Pushdown Automata

Announcements

- Problem Set 6 due Friday at 2:15PM.
  - Stop by OH with questions!
  - Email **cs103@cs.stanford.edu** with questions!

Generation vs. Recognition

- We have seen two general approaches to describing regular languages:
  - Build **automata** that accept precisely the strings in the language.
  - Design **regular expressions** that describe precisely the strings in the language.
- Regular expressions **generate** all of the strings in the language.
- Finite automata **recognize** all of the strings in the language.

Generation vs. Recognition

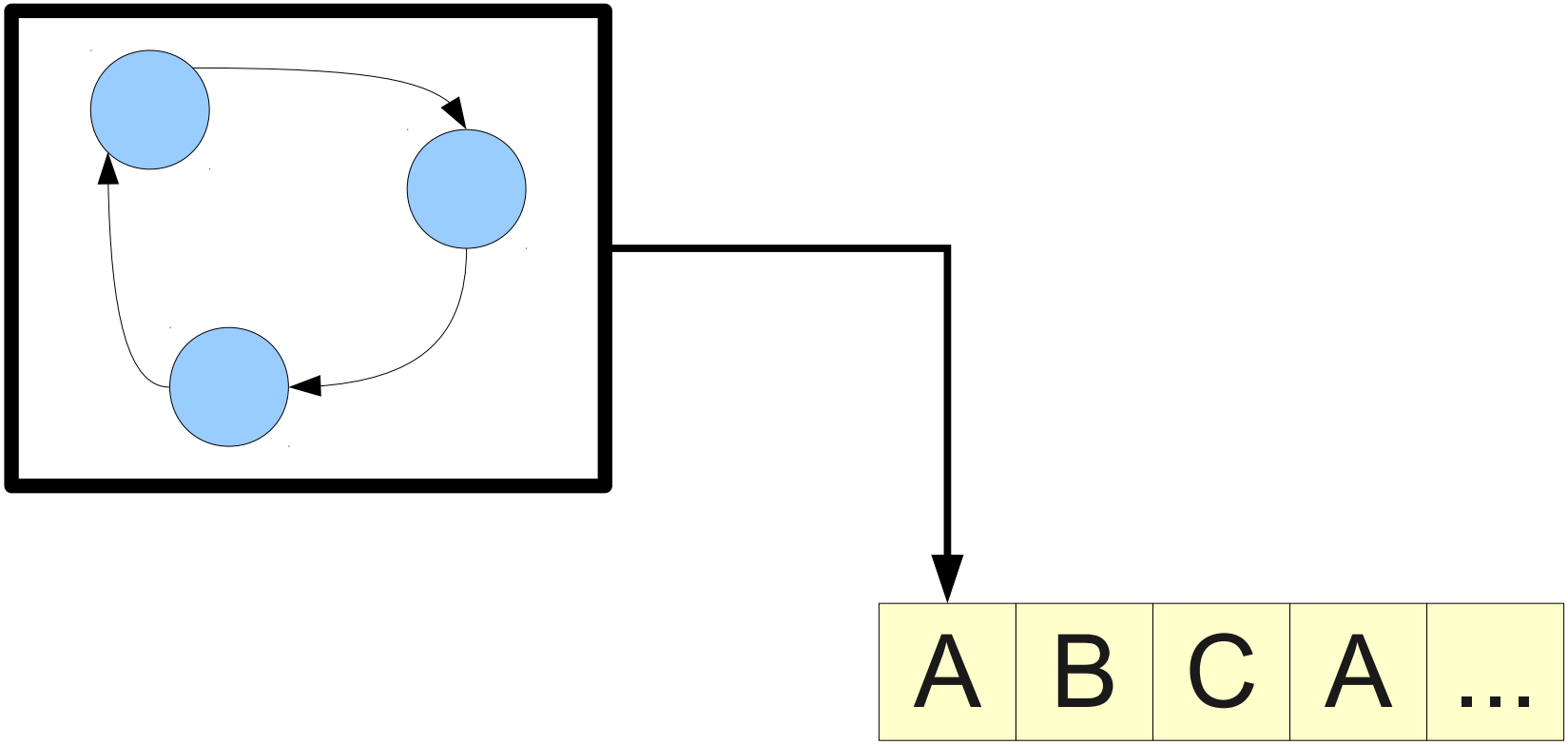
- Generation is useful
  - to list off the strings in the language.
  - to describe how those strings are constructed.
- Recognition is useful
  - for testing whether some particular string is in the language.
  - for reconstructing how those strings were assembled.

Context-Free Languages

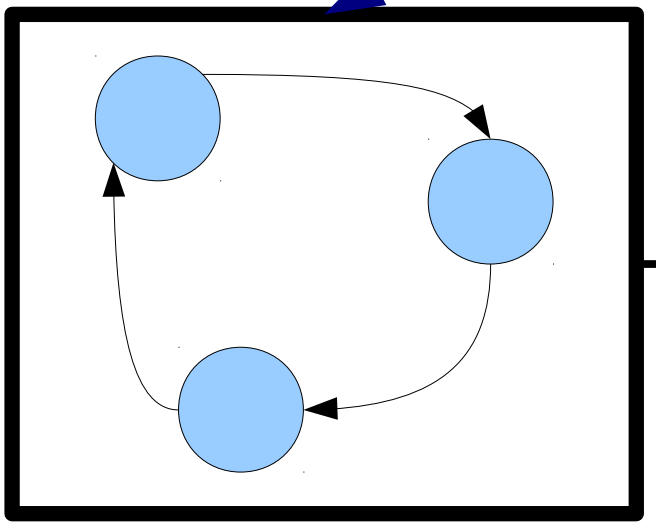
- Yesterday, we saw the **context-free languages**, which are those that can be generated by **context-free grammars**.
- Is there some way to build an automaton that can **recognize** the context-free languages?

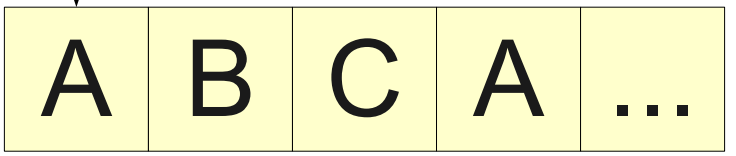
The Problem

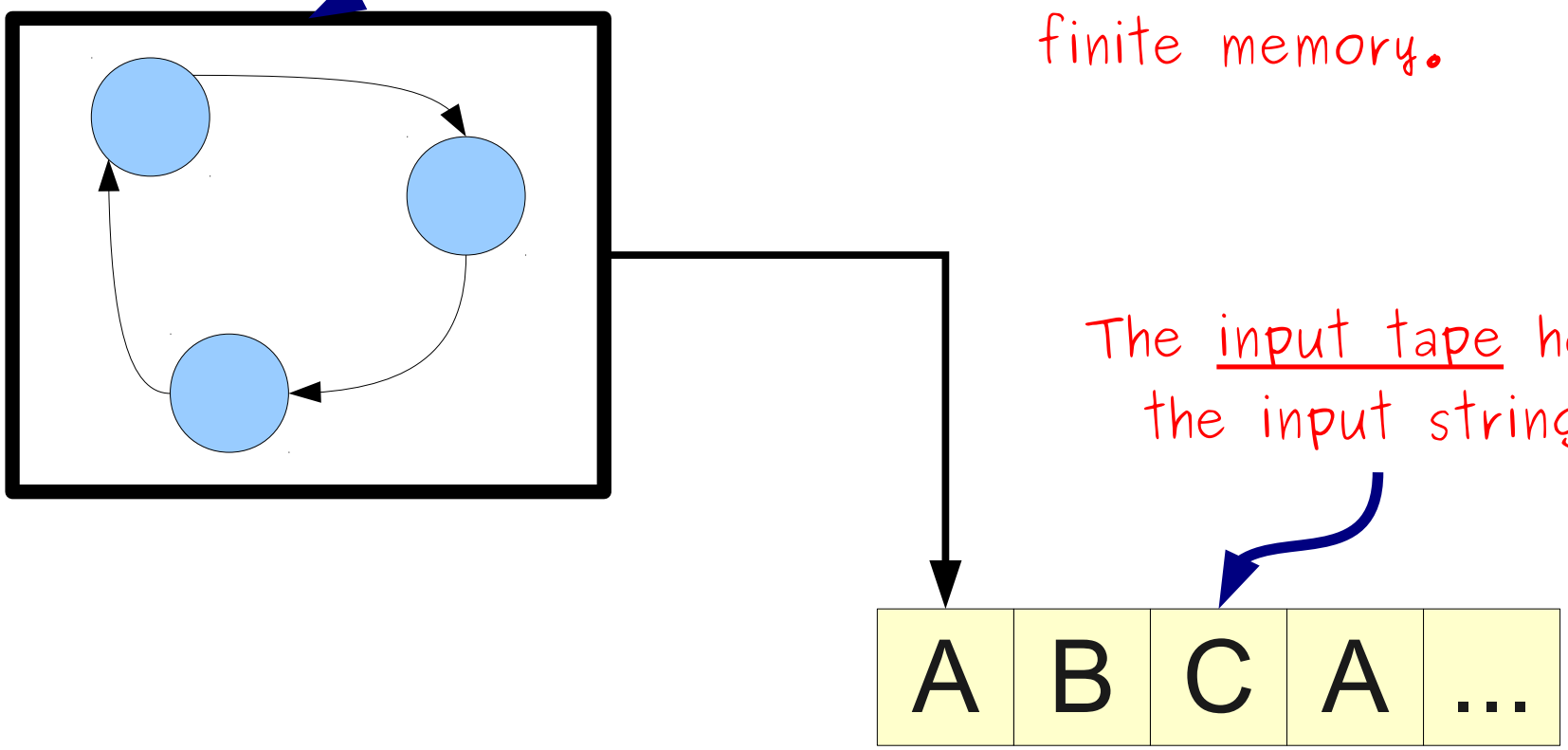
- Finite automata accept precisely the regular languages.
- We may need unbounded memory to recognize context-free languages.
  - e.g. $\{ 0^n1^n \mid n \in \mathbb{N} \}$ requires unbounded counting.
- How do we build an automaton with finitely many states but unbounded memory?


A
B
C
A
...

The finite-state control acts as a finite memory.



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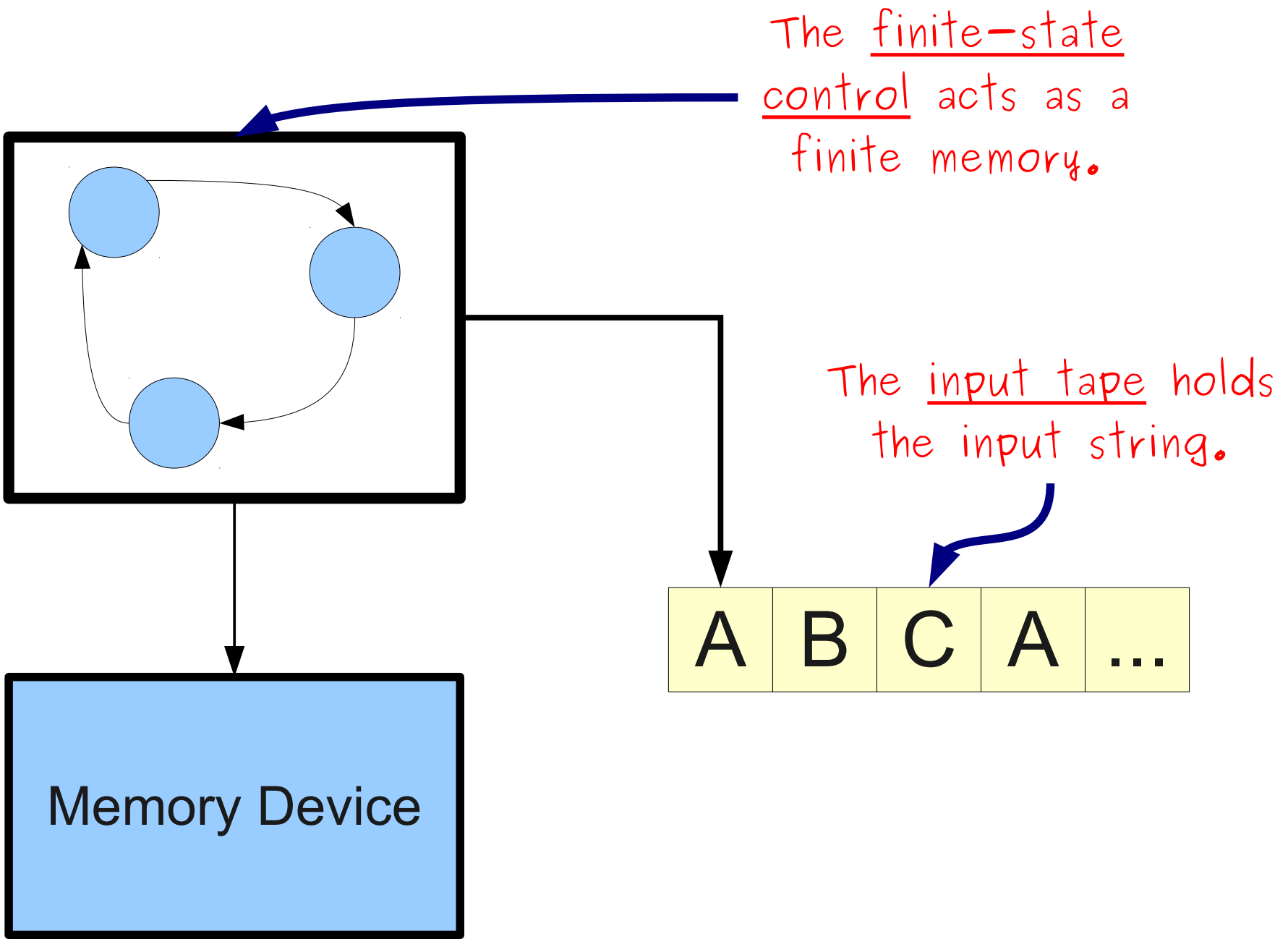
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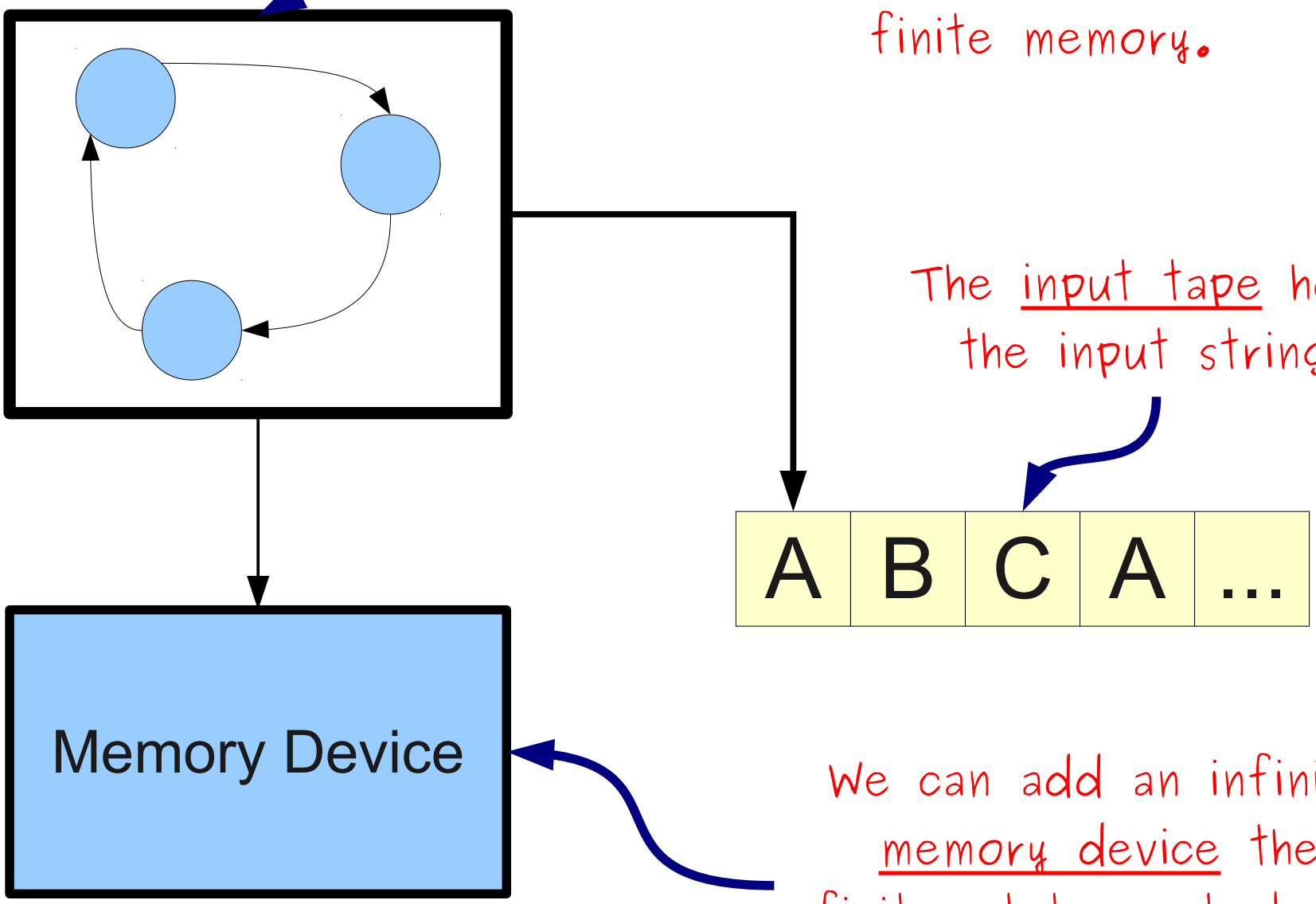
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Memory Device
We can add an infinite memory device the finite-state control can use to store information.

Adding Memory to Automata

- We can augment a finite automaton by adding in a **memory device** for the automaton to store extra information.
- The finite automaton now can base its transition on both the current symbol being read and values stored in memory.
- The finite automaton can issue commands to the memory device whenever it makes a transition.
  - e.g. add new data, change existing data, etc.

Stack-Based Memory

- There are **many** types of memory that we might give to an automaton.
  - We'll see at least two this quarter.
- One of the simplest types of memory is a **stack**.

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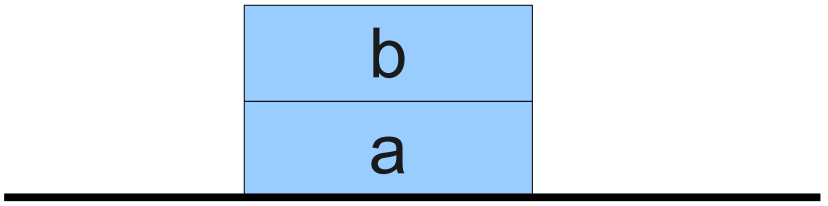

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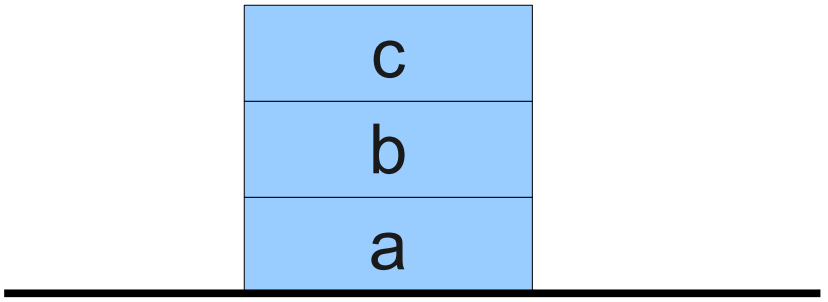

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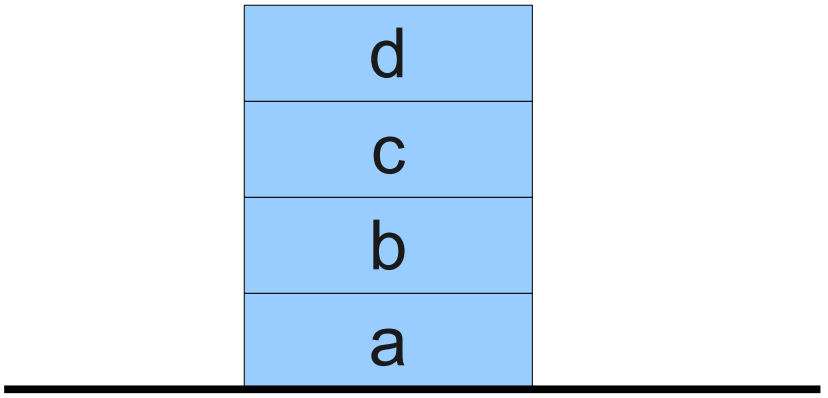

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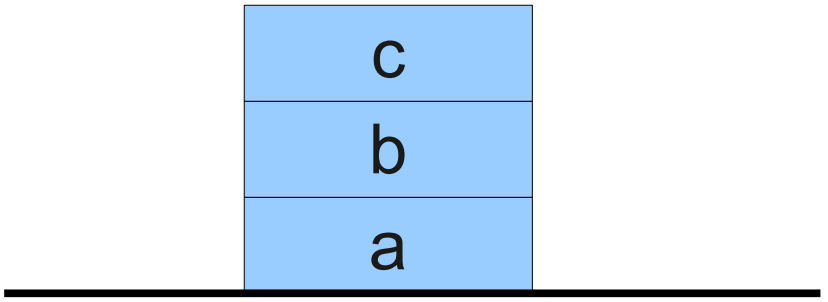

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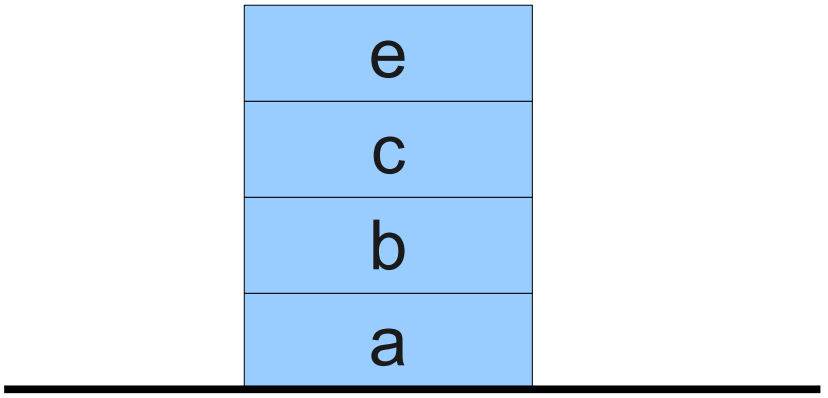

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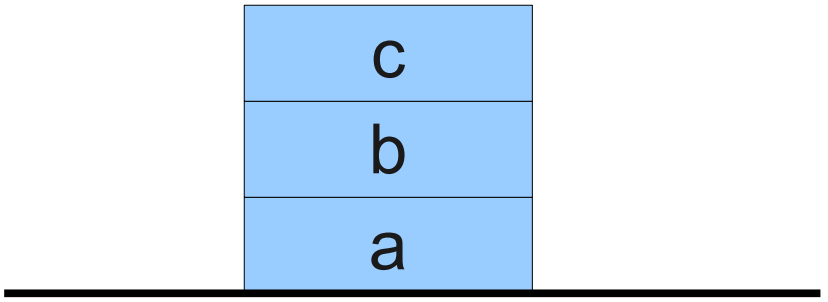

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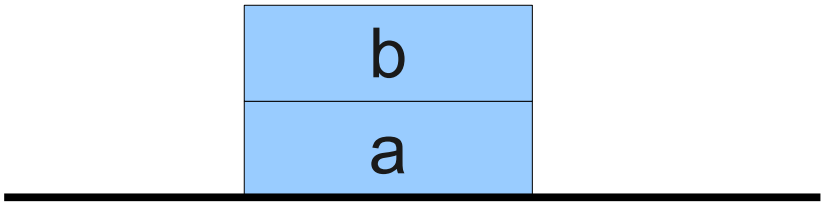

Stack-Based Memory

- Only the top of the stack is visible at any point in time.
- New symbols may be **pushed** onto the stack, which cover up the old stack top.
- The top symbol of the stack may be **popped**, exposing the symbol below it.

