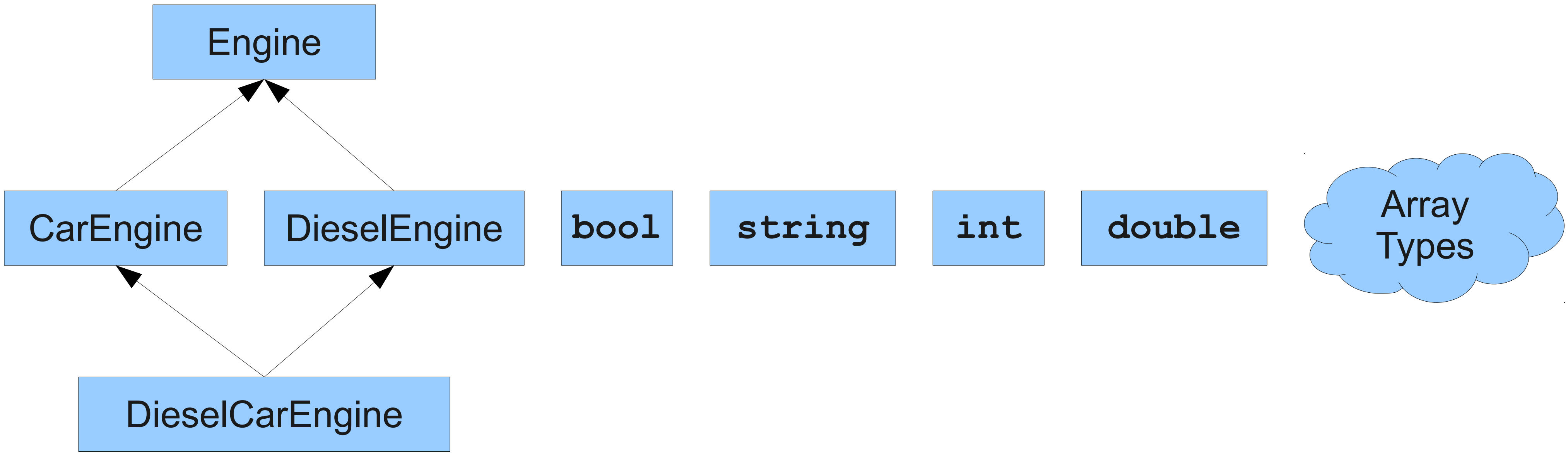
Updated Rule for Function Calls

$$\frac{\begin{array}{c} f \text{ is an identifier.} \\ f \text{ is a non-member function in scope S.} \\ f \text{ has type } (T_1, \ldots, T_n) \rightarrow U \\ S \vdash e_i : R_i \ \text{ for } 1 \le i \le n \\ R_i \le T_i \ \text{ for } 1 \le i \le n \end{array}}{S \vdash f(e_1, \ldots, e_n) \ : U}$$