Pushdown Automata

- A **pushdown automaton** (PDA) is a finite automaton equipped with a stack-based memory.
- Each transition
  - is based on the current input symbol and the top of the stack,
  - optionally pops the top of the stack, and
  - optionally pushes new symbols onto the stack.
- Initially, the stack holds a special symbol Z_0 that indicates the bottom of the stack.

A Note on Terminology

- Finite automata are highly standardized.
- There are many equivalent but different definitions of PDAs.
- The one we will use is a slight variant on the one described in Sipser.
  - Sipser does not have a start stack symbol.
  - Sipser does not allow transitions to push multiple symbols onto the stack.
- Feel free to use either this version or Sipser's; the two are equivalent to one another.

Our First PDA

- Consider the language
  L = { w | w is a string of balanced parentheses }
  over Σ = { (,) }
- We can exploit the stack to our advantage:
  - Whenever we see a (, push it onto the stack.
  - Whenever we see a), pop the corresponding (from the stack (or fail if not matched)
  - When input is consumed, if the stack is empty, accept.

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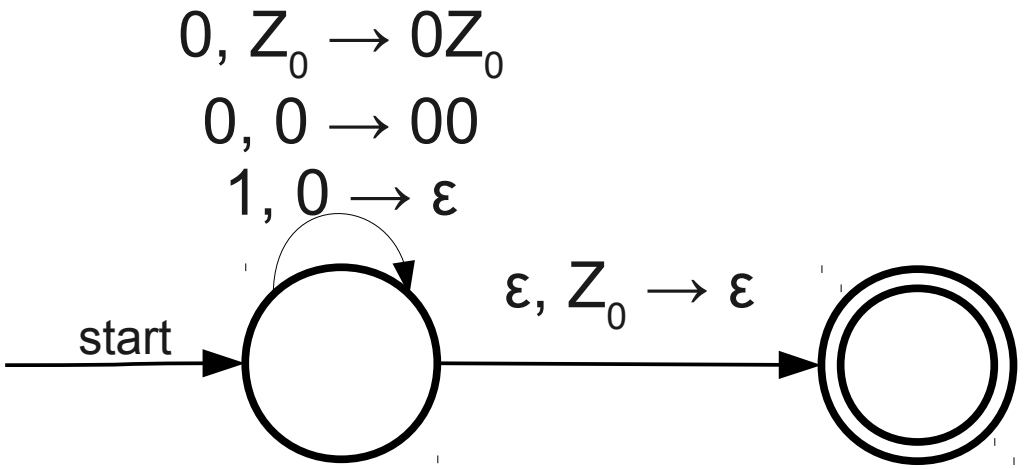
Our First PDA

- Consider the language
  L = { w | w is a string of balanced **digits** }
  over Σ = { **0**, **1** }
- We can exploit the stack to our advantage:
  - Whenever we see a **0**, push it onto the stack.
  - Whenever we see a **1**, pop the corresponding **0** from the stack (or fail if not matched)
  - When input is consumed, if the stack is empty, accept.

A Simple Pushdown Automaton



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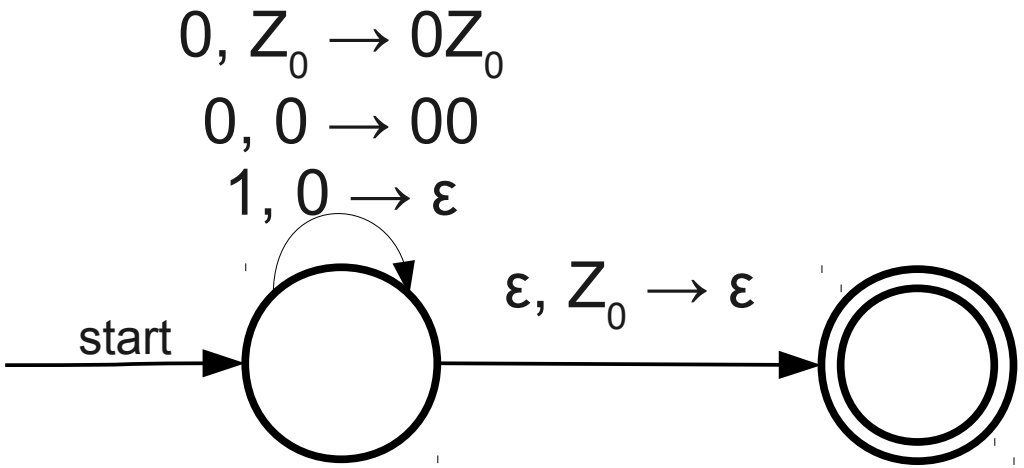

A transition of the form

$$a, b \rightarrow z$$

Means "If the current input symbol is a and the current stack symbol is b, then follow this transition, pop b, and push the string z.

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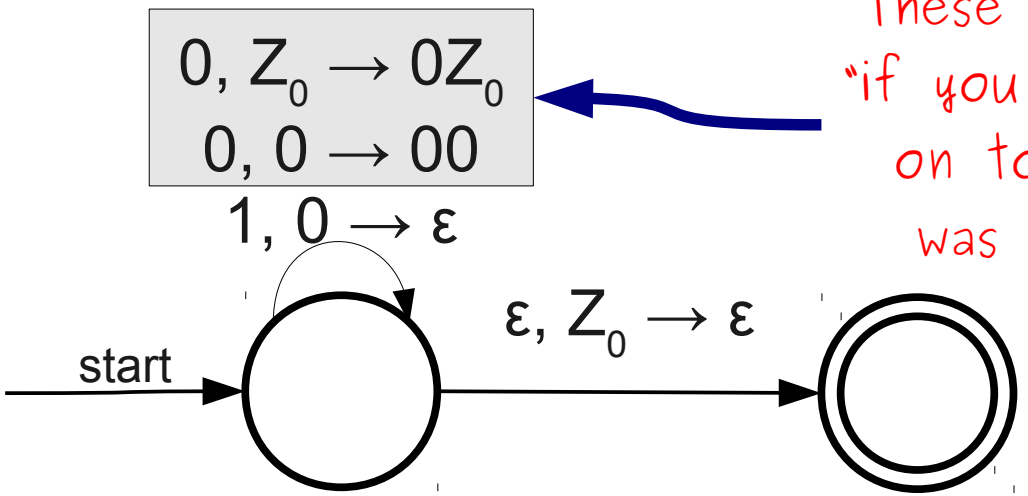

These transitions mean "if you see 0, push a 0 on top of whatever was already there"

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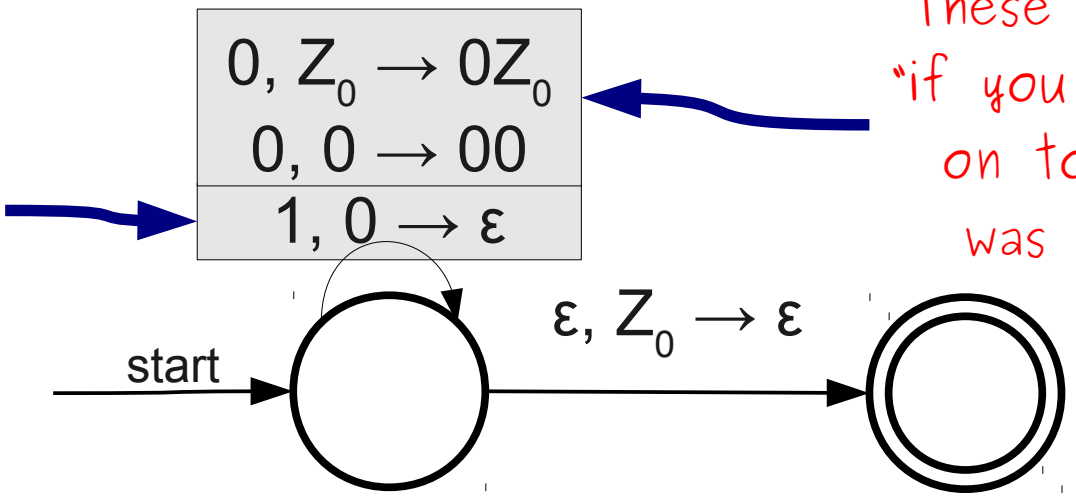
This one says "if you read a 1 and the stack top is 0, pop the stack."



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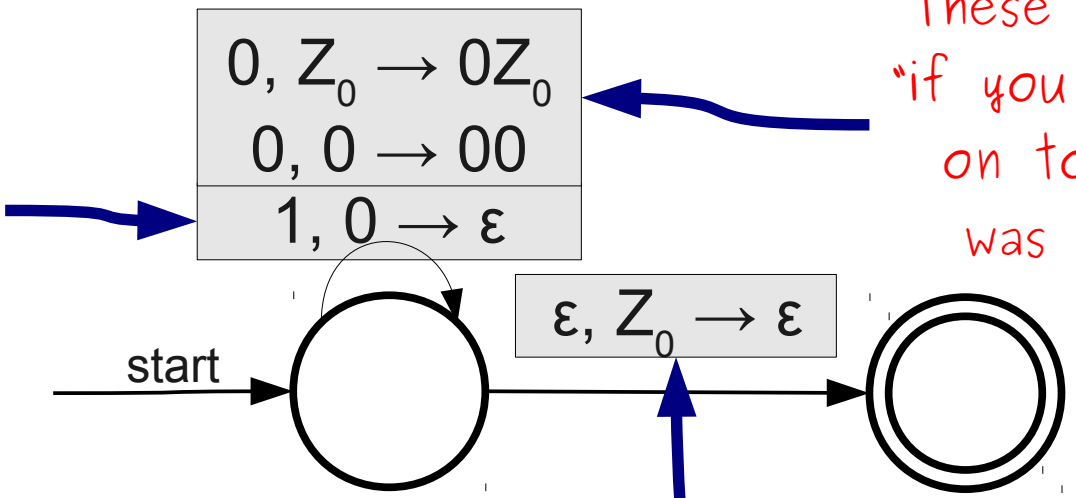

These transitions mean "if you see 0, push a 0 on top of whatever was already there"

A transition of the form

$$a, b \rightarrow z$$

This one says "at any time when the stack has no zeros on it, you can nondeterministically guess whether to transition into the accept state.

Means "If the current input symbol is a and the current stack symbol is b, then follow this transition, pop b, and push the string z.

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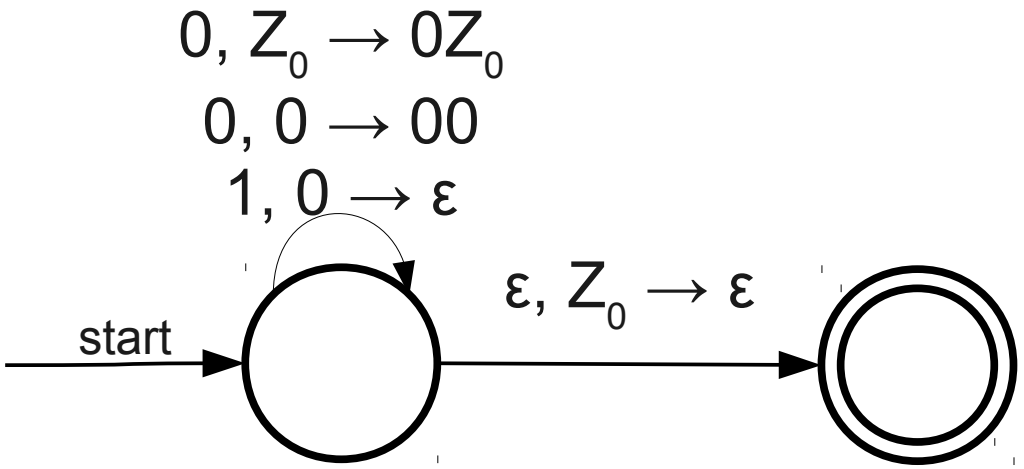

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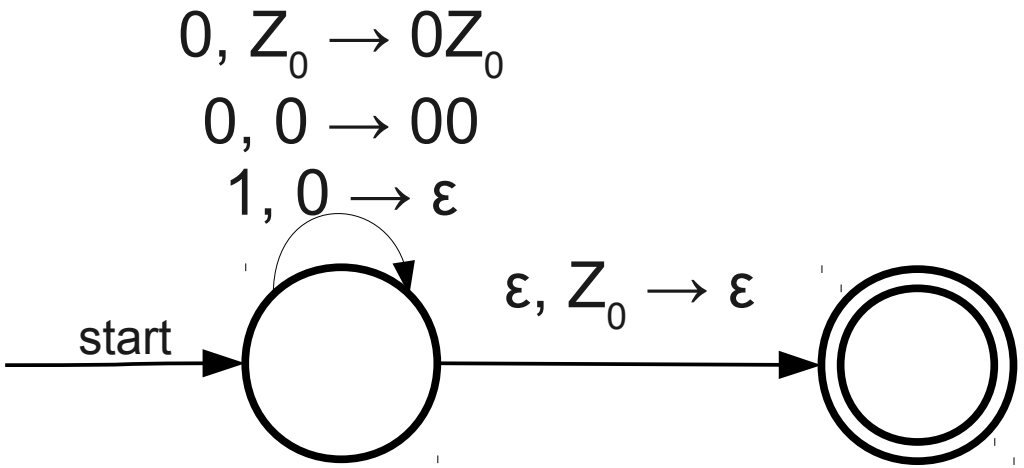

0 0 0 1 1 1

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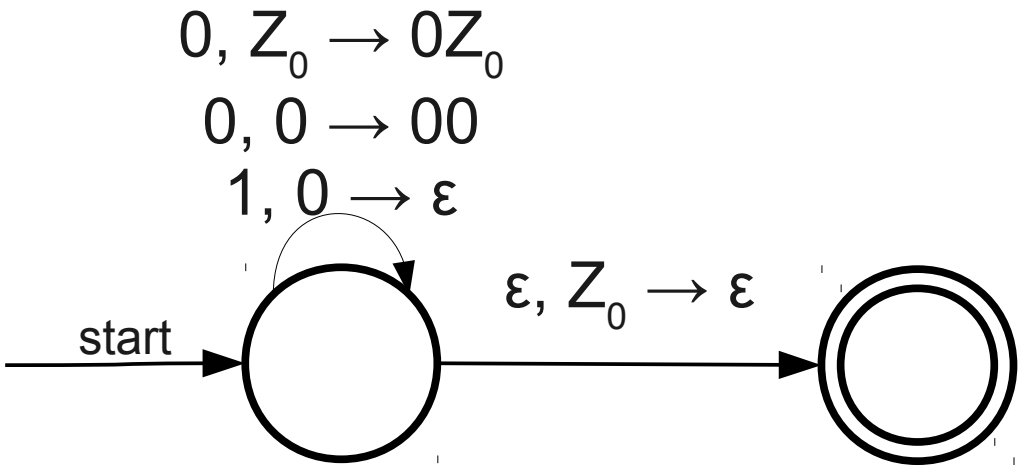

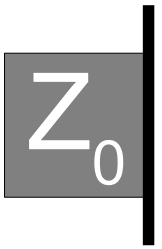

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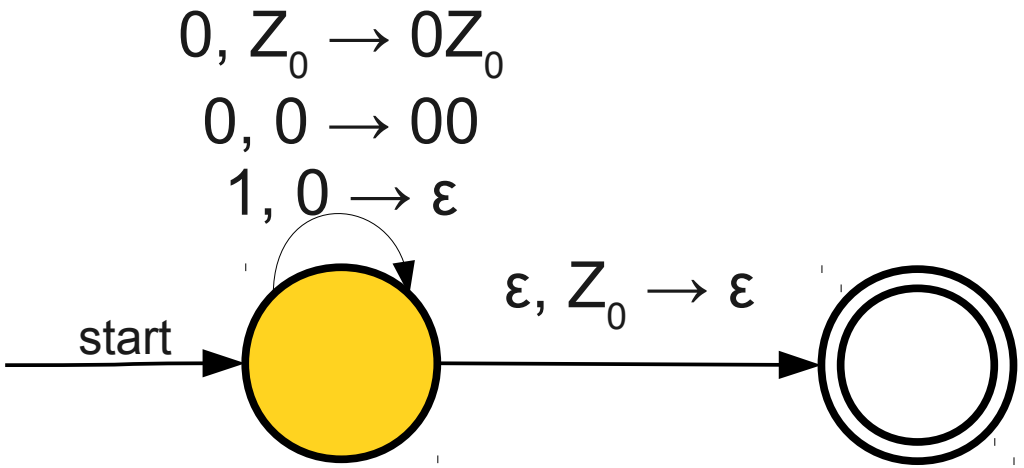

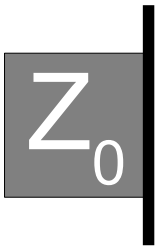

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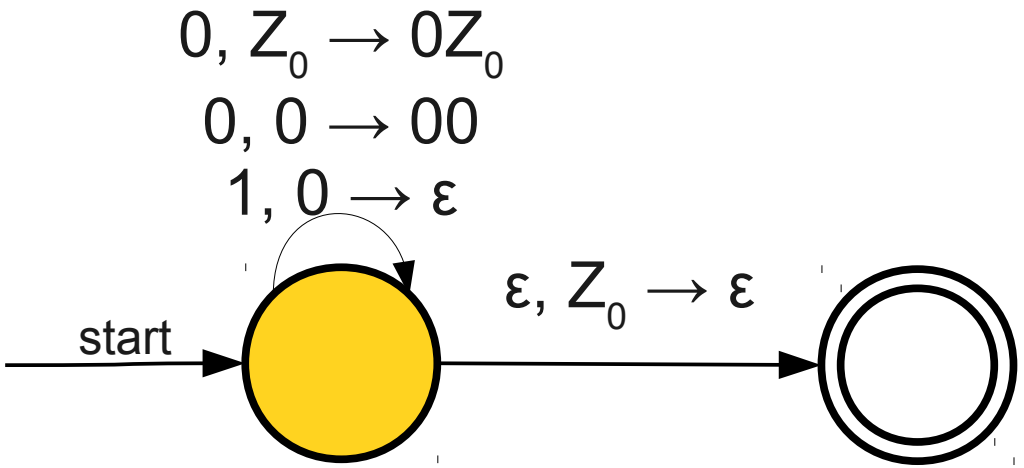

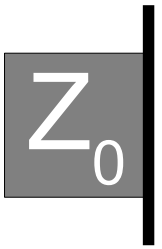

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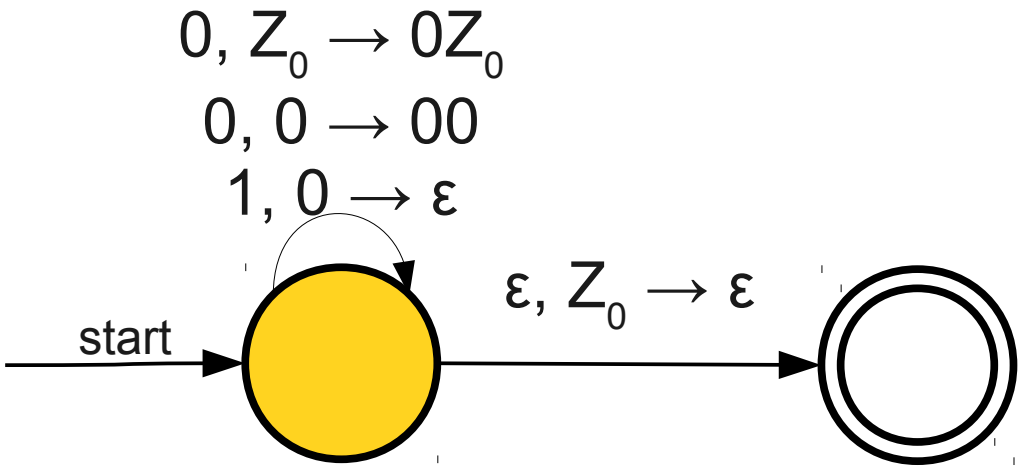

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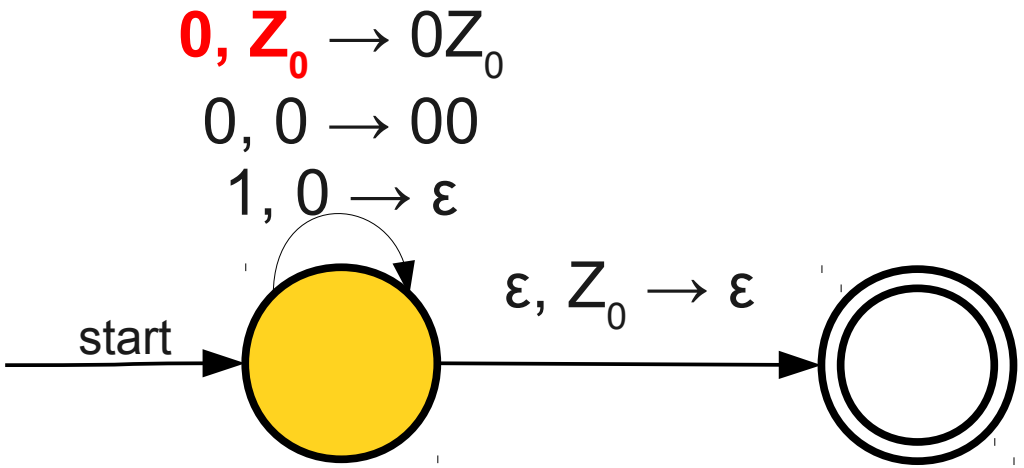

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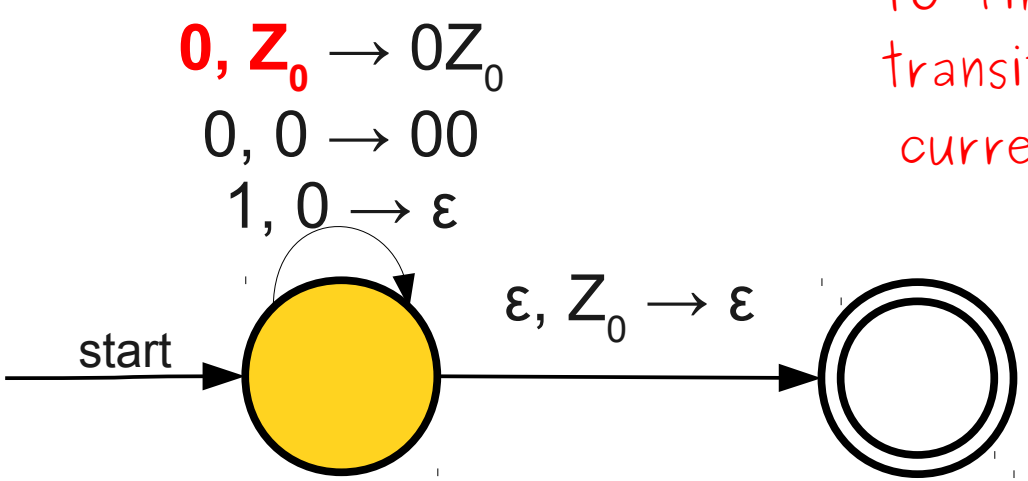

To find an applicable transition, match the current input/stack pair.

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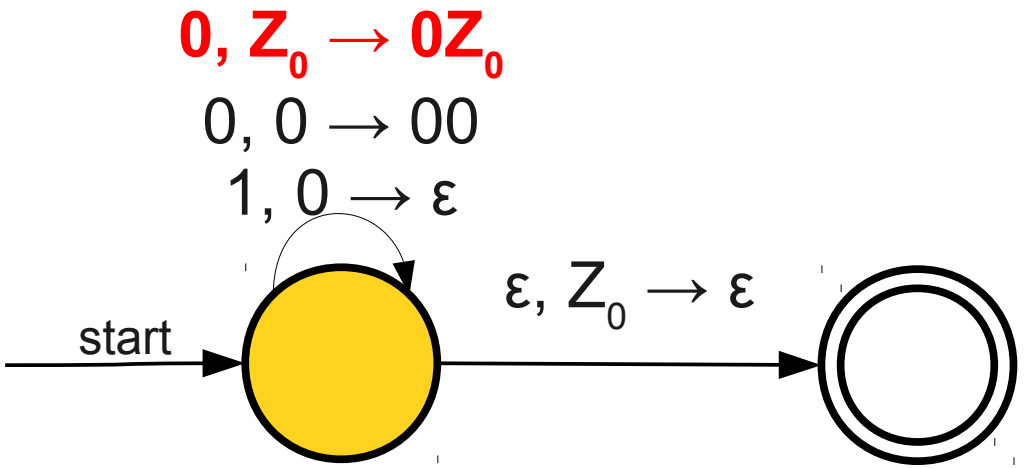

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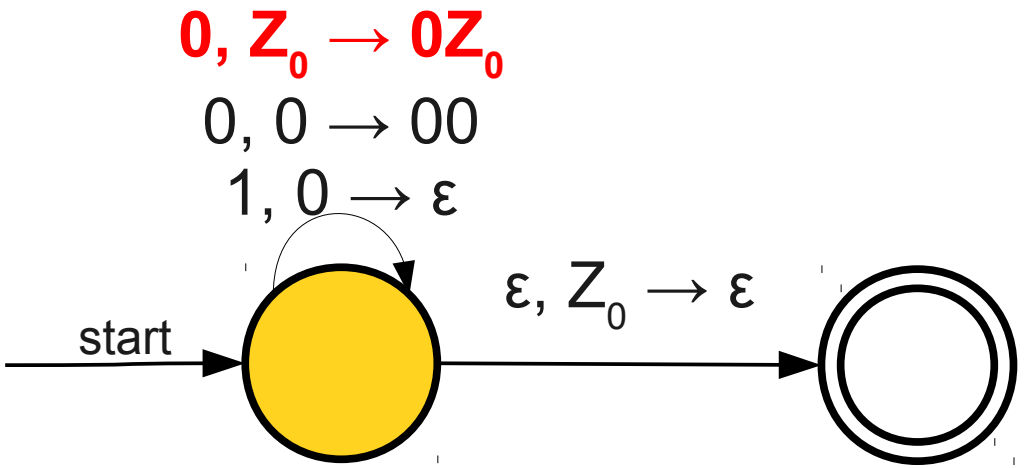

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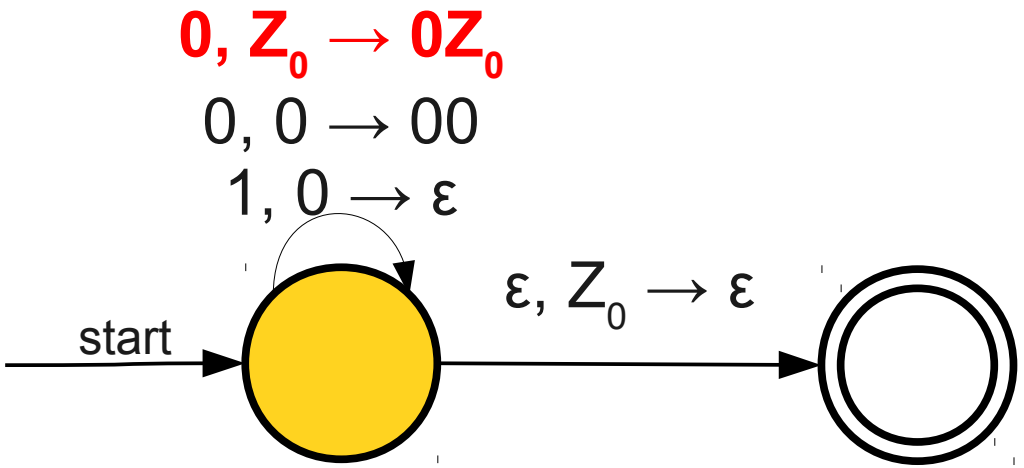

If a transition reads the top symbol of the stack, it always pops that symbol (though it might replace it)

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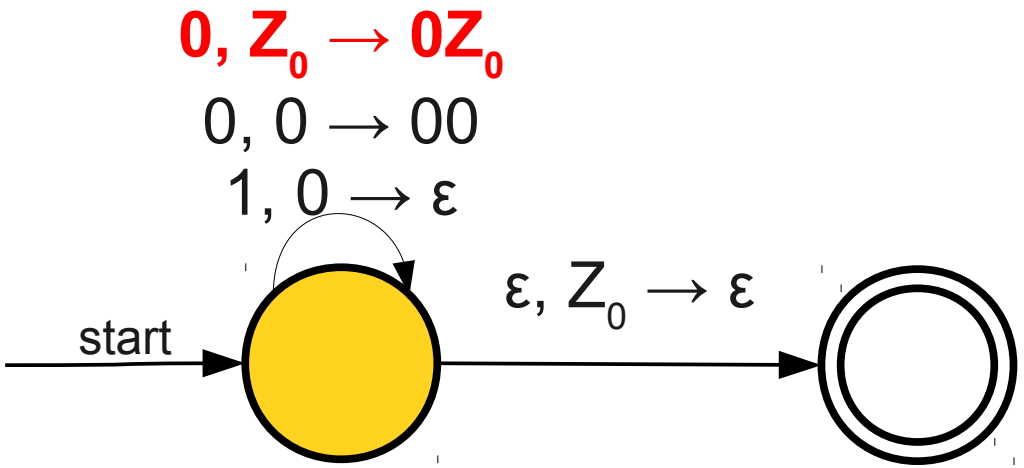

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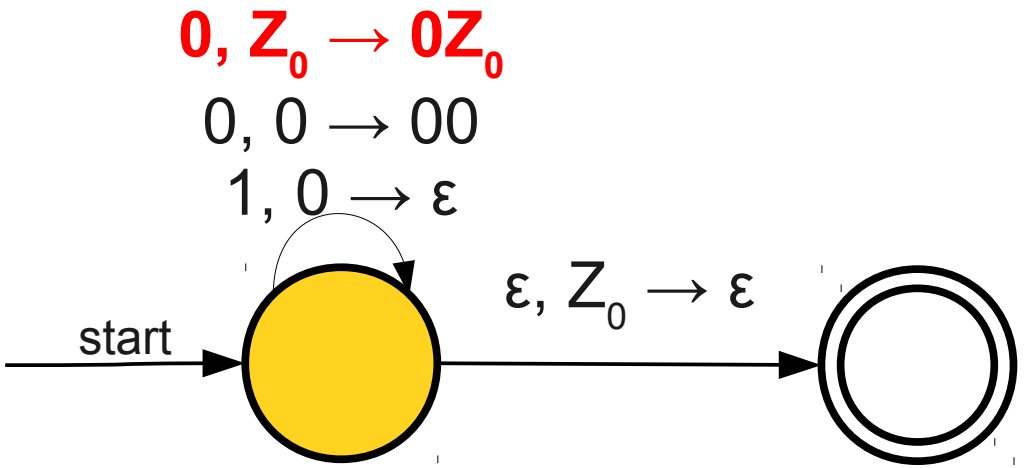

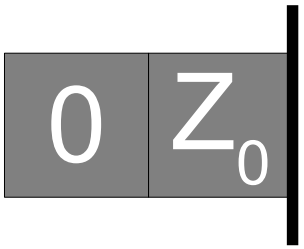

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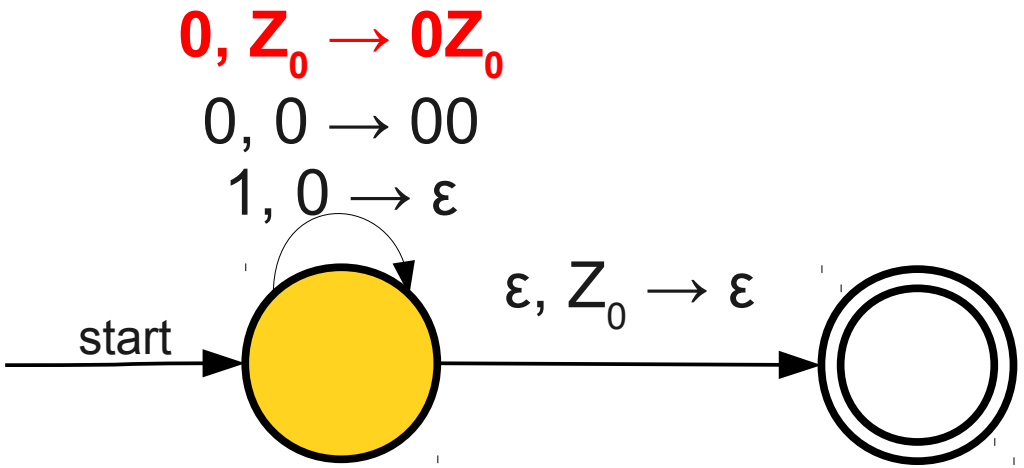
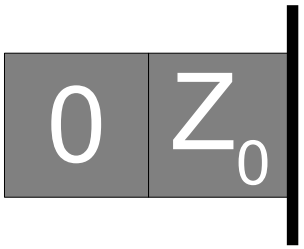


Each transition then pushes some (possibly empty) string back onto the stack. Notice that the leftmost symbol is pushed onto the top.



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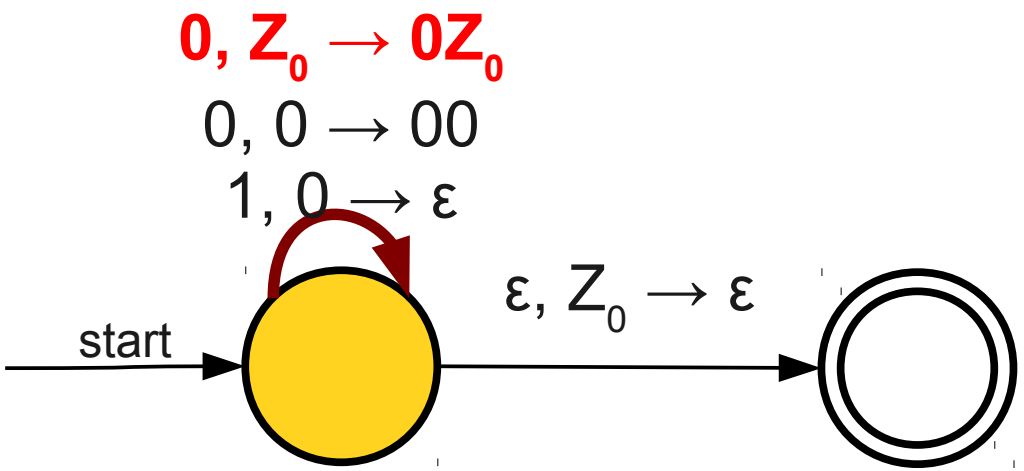

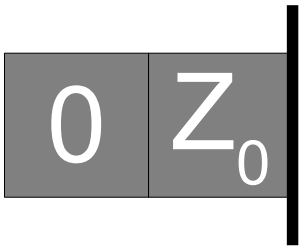

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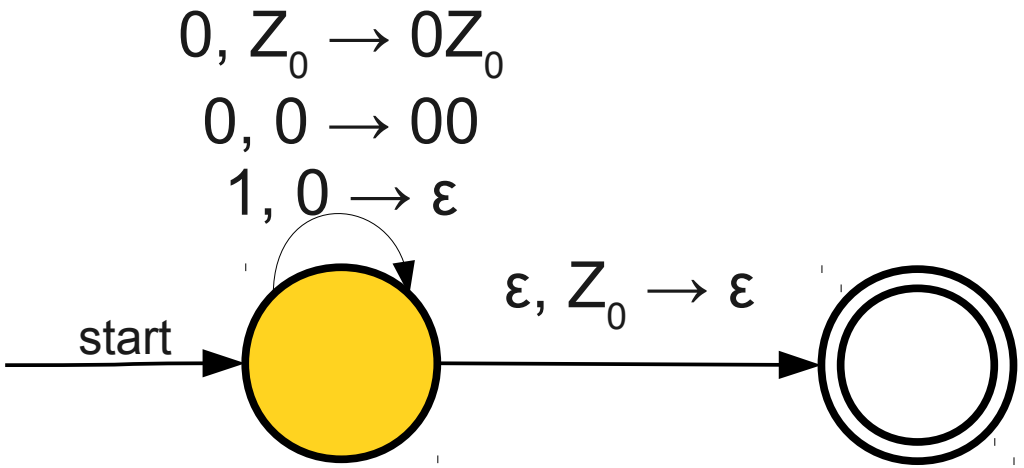

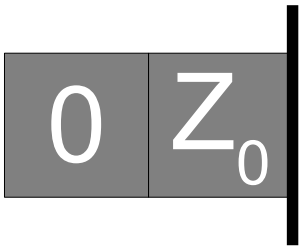

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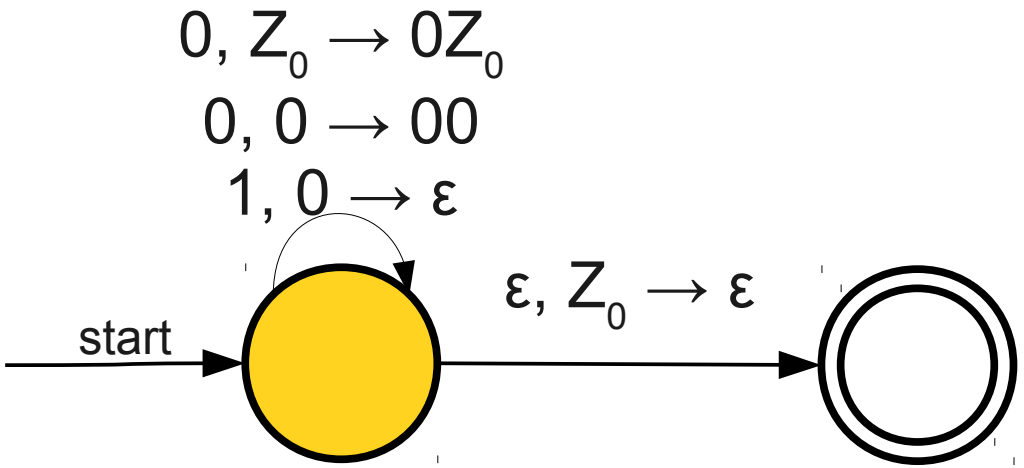

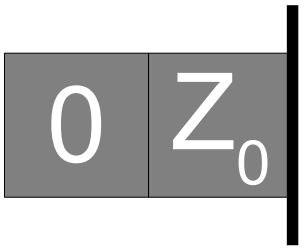

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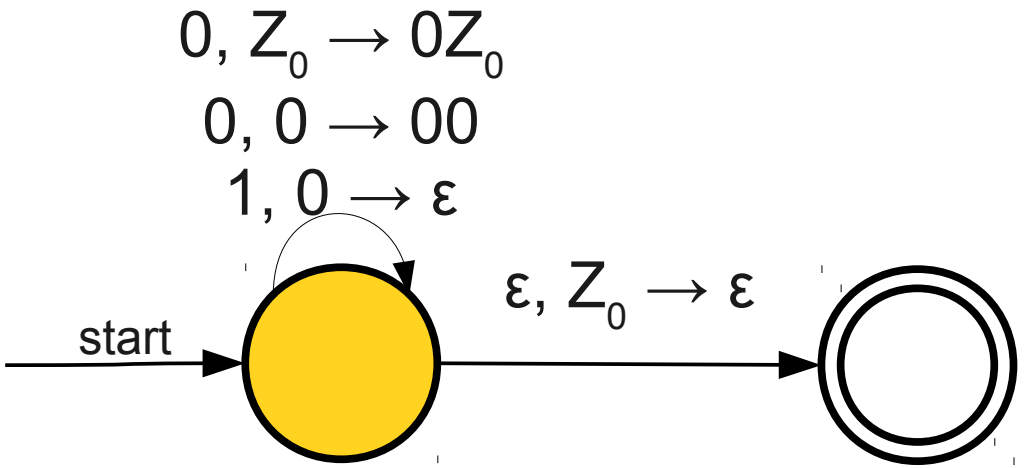

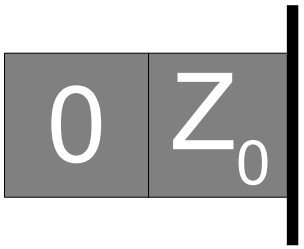

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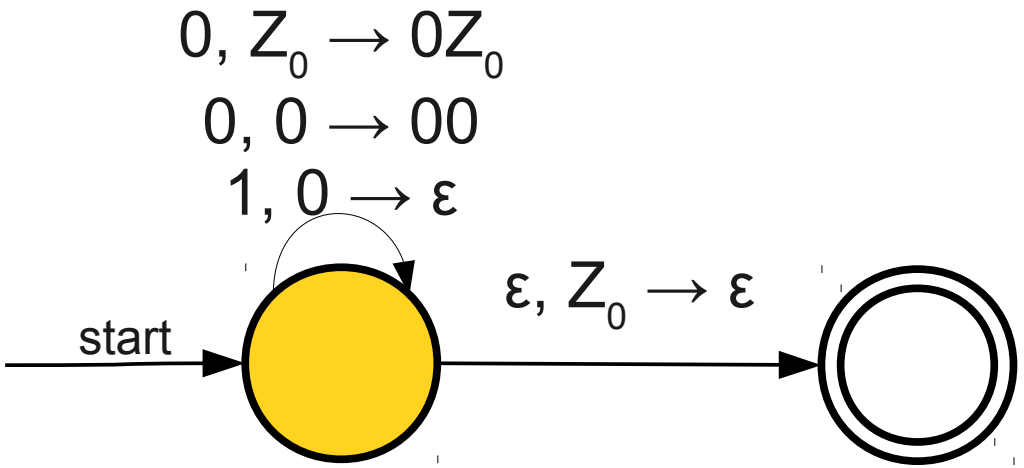

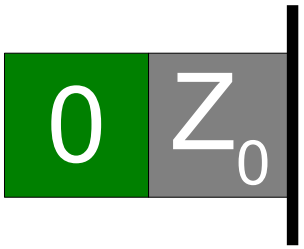

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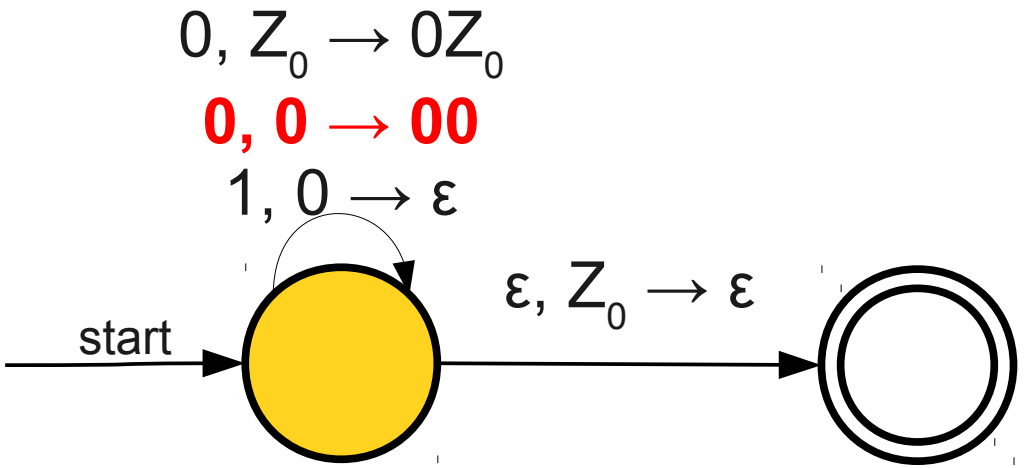

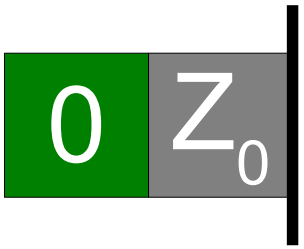

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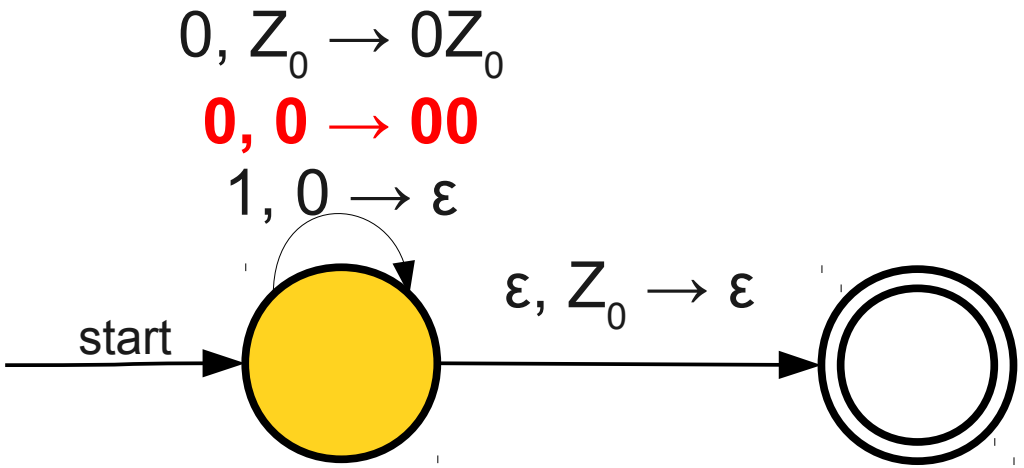