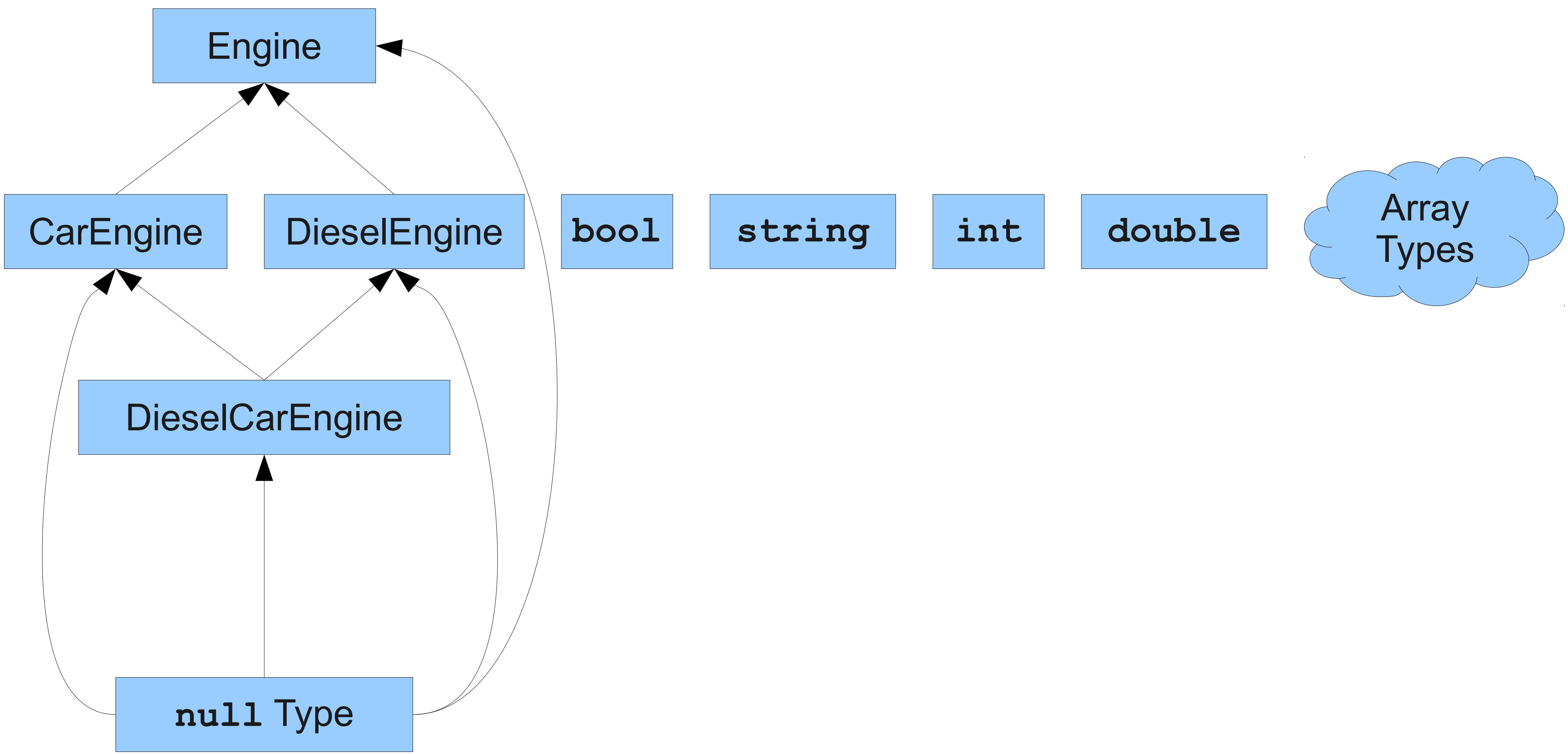
A Tricky Case

$$\frac{}{S \vdash \texttt{null} : ??}$$

Back to the Drawing Board



Back to the Drawing Board



Handling `null`

- Define a new type corresponding to the type of the literal **`null`**; call it "**`null`** type."
- Define **`null`** type ≤ A for any class type A.
- The **`null`** type is not accessible to programmers; it's only used internally inside the compiler.
- Many programming languages have types like these.

A Tricky Case

$$\frac{}{S \vdash \texttt{null} : ??}$$

A Tricky Case

$$\frac{}{S \vdash \texttt{null} : \texttt{null}\ \text{type}}$$

A Tricky Case

$$\frac{}{S \vdash \texttt{null} : \texttt{null}\ \text{type}}$$

Object-Oriented Considerations

$$\frac{\text{S is in scope of class T.}}{S \vdash \texttt{this} : T}$$

$$\frac{\text{T is a class type.}}{S \vdash \texttt{new}\ T : T}$$

$$\frac{S \vdash e : \texttt{int}}{S \vdash \texttt{NewArray(}e, T\texttt{)} : T\texttt{[]}}$$

Object-Oriented Considerations

$$\frac{\text{S is in scope of class T.}}{S \vdash \textbf{this} : T}$$

$$\frac{\text{T is a class type.}}{S \vdash \textbf{new}\ T : T}$$

$$\frac{S \vdash e : \textbf{int}}{S \vdash \textbf{NewArray}(e, T) : T\textbf{[]}}$$

Why don't we need to check if T is **void**?