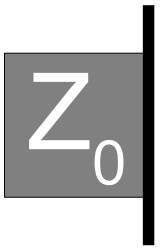

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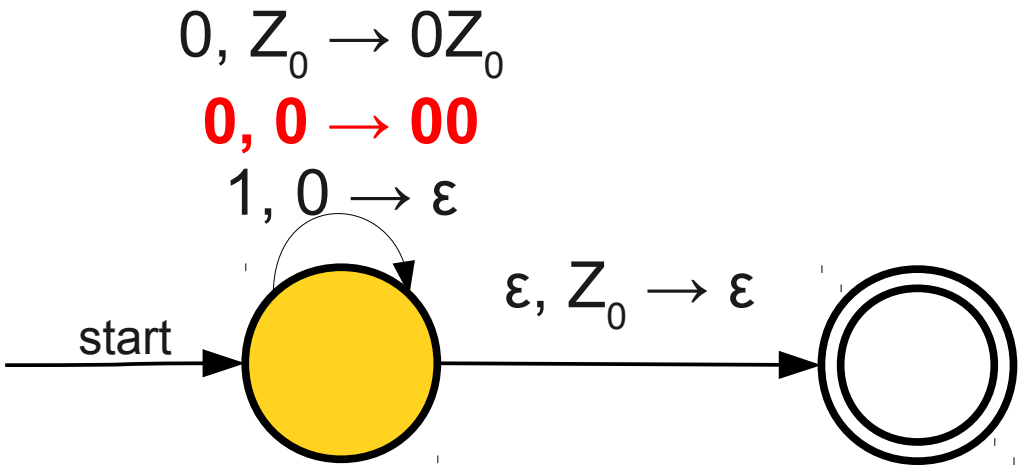

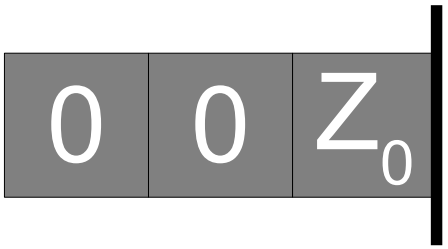

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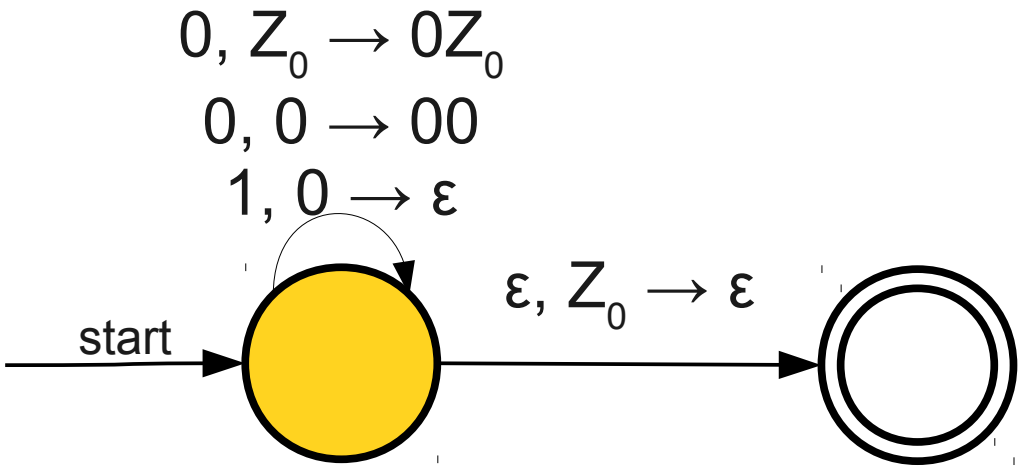

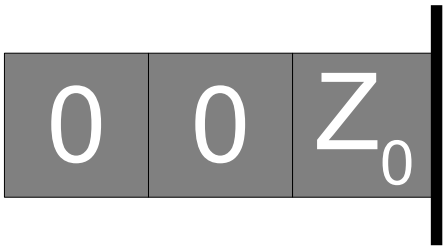

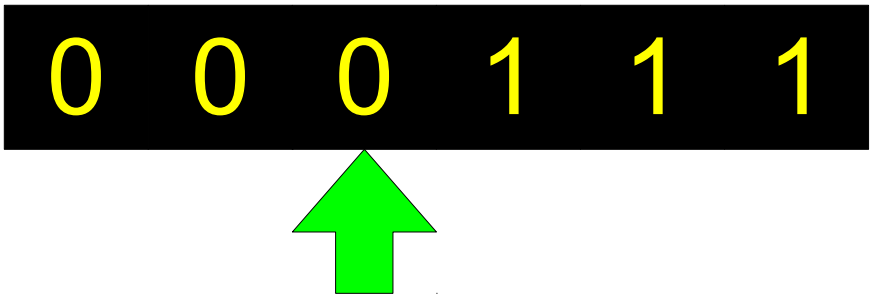

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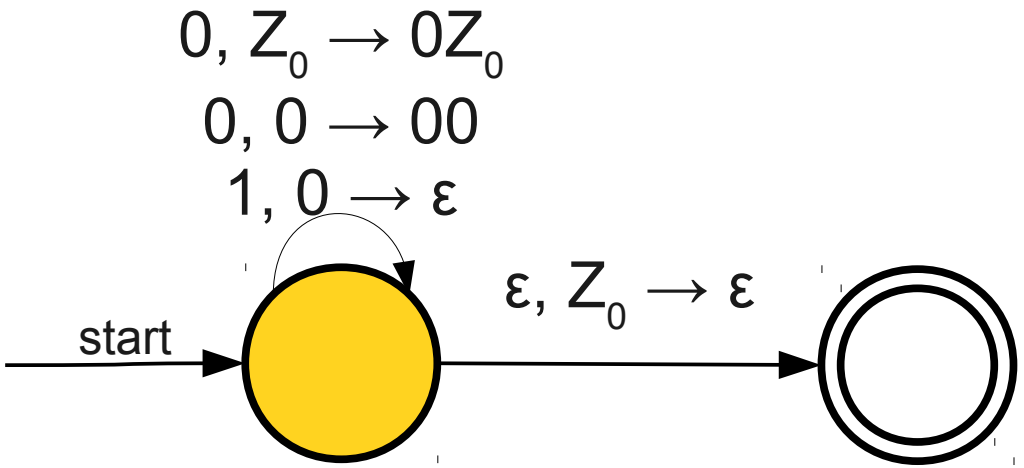

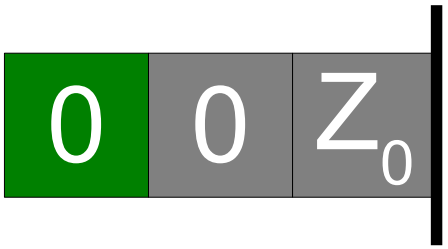

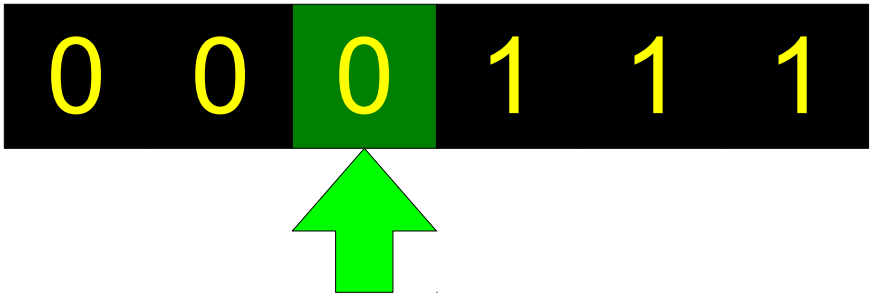

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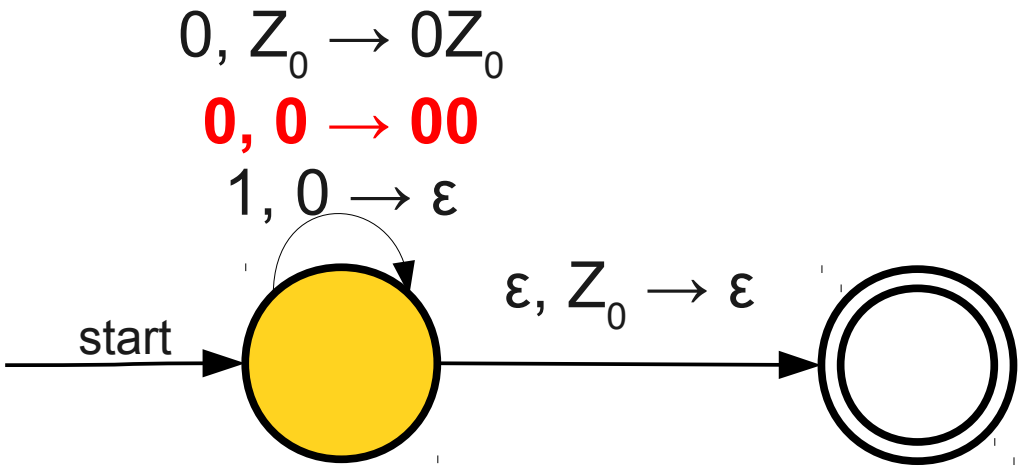

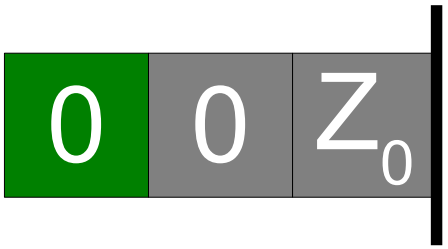

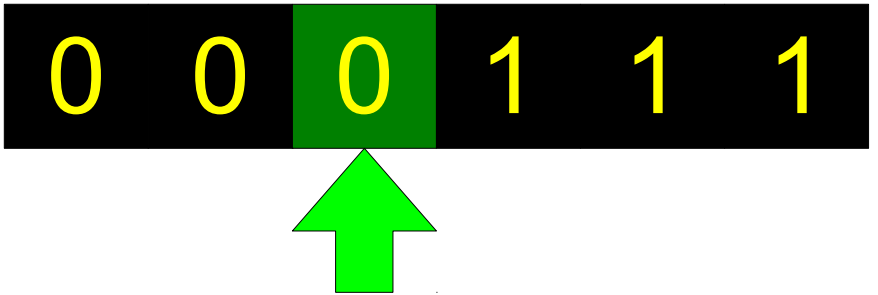

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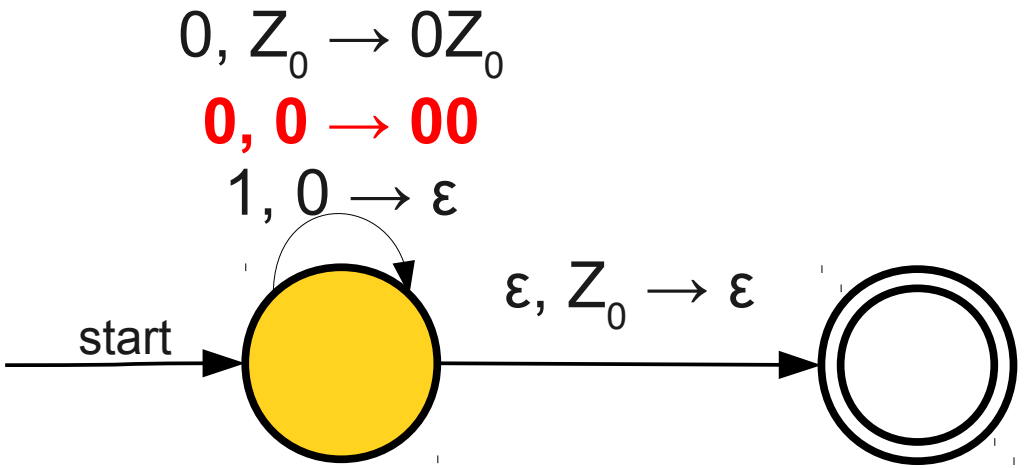

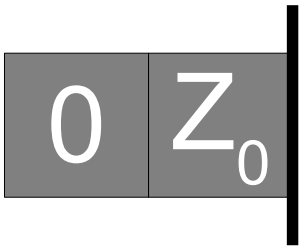

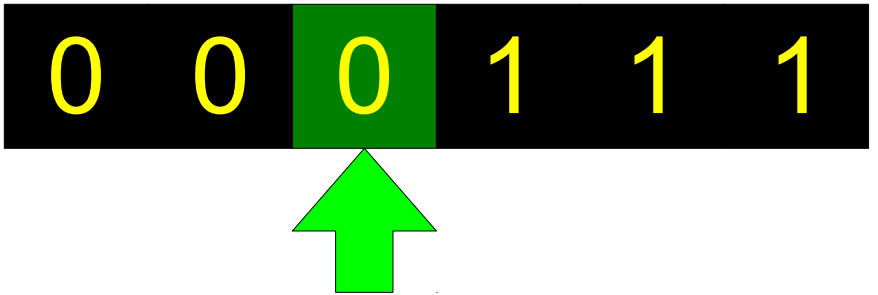

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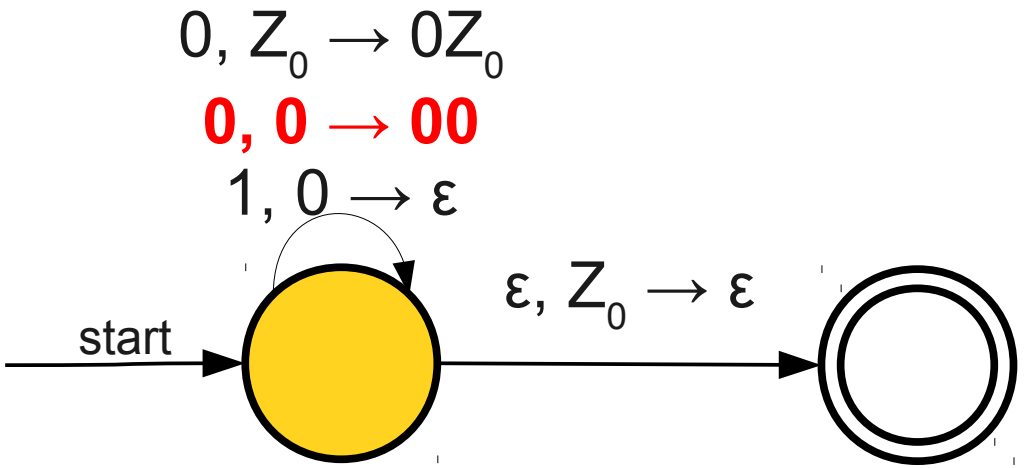

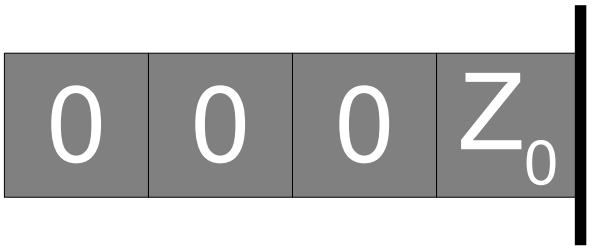

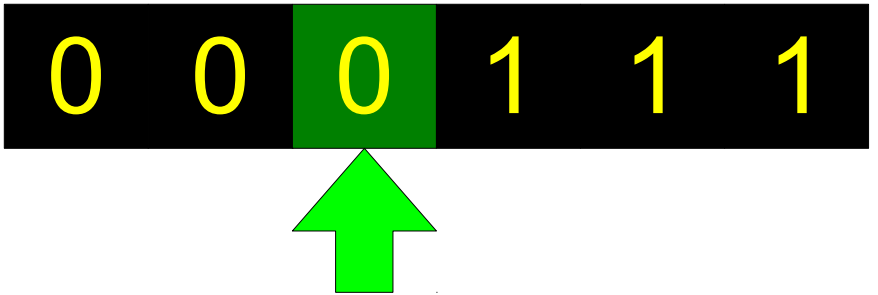

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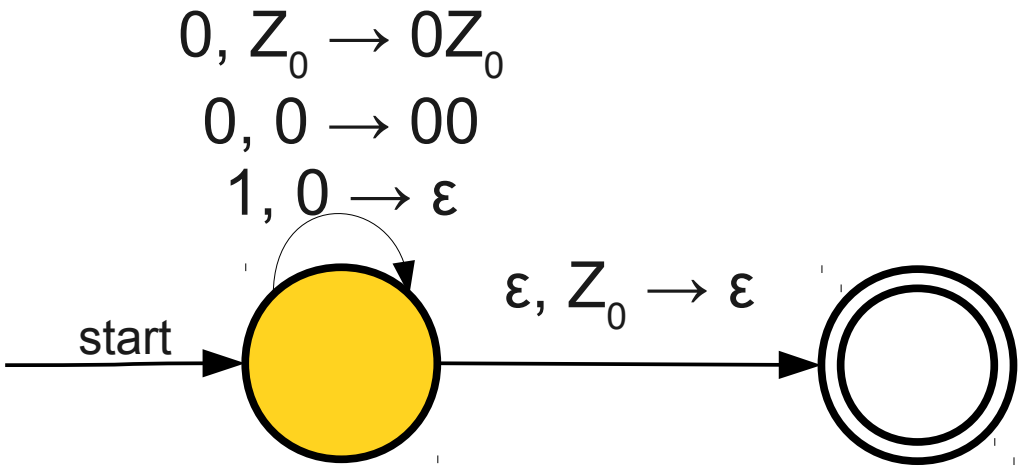

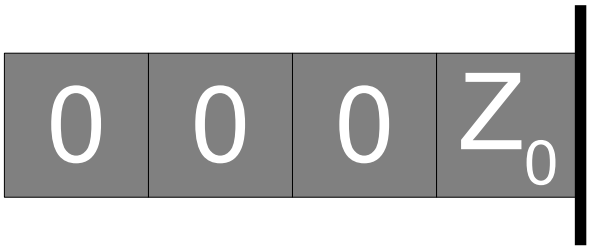

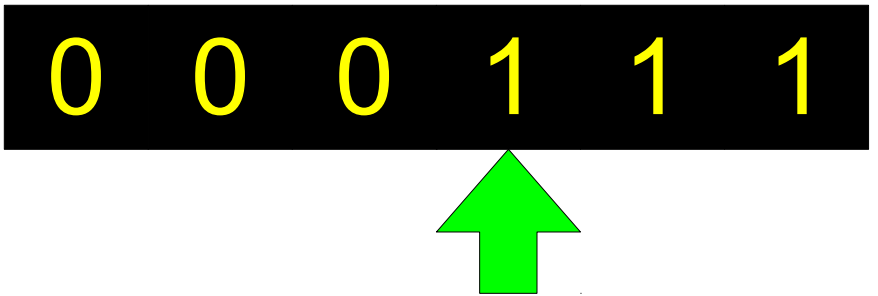

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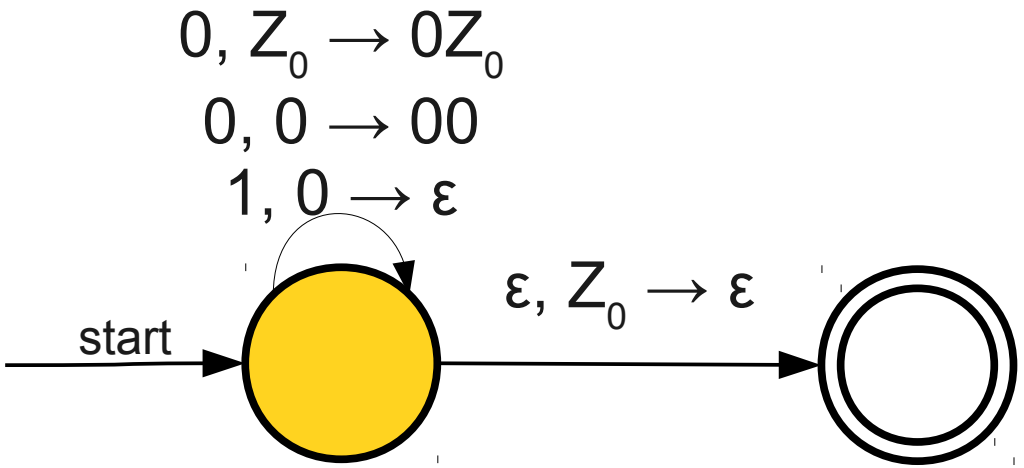

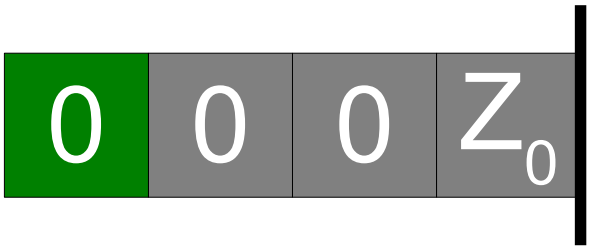

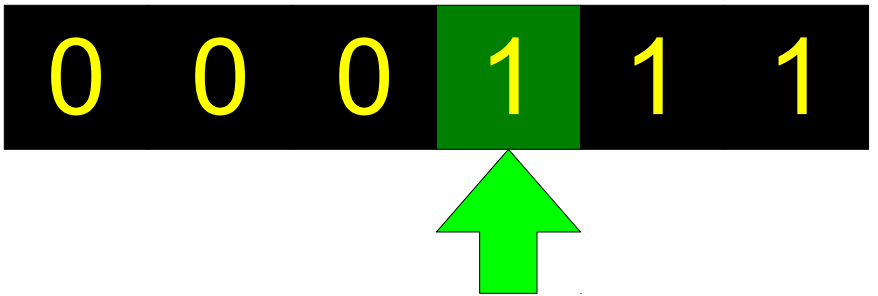

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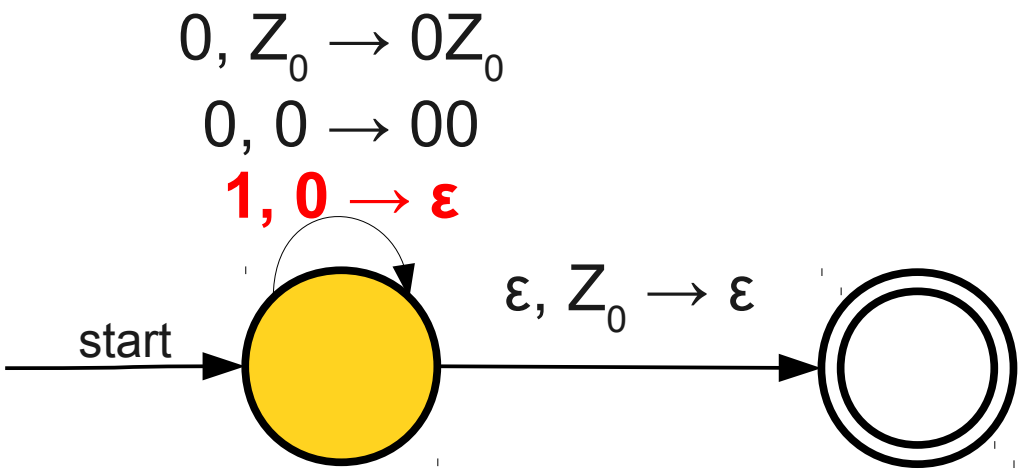

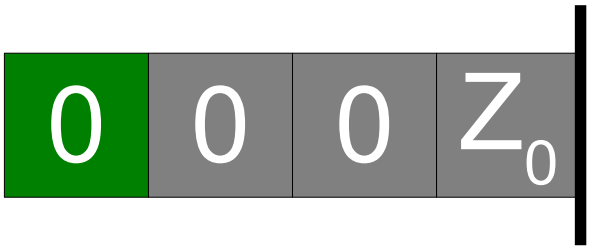

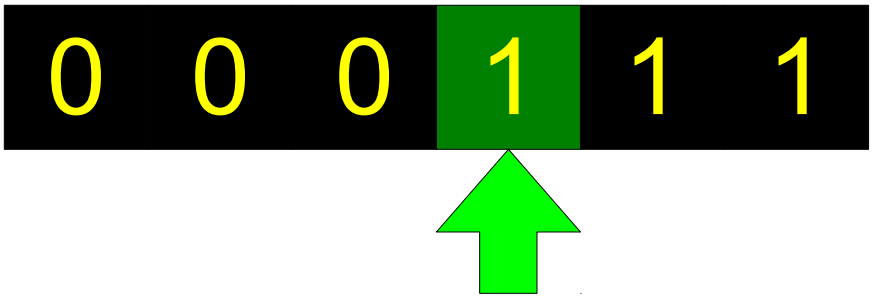

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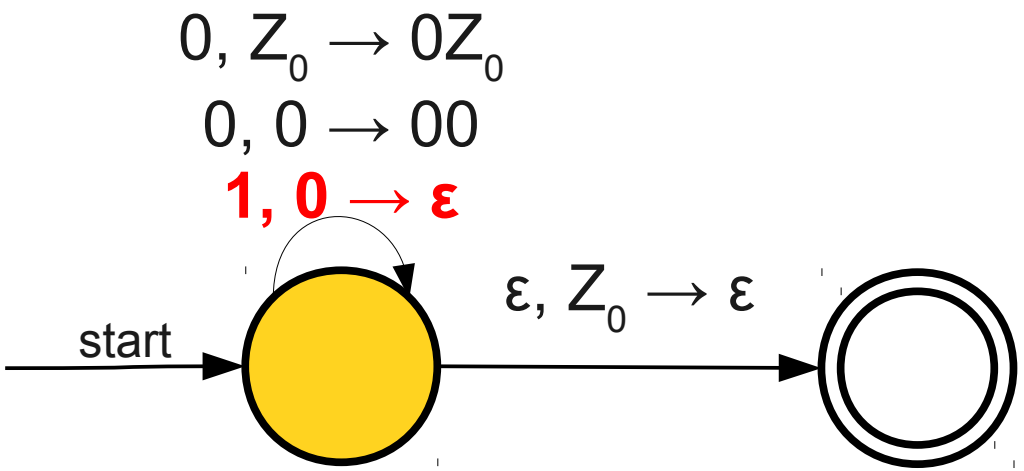

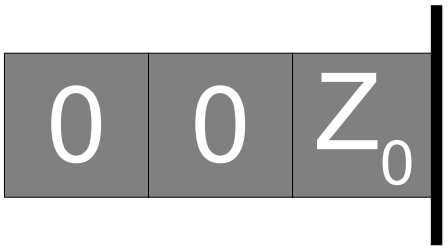

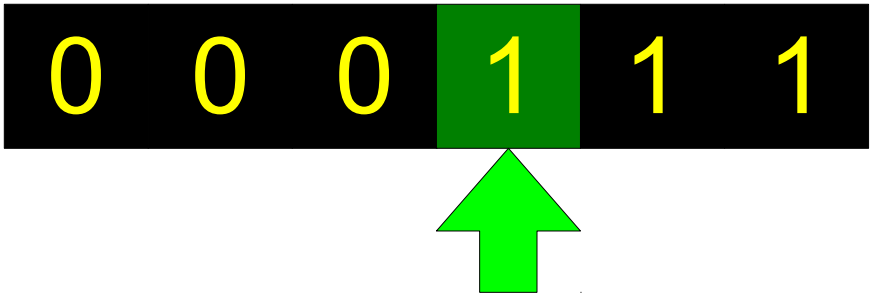

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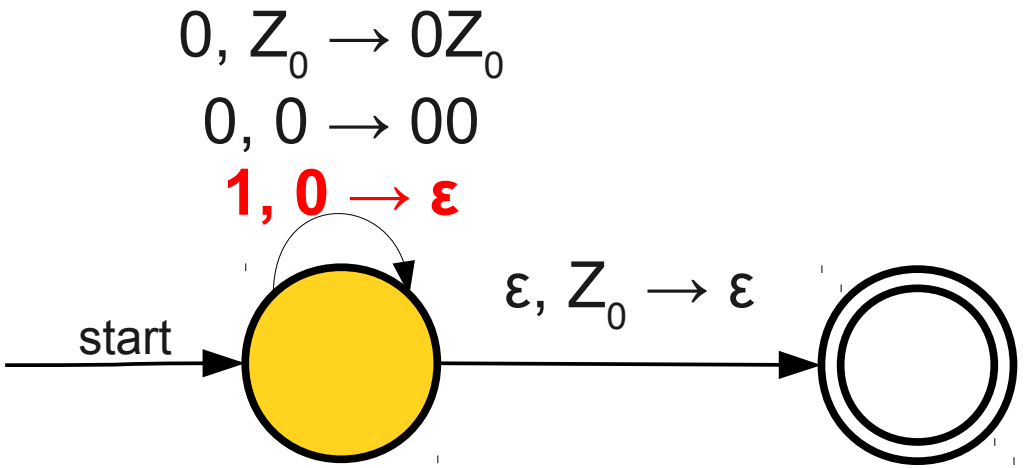

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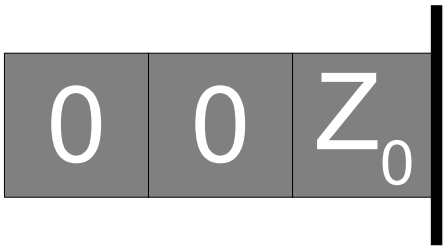

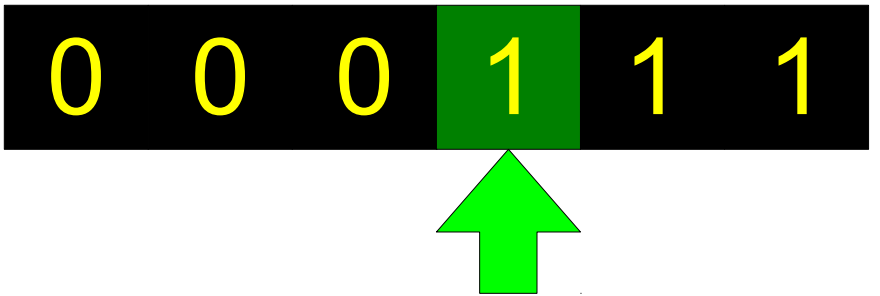

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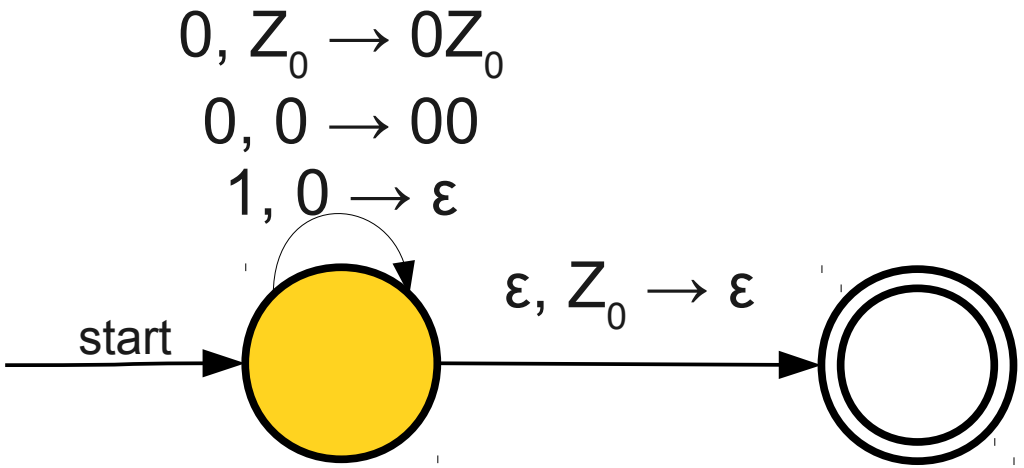

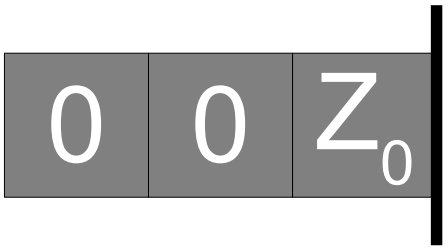

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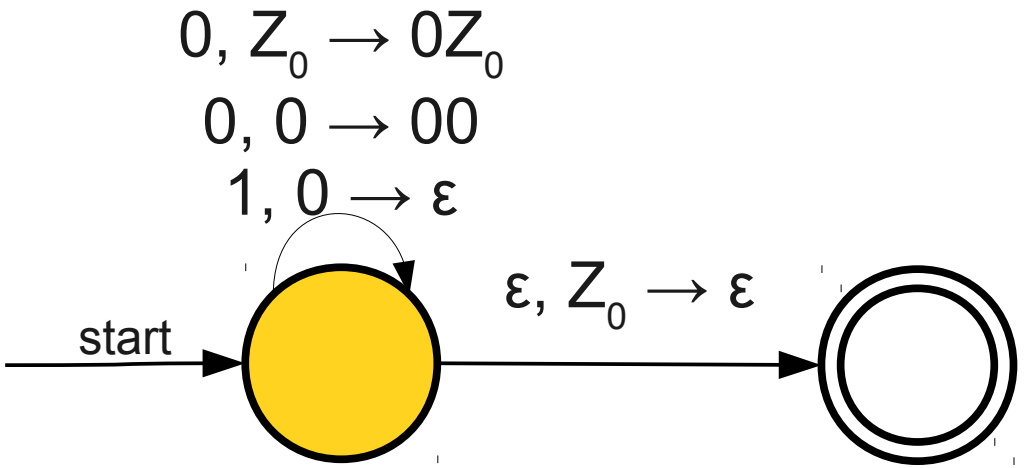

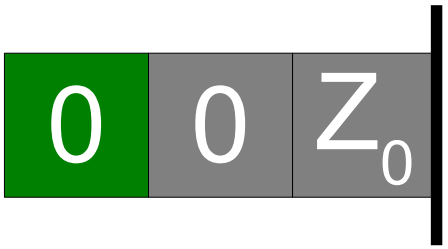

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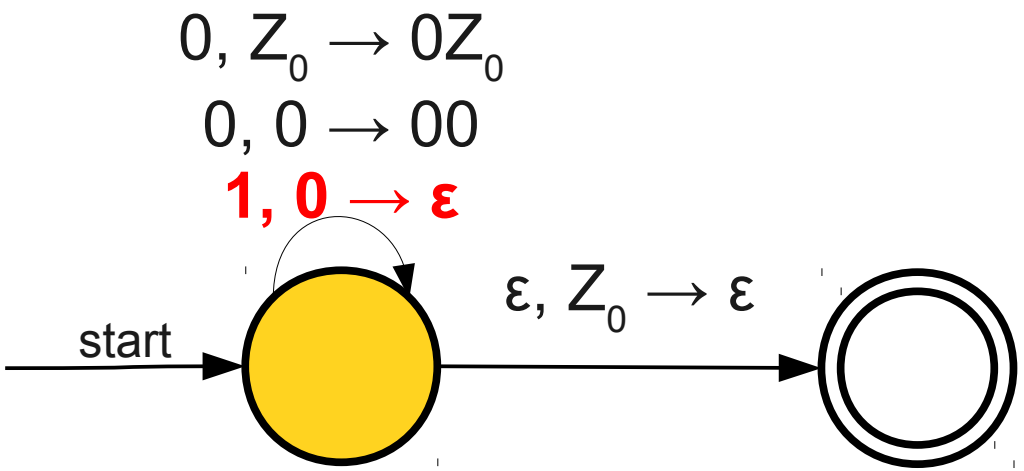

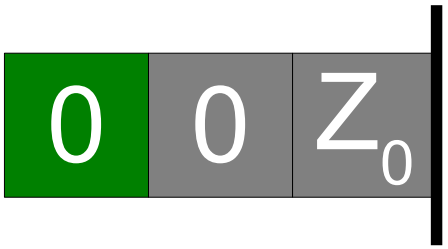

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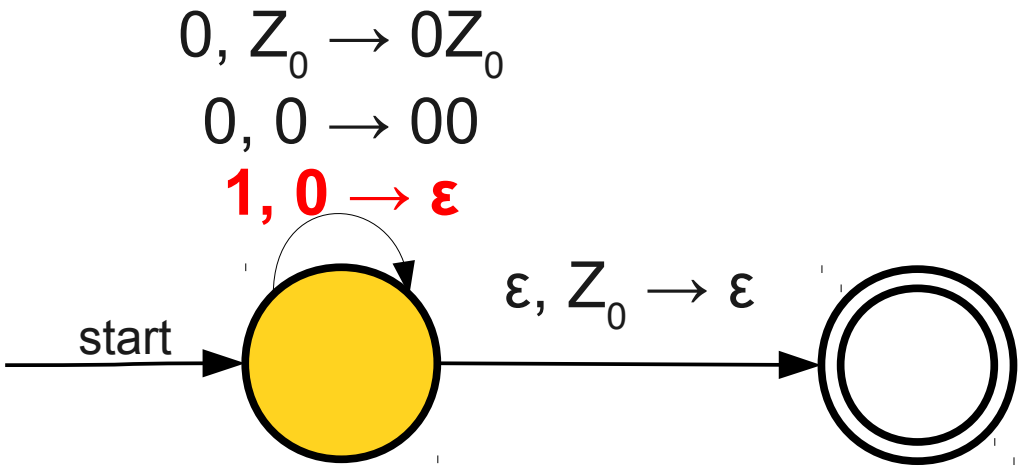

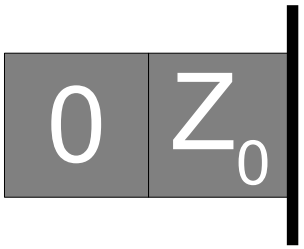

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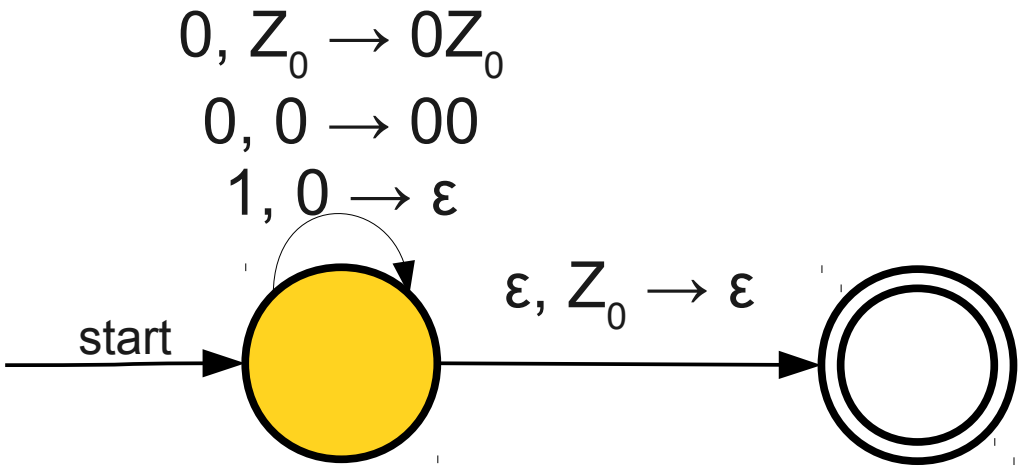

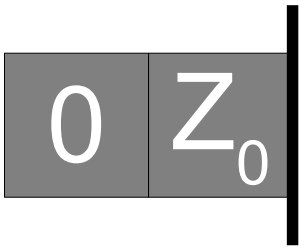

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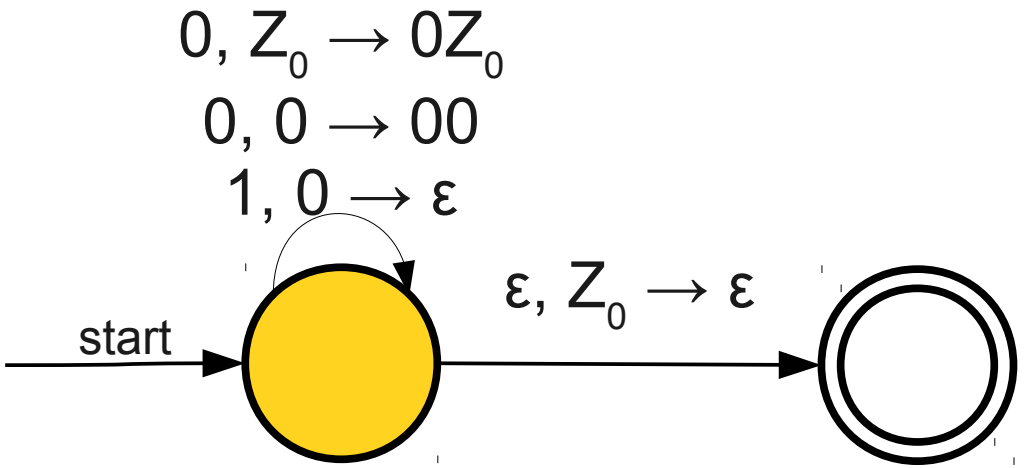

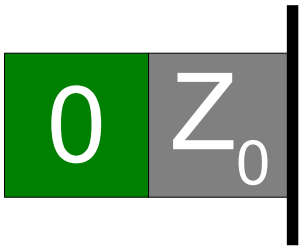

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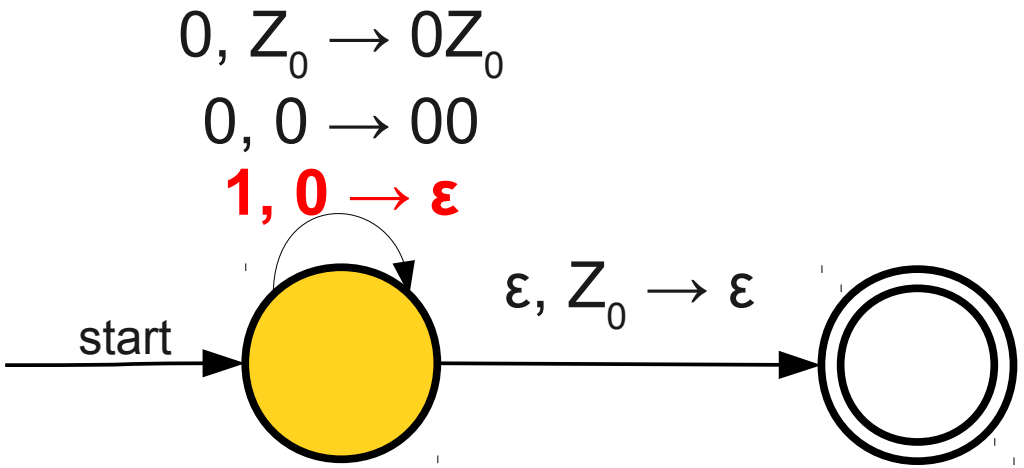

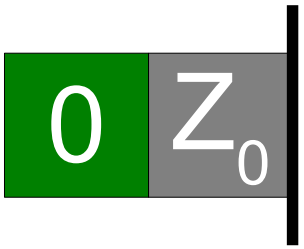

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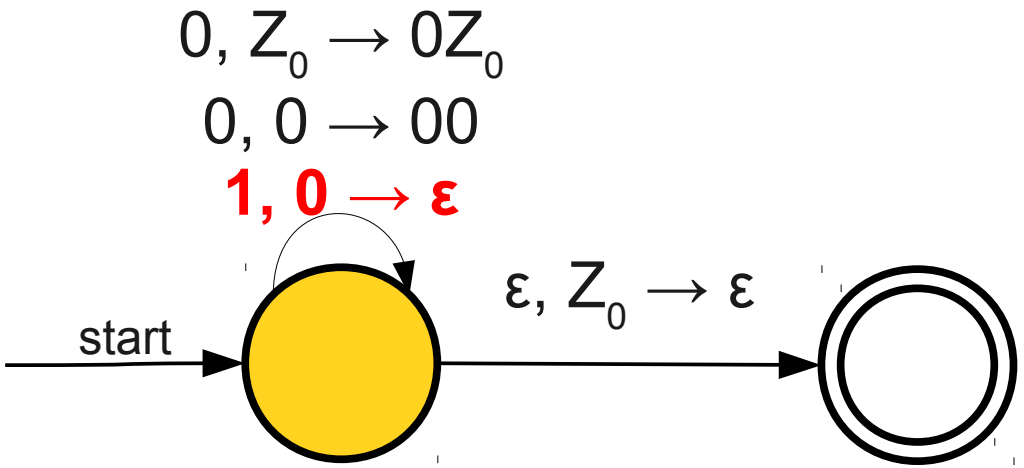

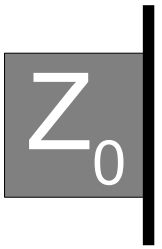

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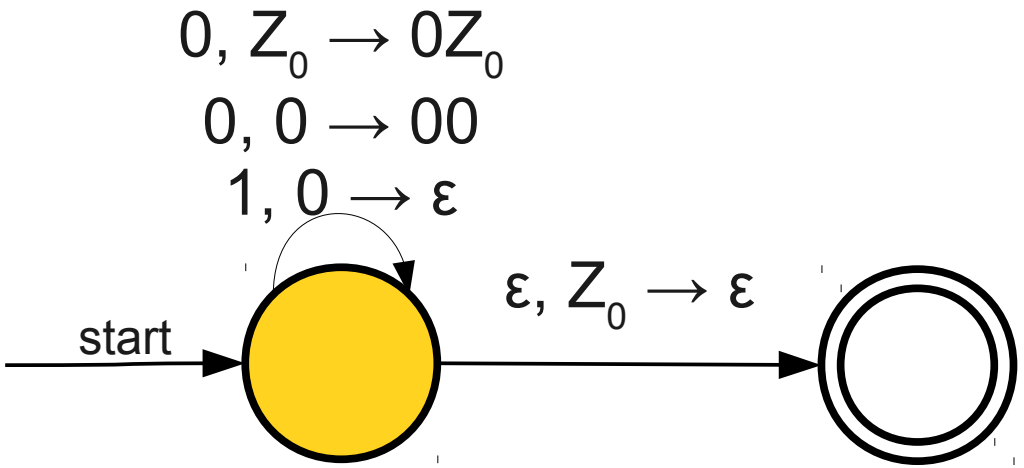