What's Left?

- We're missing a few language constructs:
  - Member functions.
  - Field accesses.
  - Miscellaneous operators.
- Good practice to fill these in on your own.

Typing is Nuanced

- The **ternary conditional operator ? :** evaluates an expression, then produces one of two values.
- Works for primitive types:
  - `int x = random()? 137 : 42;`
- Works with inheritance:
  - `Base b = isB? new Base : new Derived;`
- What might the typing rules look like?

A Proposed Rule

$$\frac{}{S \vdash \mathit{cond}\ ?\ e_1 : e_2 : ??}$$

A Proposed Rule

$$\frac{S \vdash \textit{cond} : \texttt{bool}}{S \vdash \textit{cond}\ ?\ e_1 : e_2 : \,??}$$

A Proposed Rule

$$\frac{\begin{array}{c} S \vdash \mathit{cond} : \mathtt{bool} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \end{array}}{S \vdash \mathit{cond} \text{ ? } e_1 : e_2 : \text{??}}$$

A Proposed Rule

$$\frac{\begin{array}{c} S \vdash \textit{cond} : \mathtt{\textbf{bool}} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \leq T_2 \text{ or } T_2 \leq T_1 \end{array}}{S \vdash \textit{cond} \ ?\ e_1 : e_2 : \ ??}$$

A Proposed Rule

$$\frac{\begin{array}{c} S \vdash \textit{cond} : \mathtt{bool} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \leq T_2 \text{ or } T_2 \leq T_1 \end{array}}{S \vdash \textit{cond} \; ? \; e_1 : e_2 : \max(T_1, T_2)}$$

A Proposed Rule

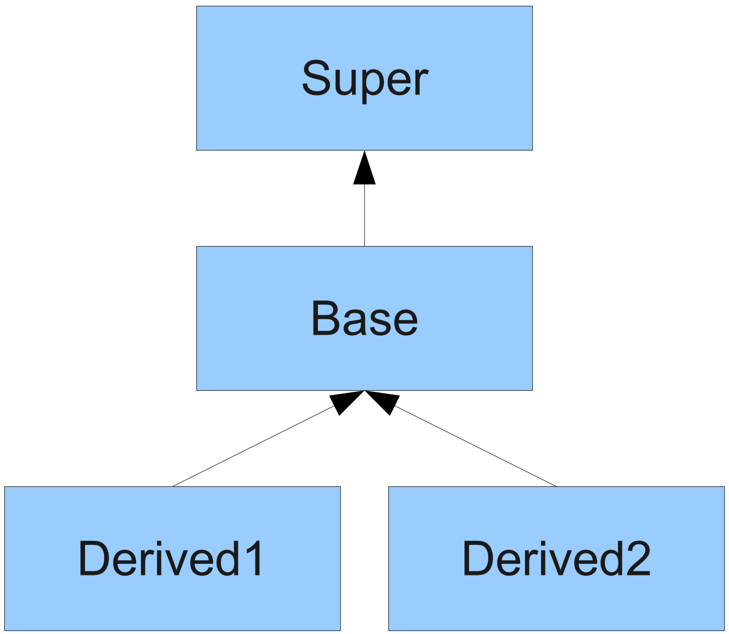
$$\frac{\begin{array}{c} S \vdash \mathit{cond} : \mathbf{\texttt{bool}} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \leq T_2 \text{ or } T_2 \leq T_1 \end{array}}{S \vdash \mathit{cond}\ ?\ e_1 : e_2 : \mathbf{max(T_1, T_2)}}$$