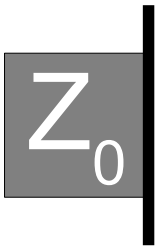

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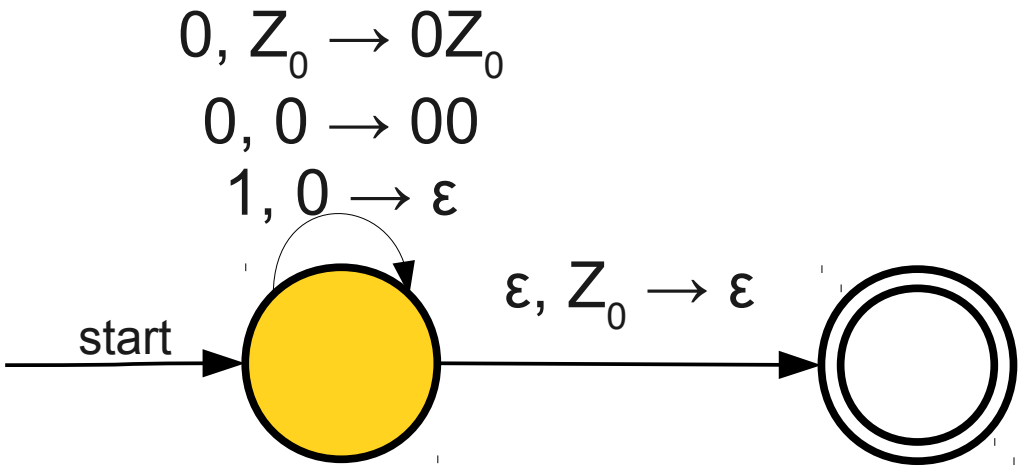

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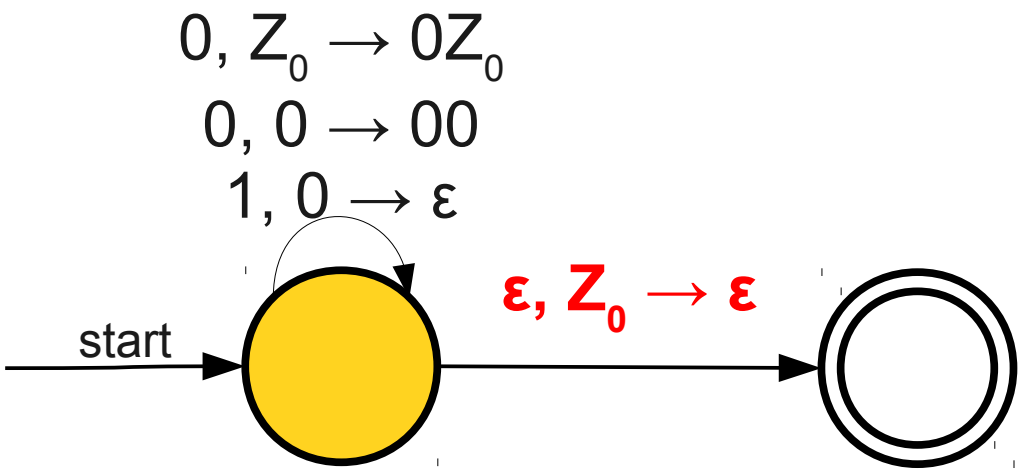

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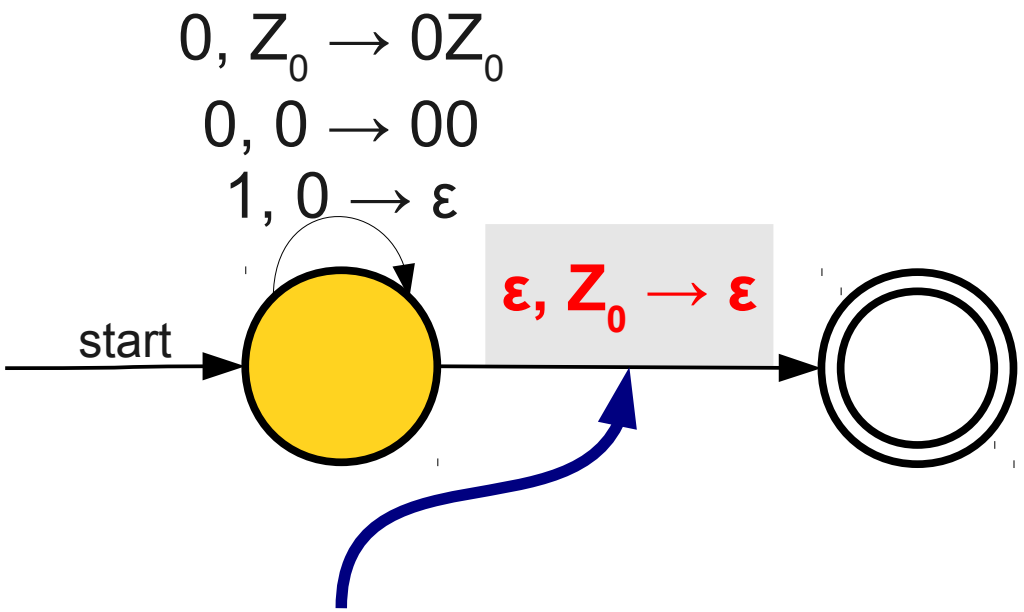
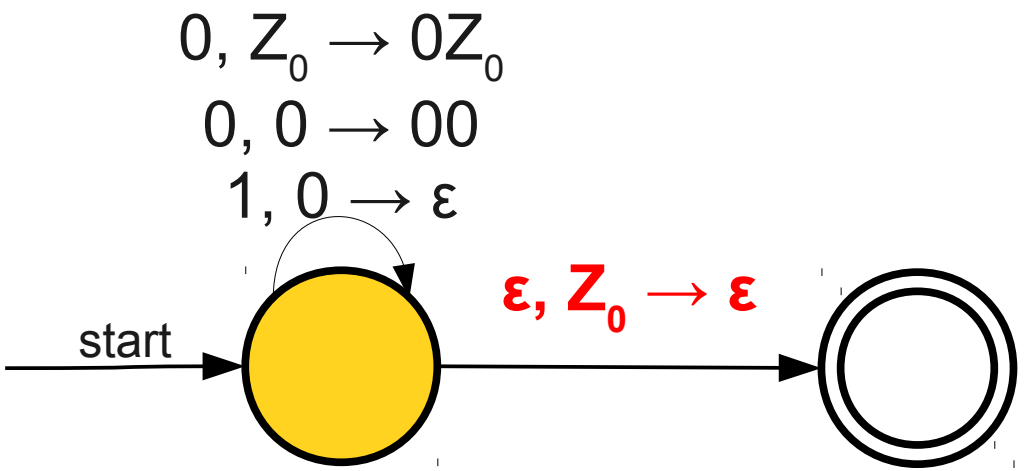


This nondeterministic transition can be taken at any time, but we've nondeterministically guessed that this would be a good time to take it.

A Simple Pushdown Automaton



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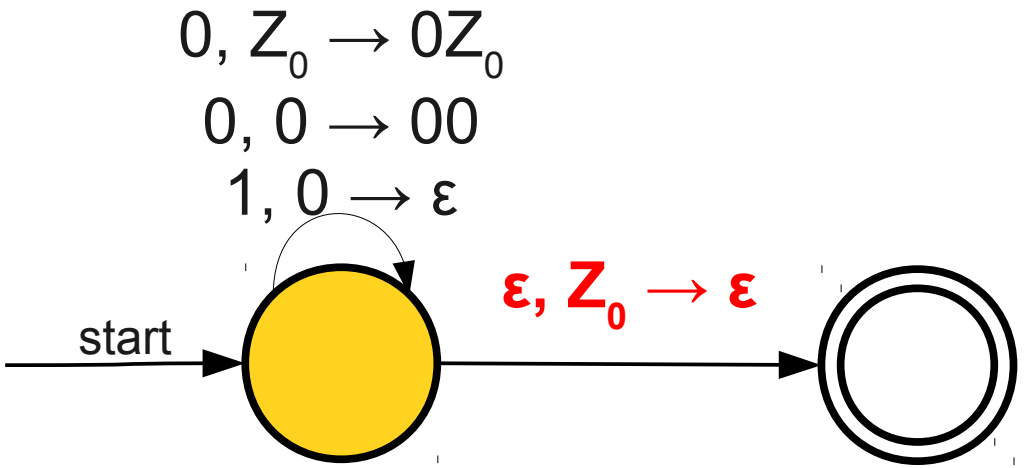

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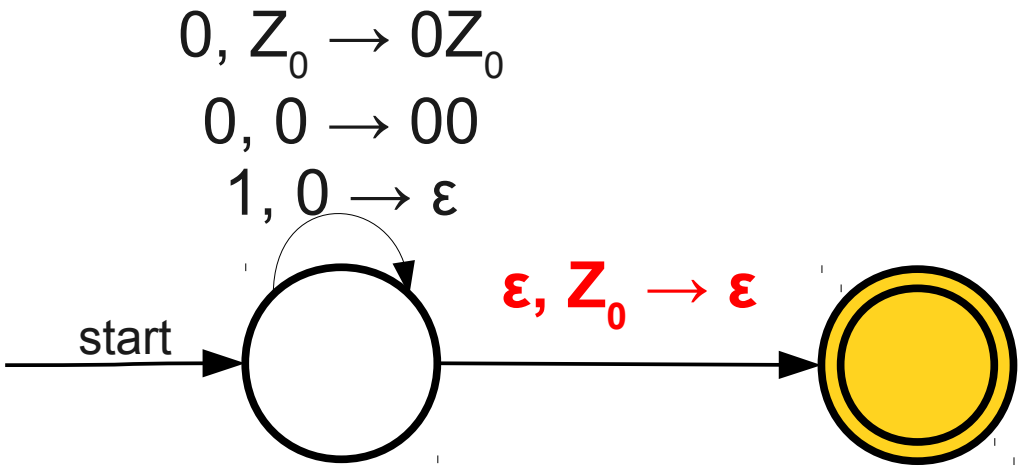

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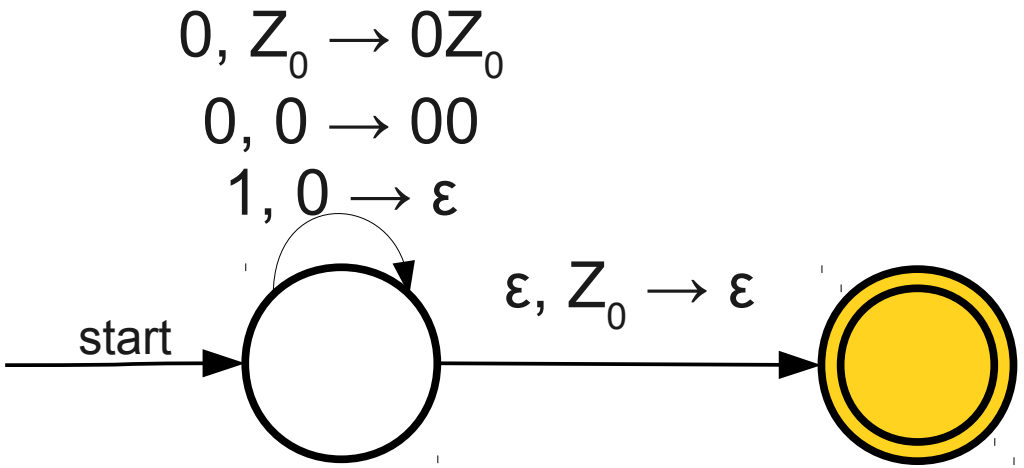
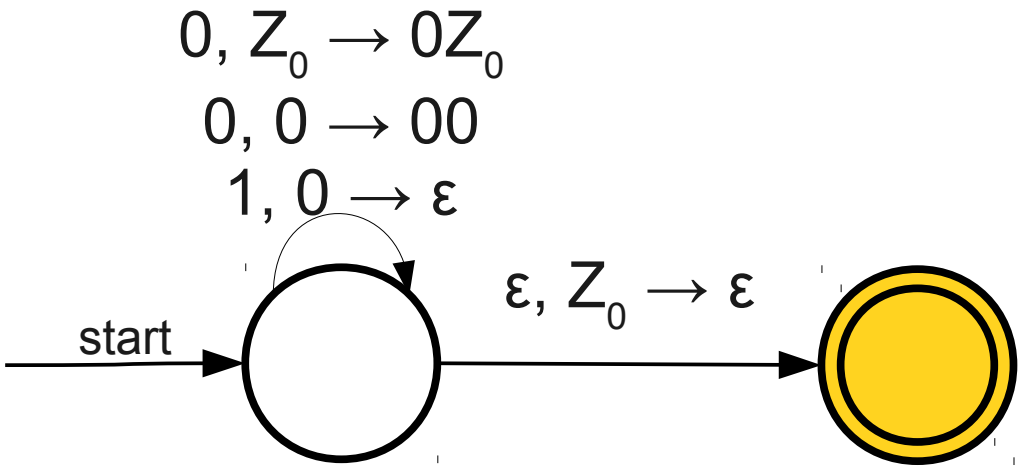


0 0 0 1 1 1

A Simple Pushdown Automaton



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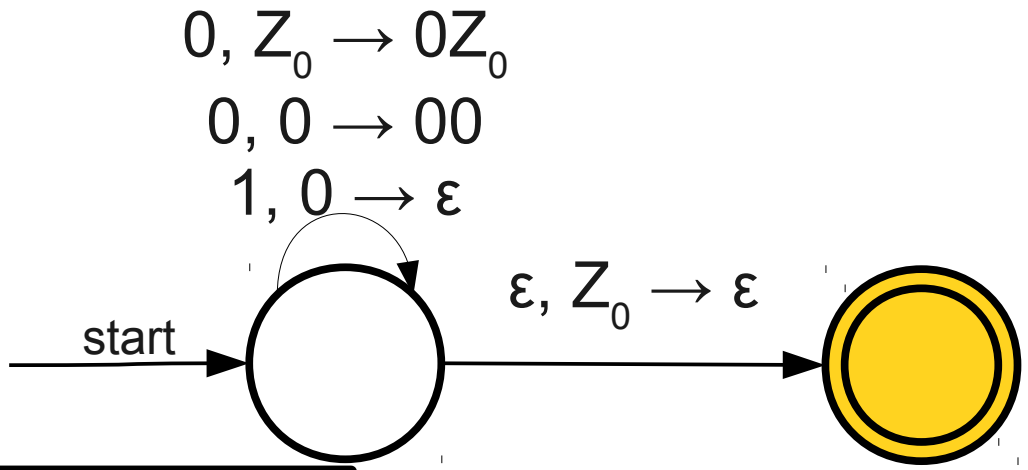

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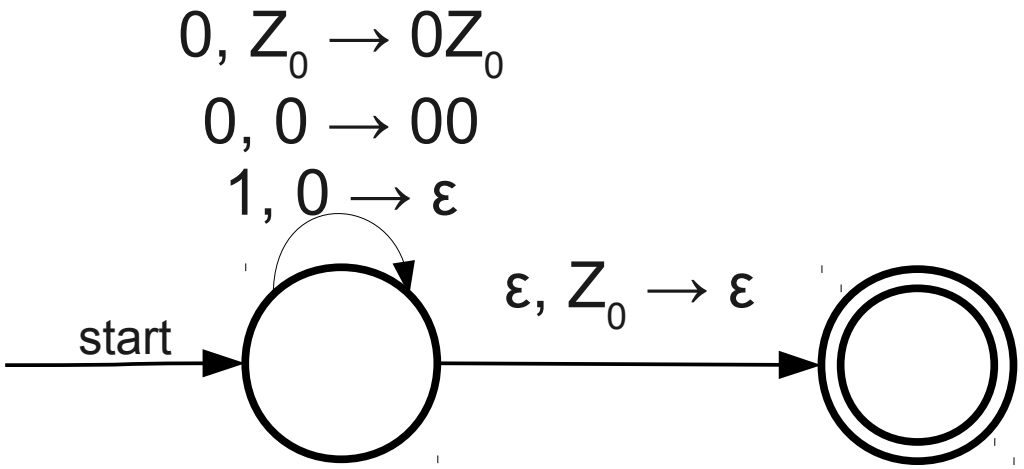

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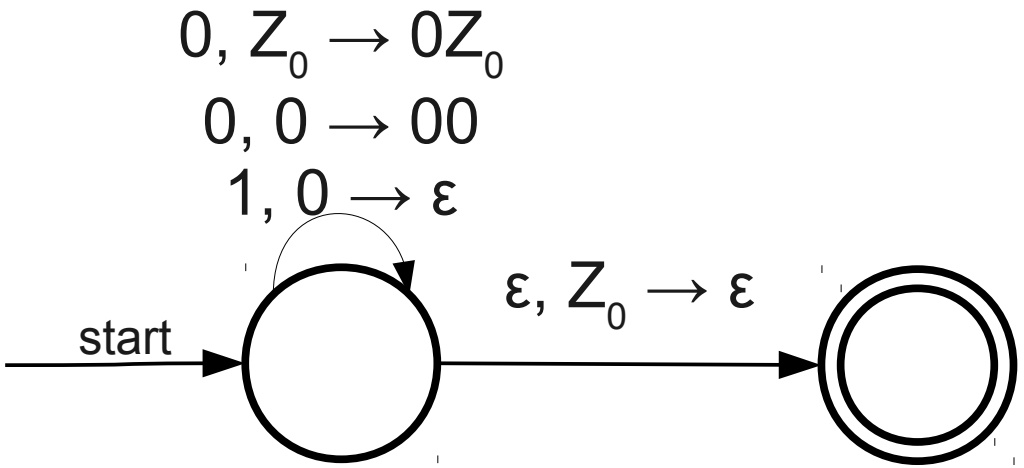

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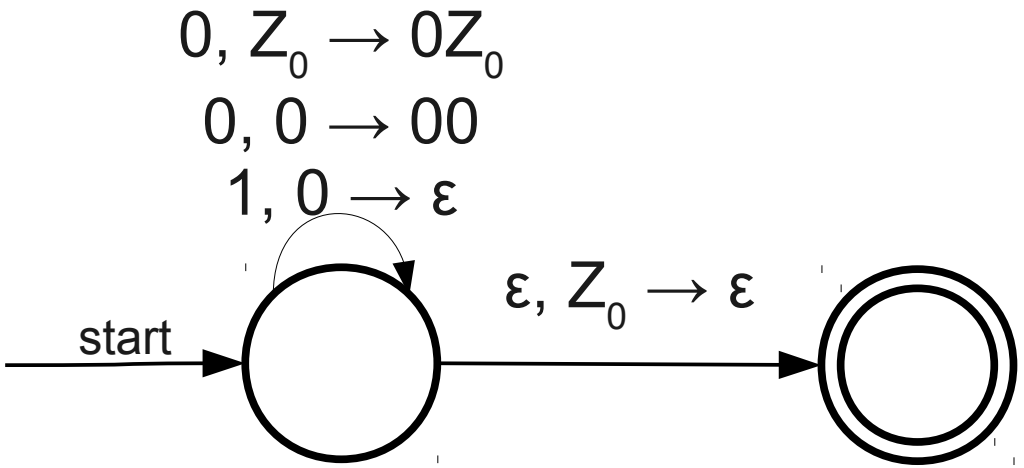

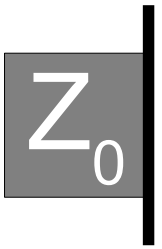

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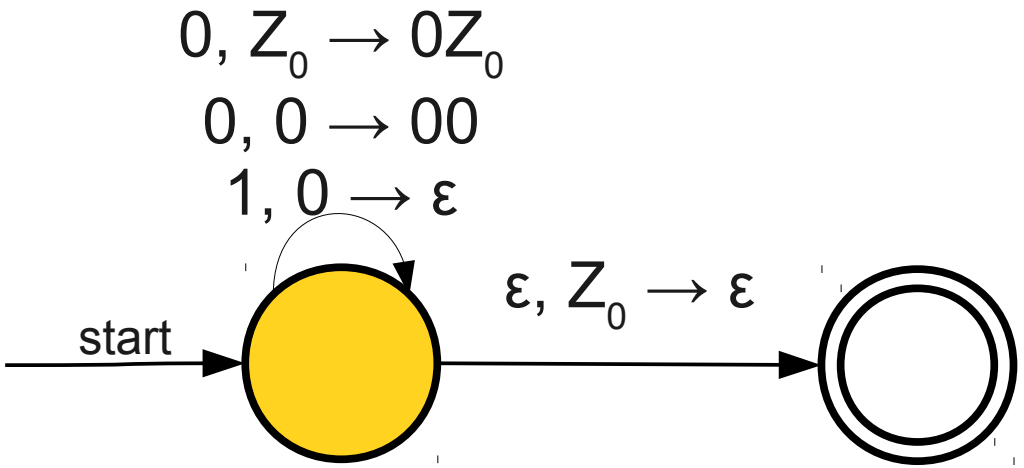

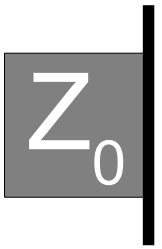

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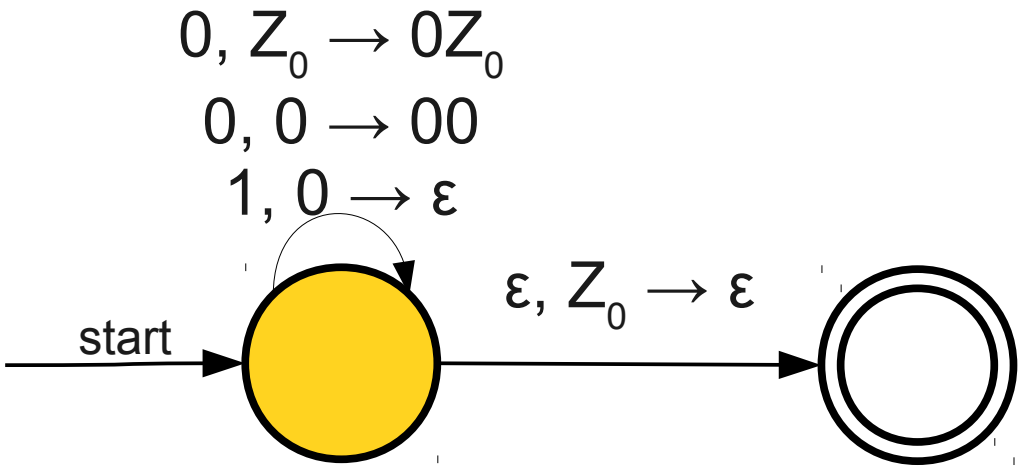