A Proposed Rule

$$\frac{\begin{array}{c} S \vdash \mathit{cond} : \mathtt{bool} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \leq T_2 \text{ or } T_2 \leq T_1 \end{array}}{S \vdash \mathit{cond} \mathbin{?} e_1 : e_2 : \boxed{\mathbf{max(T_1, T_2)}}}$$

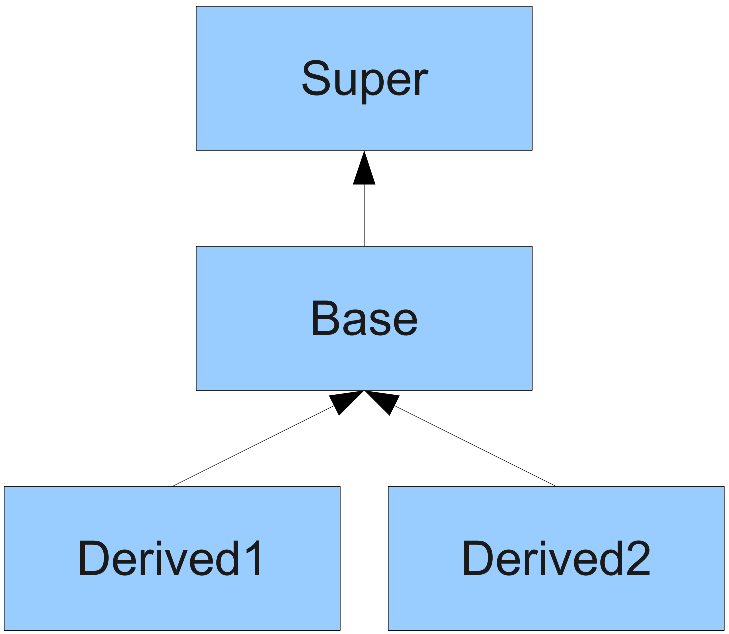
Is this **really** what we want?

A Small Problem



$$\frac{\begin{array}{c} S \vdash \mathit{cond} : \texttt{bool} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \leq T_2 \text{ or } T_2 \leq T_1 \end{array}}{S \vdash \mathit{cond}\ ?\ e_1 : e_2 : \max(T_1, T_2)}$$

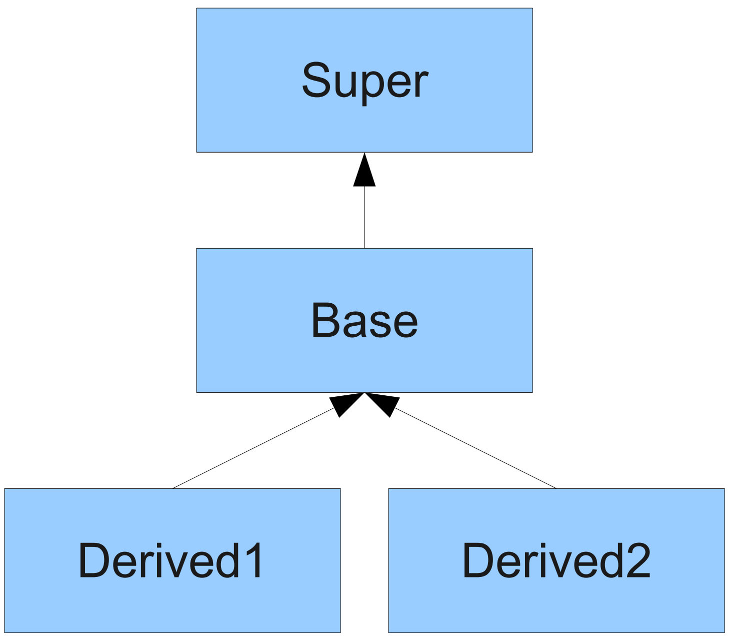
A Small Problem



$$\frac{\begin{array}{c} S \vdash \mathit{cond} : \texttt{bool} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \leq T_2 \text{ or } T_2 \leq T_1 \end{array}}{S \vdash \mathit{cond}\ ?\ e_1 : e_2 : \max(T_1, T_2)}$$

```
Base = random()?
            new Derived1 : new Derived2;
```