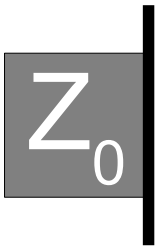

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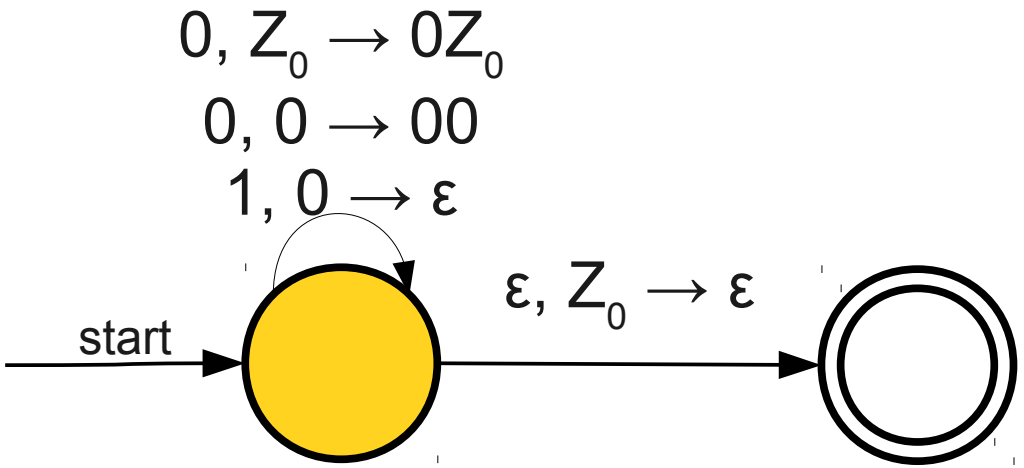

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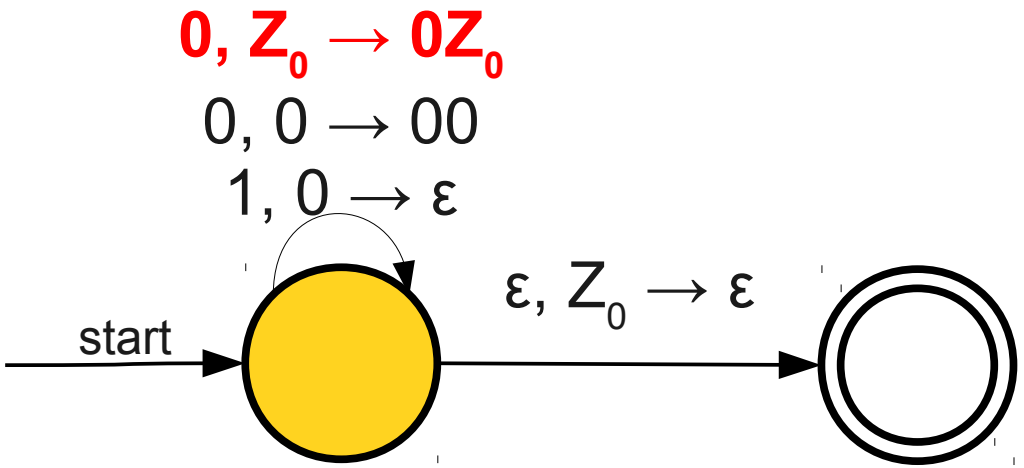

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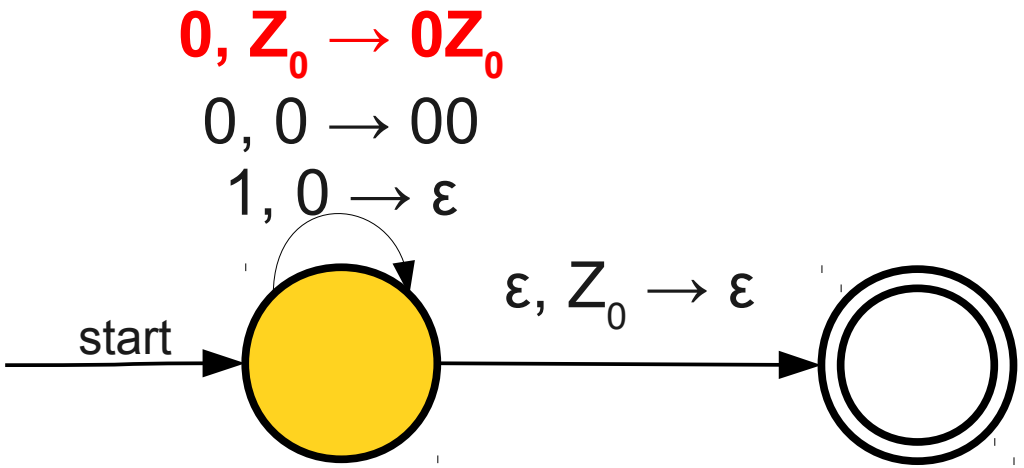

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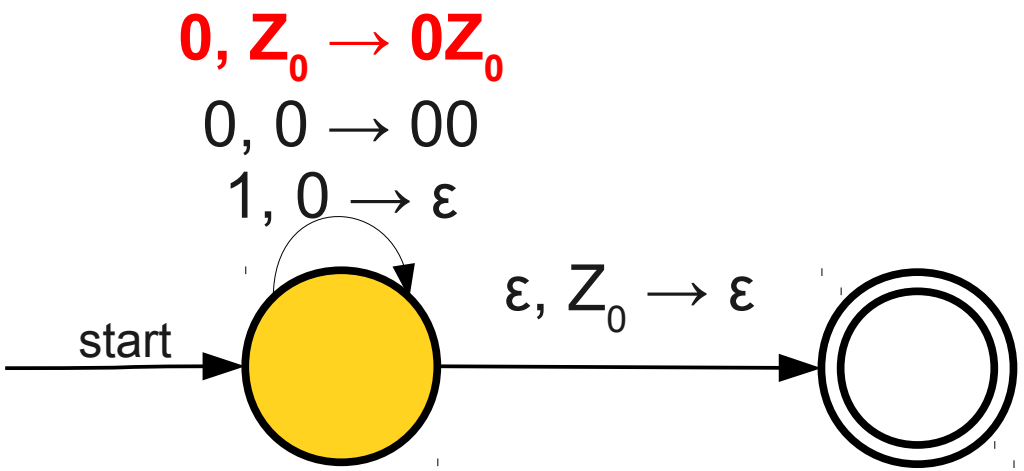

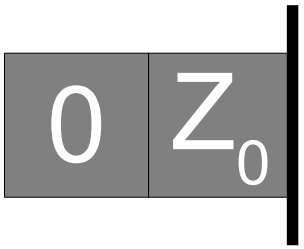

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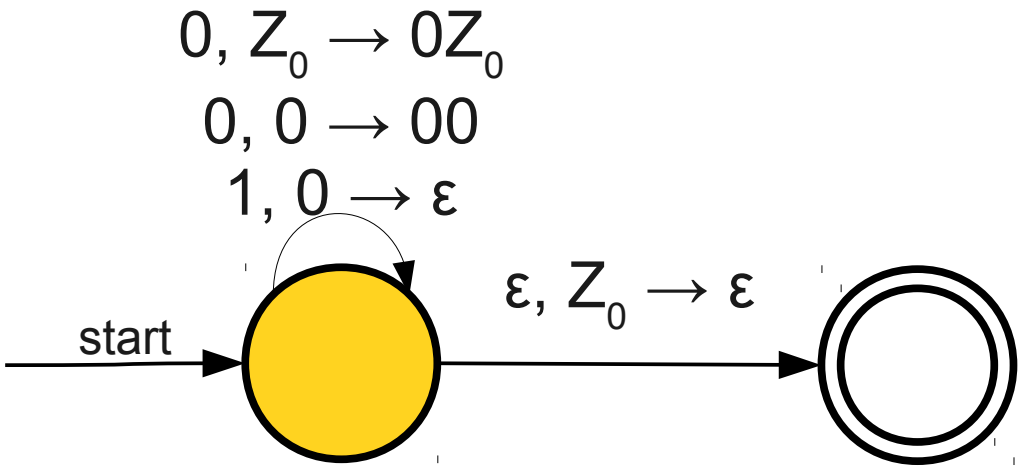

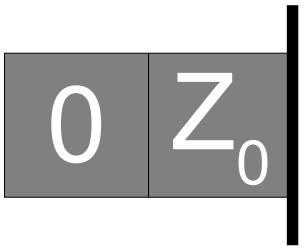

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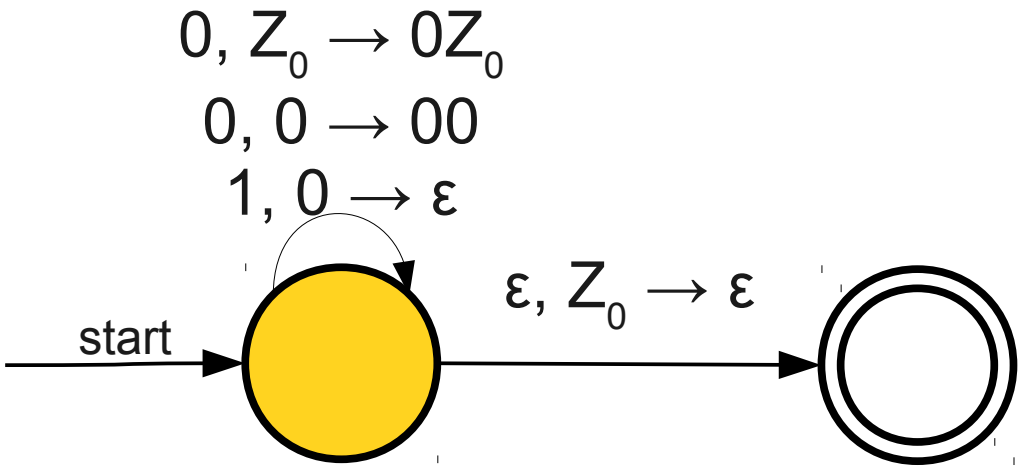

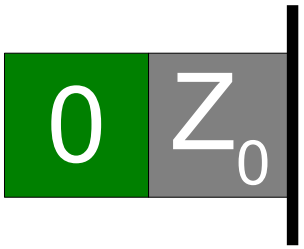

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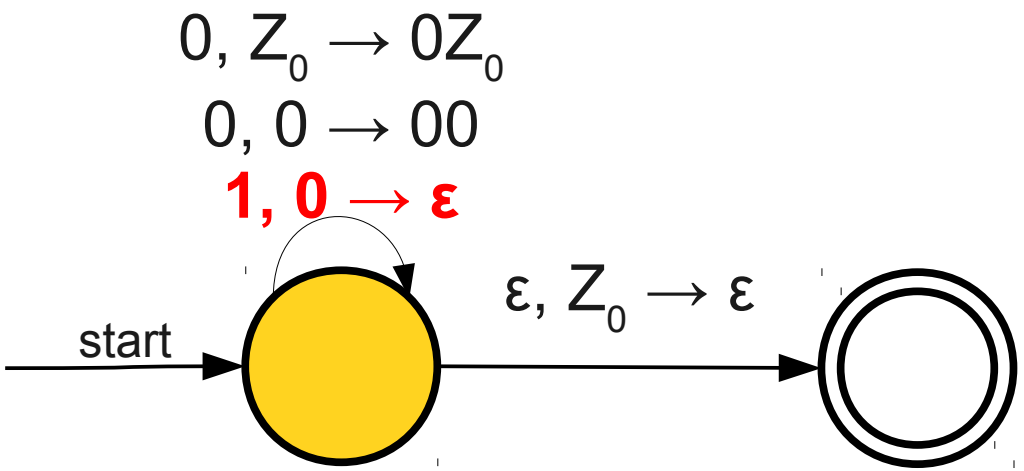

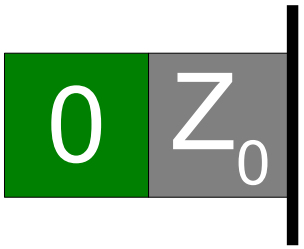

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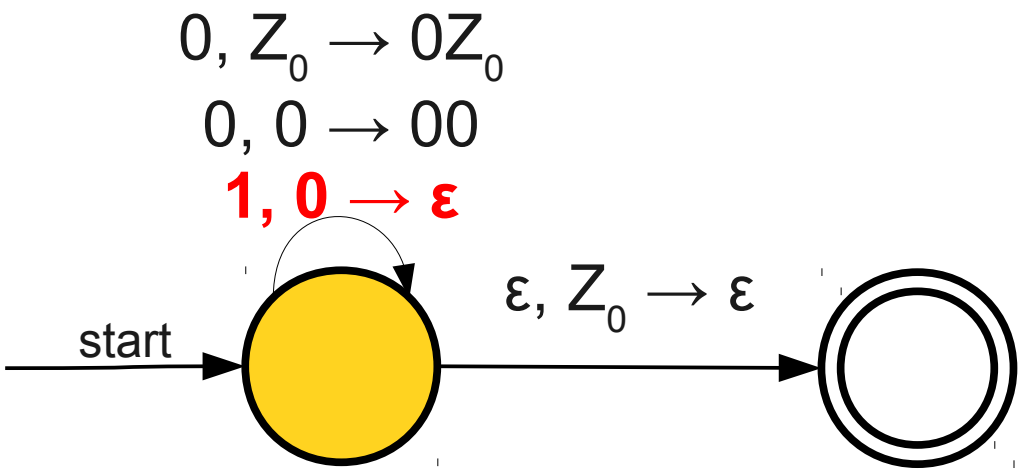

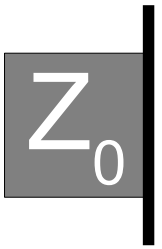

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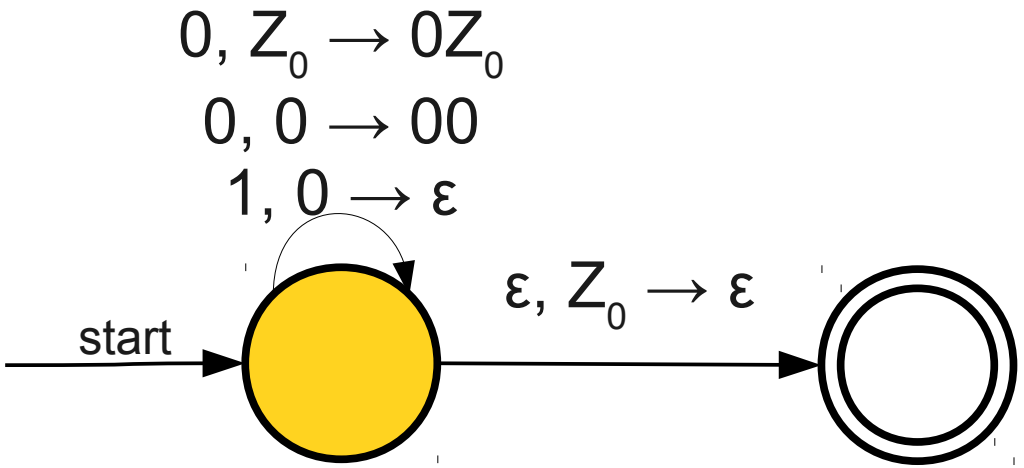

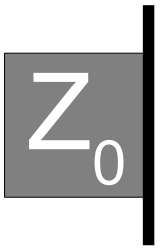

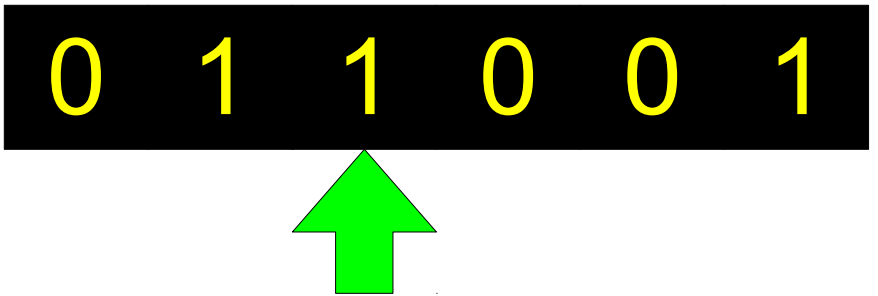

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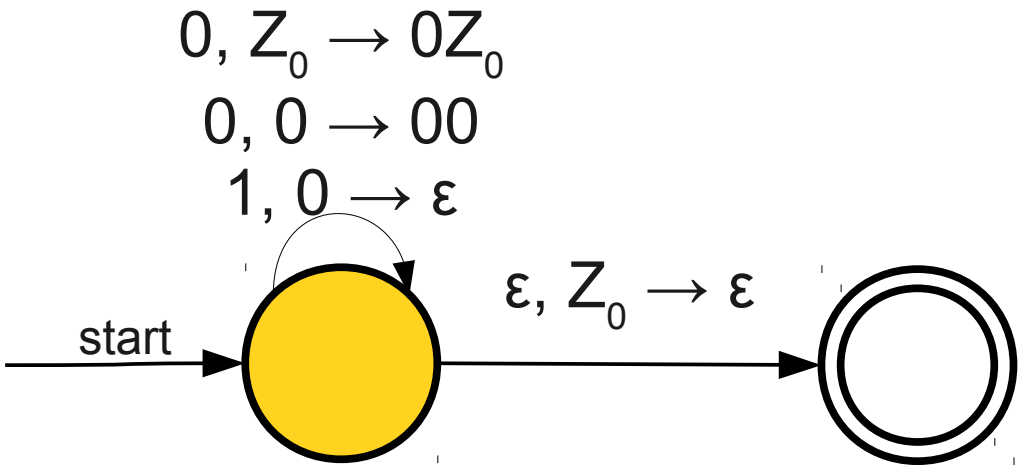

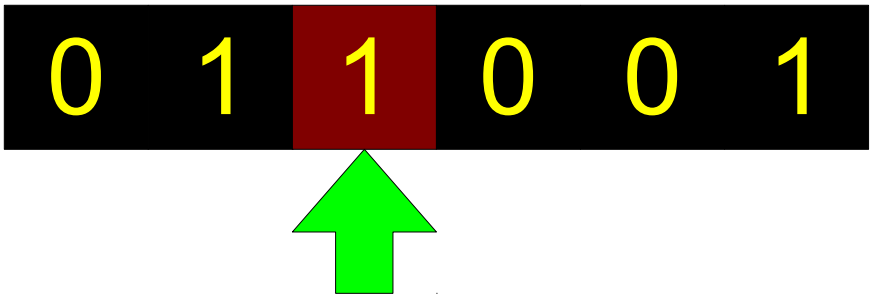

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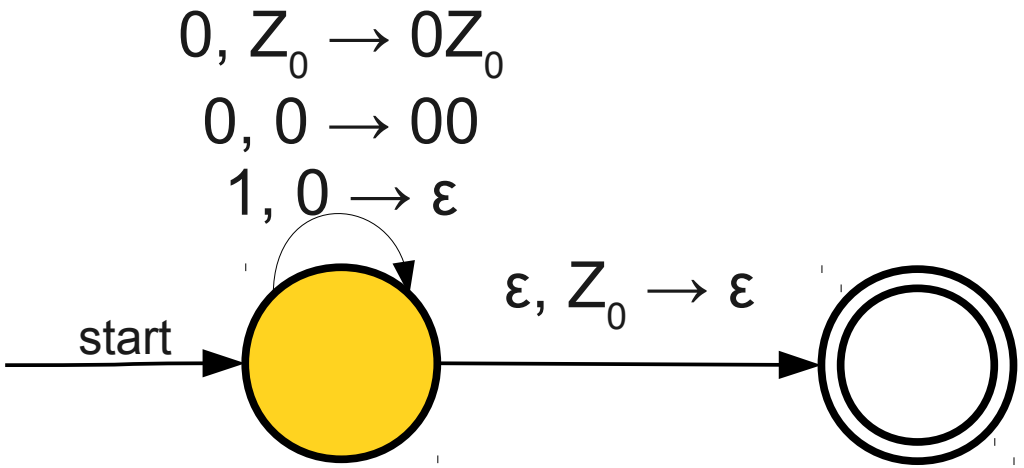

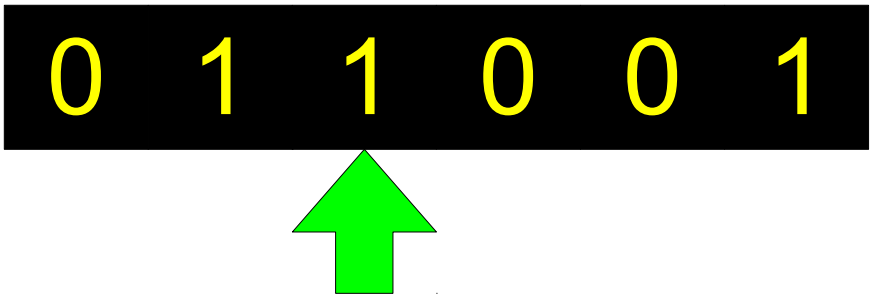

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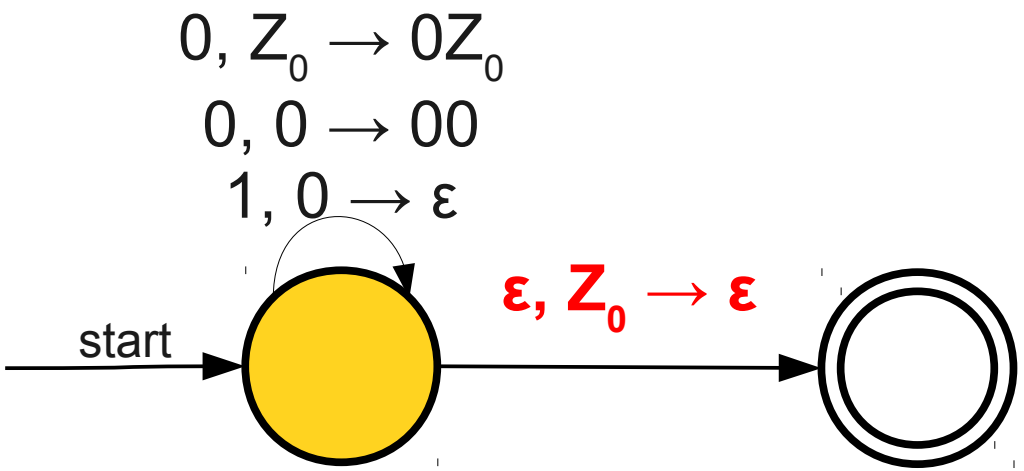