A Small Problem



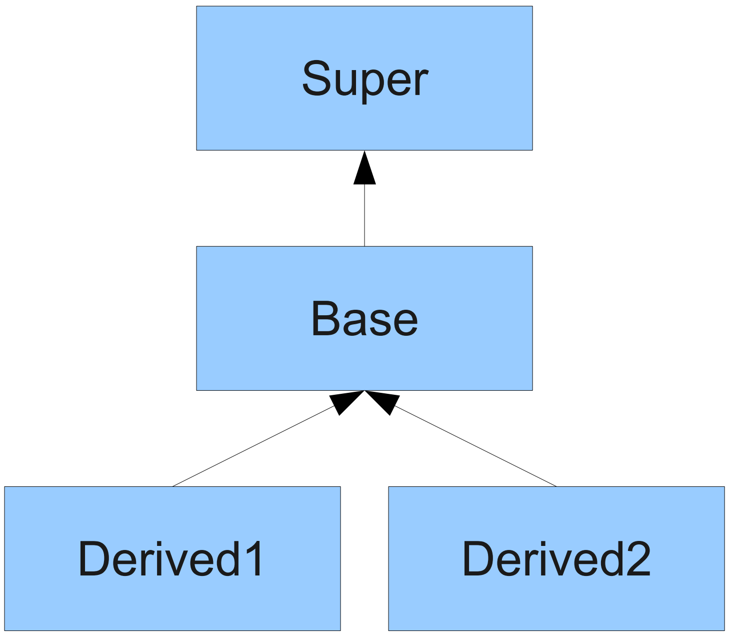
$$\frac{S \vdash \textit{cond} : \texttt{bool} \quad S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad \color{red}{\mathbf{T_1 \leq T_2 \text{ or } T_2 \leq T_1}}}{S \vdash \textit{cond} \text{ ? } e_1 : e_2 : \max(T_1, T_2)}$$

```
Base = random()?
            new Derived1 : new Derived2;
```

Least Upper Bounds

- An **upper bound** of two types A and B is a type C such that A ≤ C and B ≤ C.
- The **least upper bound** of two types A and B is a type C such that:
  - C is an upper bound of A and B.
  - If C' is an upper bound of A and B, C ≤ C'.
- When the least upper bound of A and B exists, we denote it A ⊔ B.
  - (When might it not exist?)

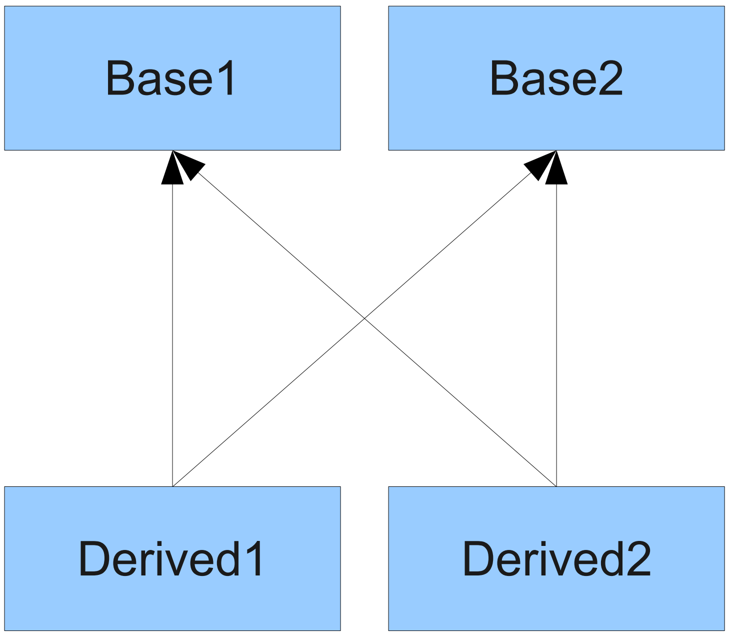
A Better Rule



$$\frac{\begin{array}{c} S \vdash \mathit{cond} : \texttt{bool} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T = T_1 \sqcup T_2 \end{array}}{S \vdash \mathit{cond} \; ? \; e_1 : e_2 : T}$$

```
Base = random()?
            new Derived1 : new Derived2;
```

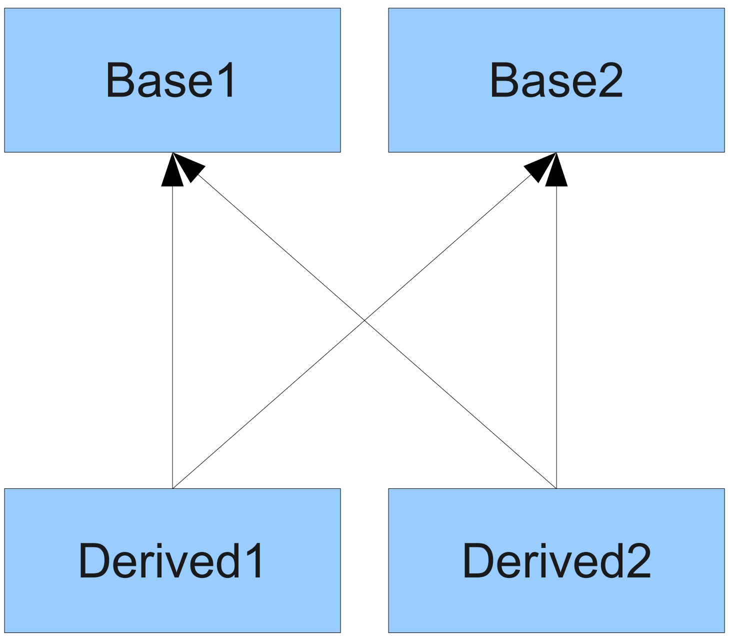
... that still has problems



$$\frac{S \vdash \mathit{cond} : \mathtt{\textbf{bool}} \quad S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad T = T_1 \sqcup T_2}{S \vdash \mathit{cond}\ ?\ e_1 : e_2 : T}$$

```
Base1 = random()?
            new Derived1 : new Derived2;
```

… that still has problems



$$\frac{\begin{array}{c} S \vdash \mathit{cond} : \mathtt{bool} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ \color{red}{\mathbf{T = T_1 \sqcup T_2}} \end{array}}{S \vdash \mathit{cond}\ ?\ e_1 : e_2 : T}$$

```
Base1 = random()?
              new Derived1 : new Derived2;
```

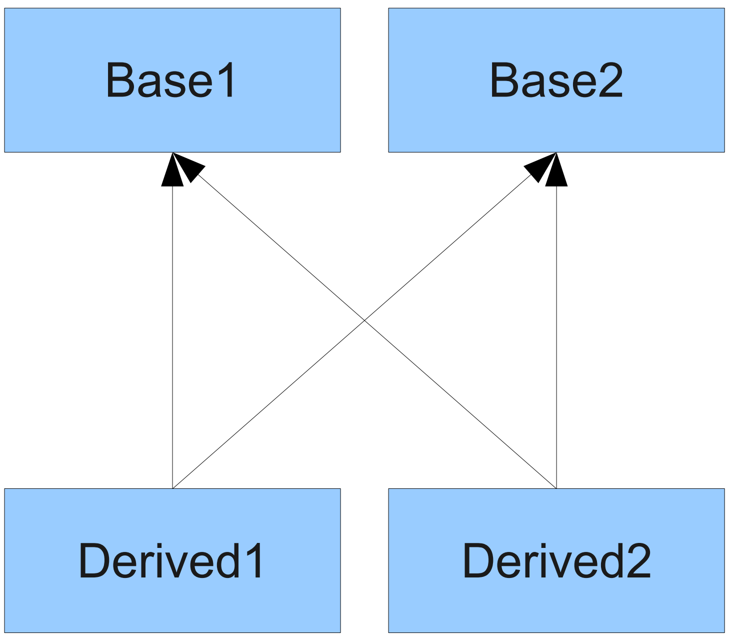
Multiple Inheritance is Messy

- Type hierarchy is no longer a tree.
- Two classes might not have a least upper bound.
- Occurs in Java due to interfaces.
- Not a problem in Decaf; there is no ternary conditional operator.
- How to fix?

Minimal Upper Bounds

- An **upper bound** of two types A and B is a type C such that $A \leq C$ and $B \leq C$.
- A **minimal upper bound** of two types A and B is a type C such that:
  - C is an upper bound of A and B.
  - If C' is an upper bound of C, then it is not true that C' < C.
- Minimal upper bounds are not necessarily unique.
- A least upper bound must be a minimal upper bound, but not the other way around.

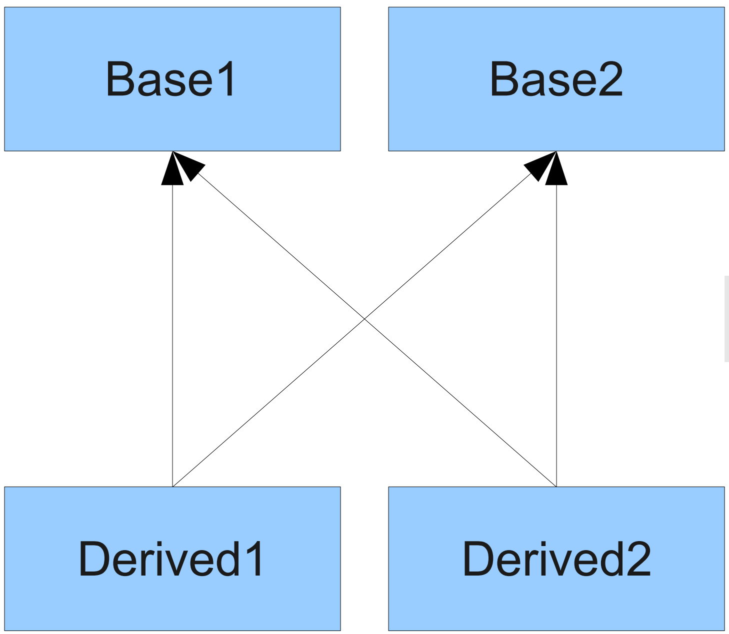
A Correct Rule



$$\frac{\begin{array}{c} S \vdash \textit{cond} : \texttt{bool} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T \text{ is a minimal upper bound of } T_1 \text{ and } T_2 \end{array}}{S \vdash \textit{cond} \; ? \; e_1 : e_2 : T}$$

```
Base1 = random()?
             new Derived1 : new Derived2;
```

A Correct Rule



$$\frac{\begin{array}{c} S \vdash \textit{cond} : \texttt{bool} \\ S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T \text{ is a minimal upper bound of } T_1 \text{ and } T_2 \end{array}}{S \vdash \textit{cond}\ ?\ e_1 : e_2 : T}$$

Can prove both that expression has type **`Base1`** and that expression has type **`Base2`**.

```
Base1 = random()?
            new Derived1 : new Derived2;
```

So What?

- **Type-checking can be tricky**.
- Strongly influenced by the choice of operators in the language.
- Strongly influenced by the legal type conversions in a language.
- In C++, the previous example **doesn't compile**.
- In Java, the previous example does compile, but the language spec is **enormously** complicated.
  - See §15.12.2.7 of the Java Language Specification.