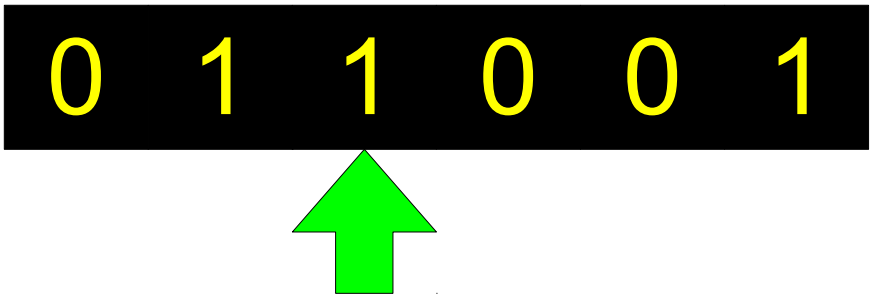

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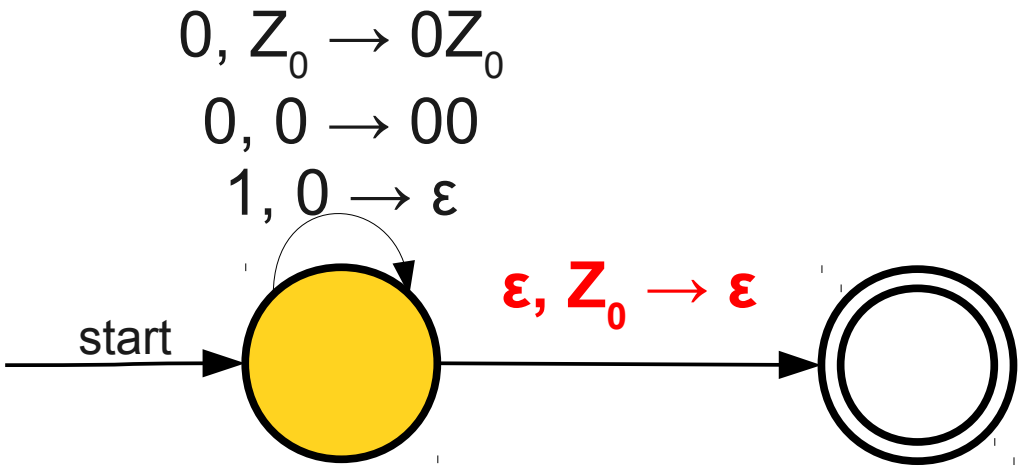

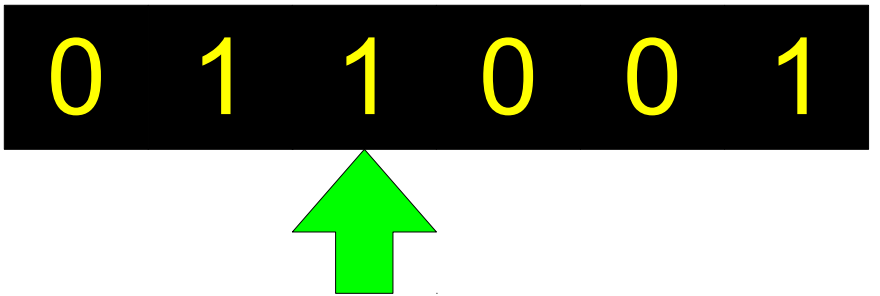

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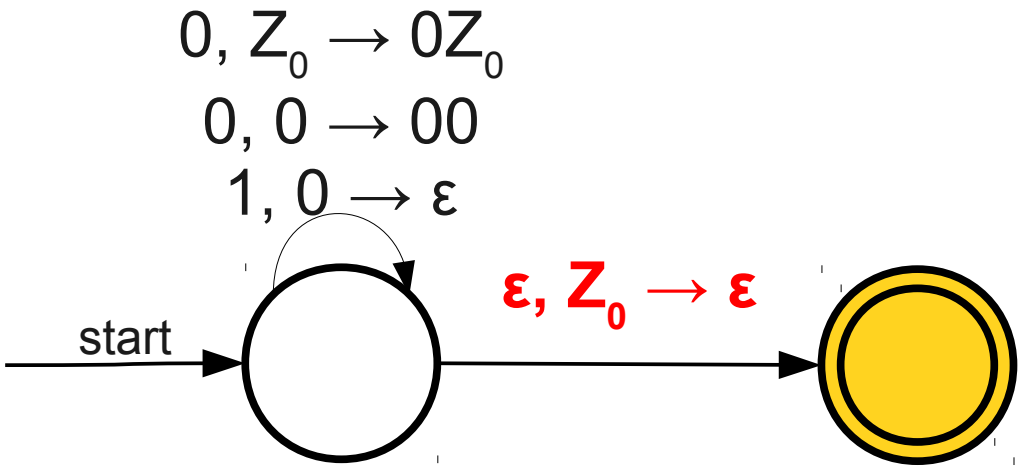

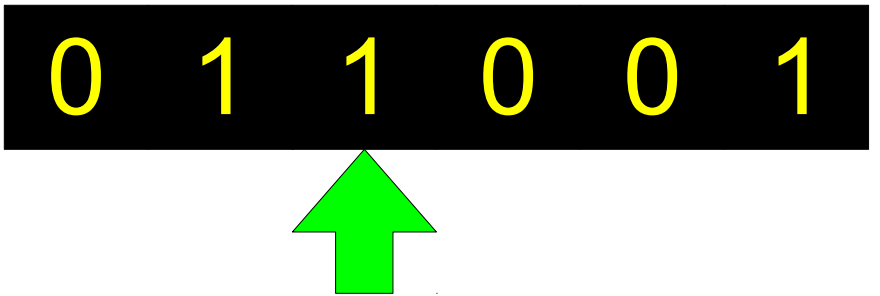

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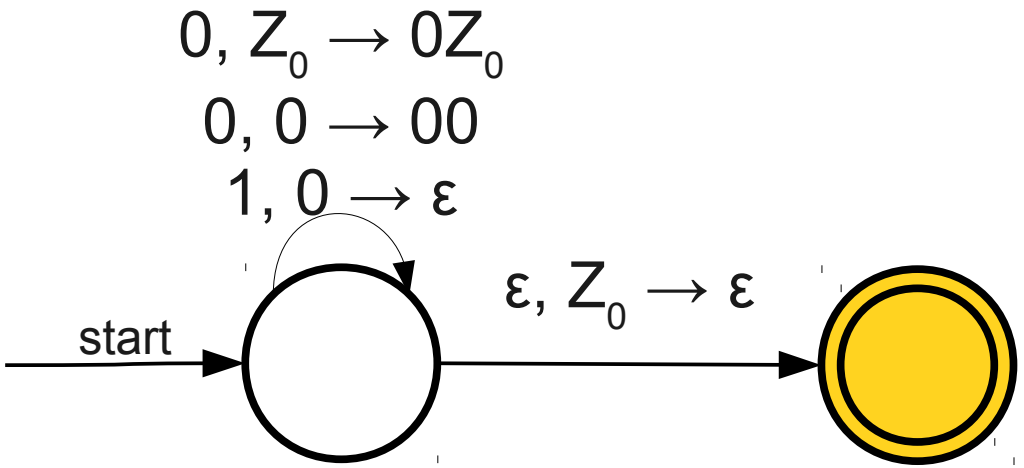
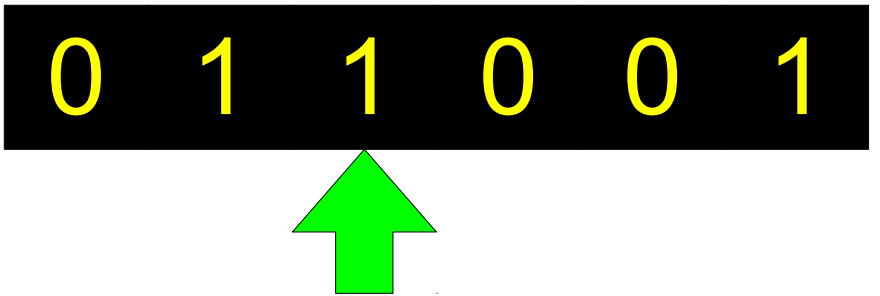
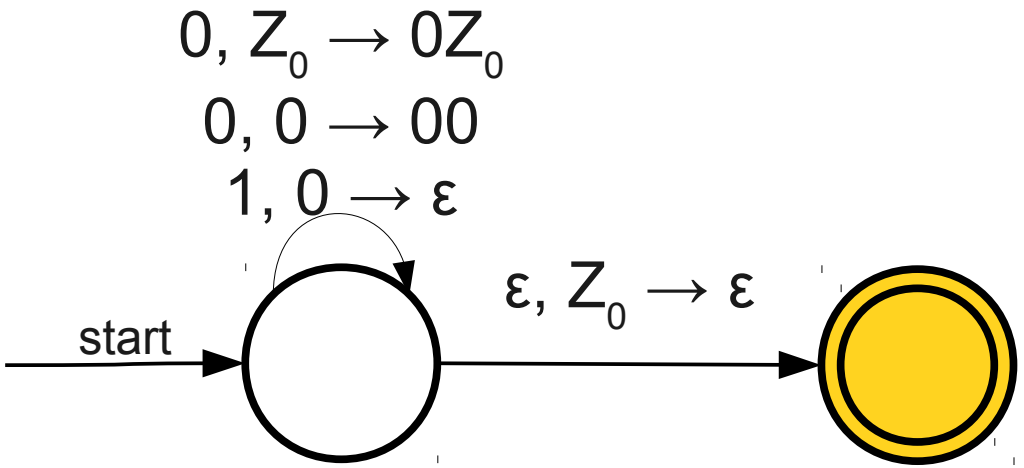
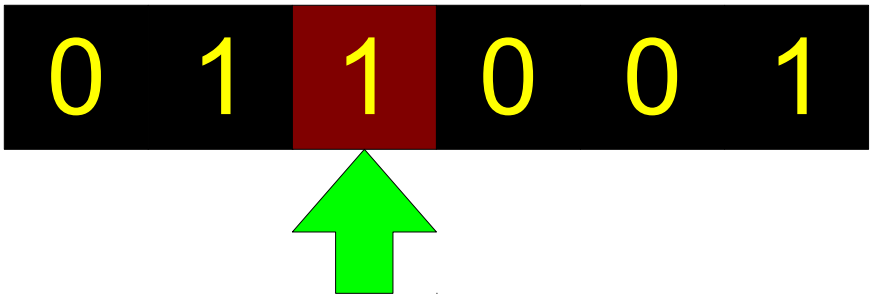




A Simple Pushdown Automaton





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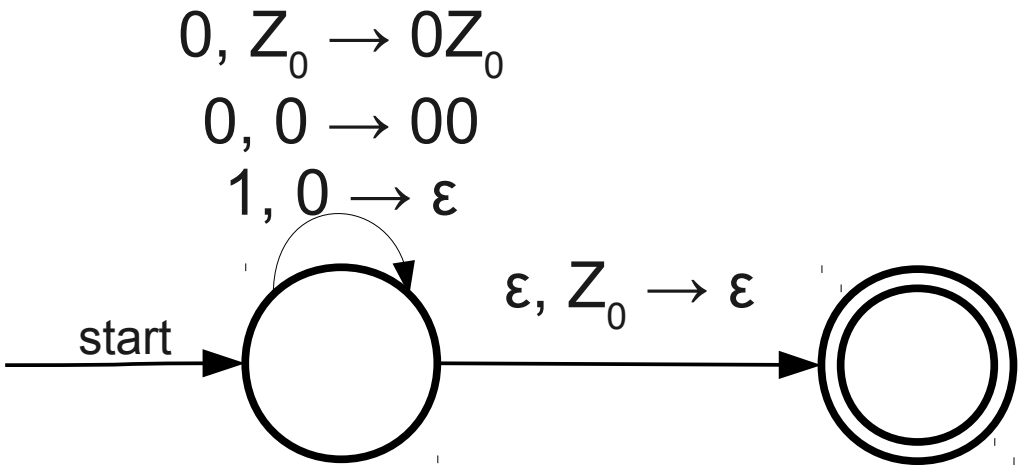

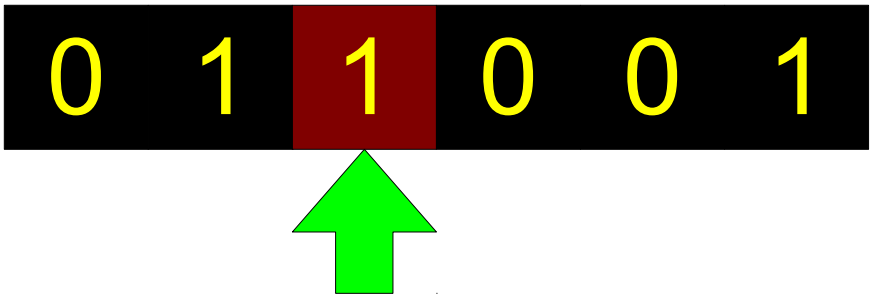

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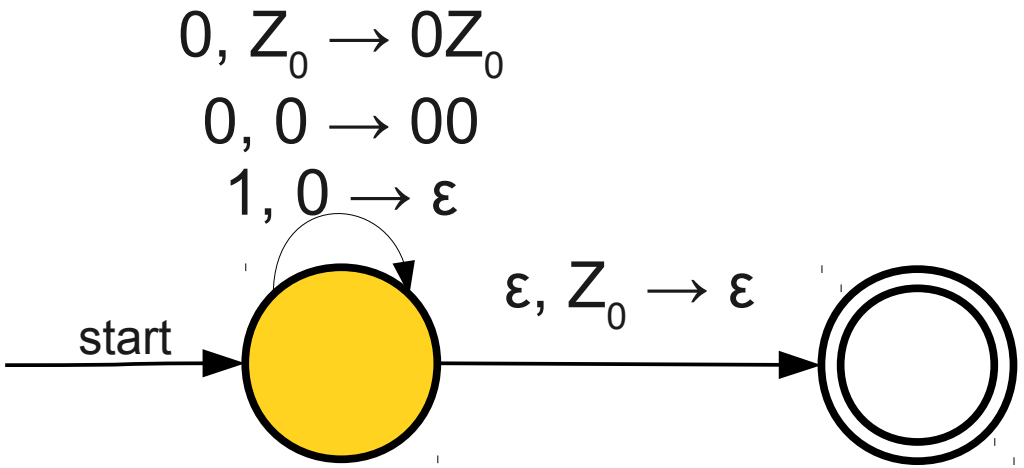

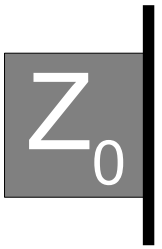

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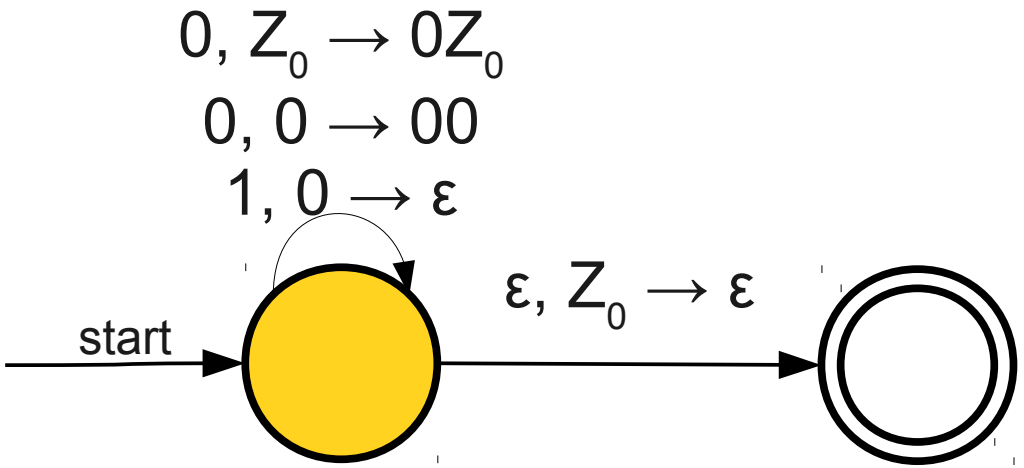

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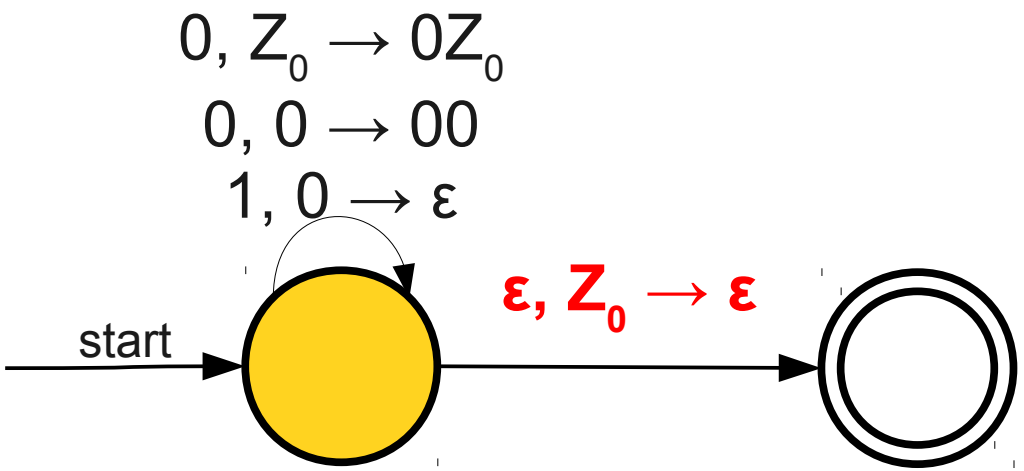

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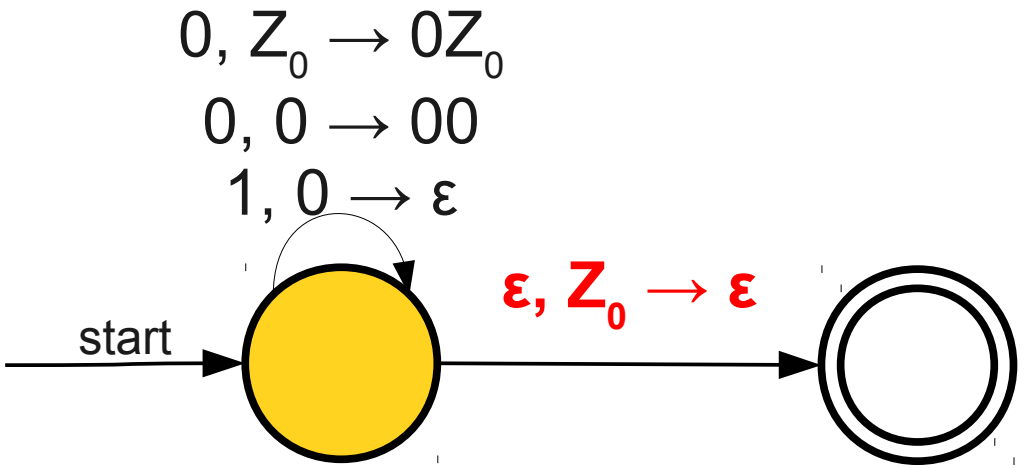

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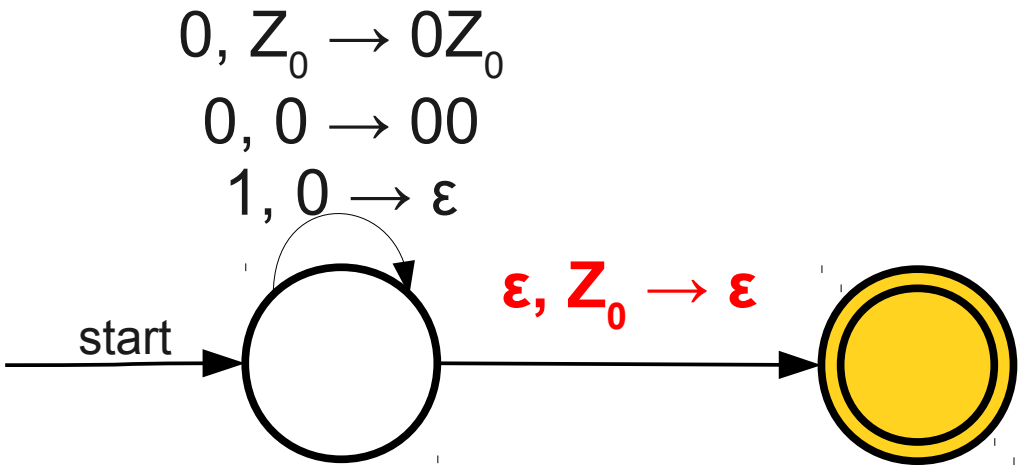

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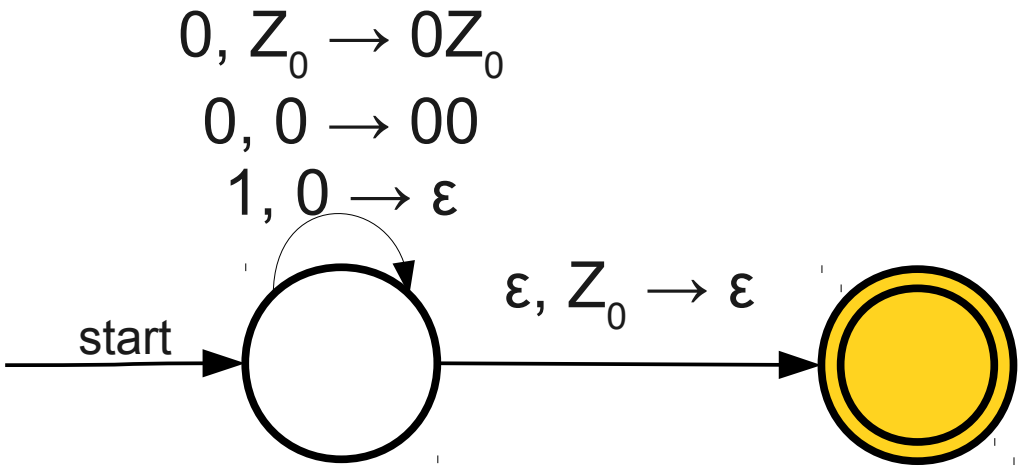

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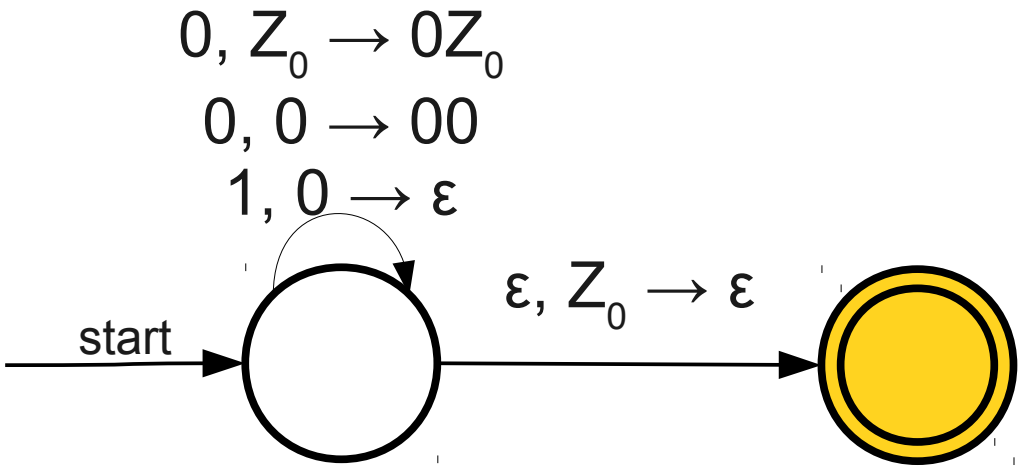

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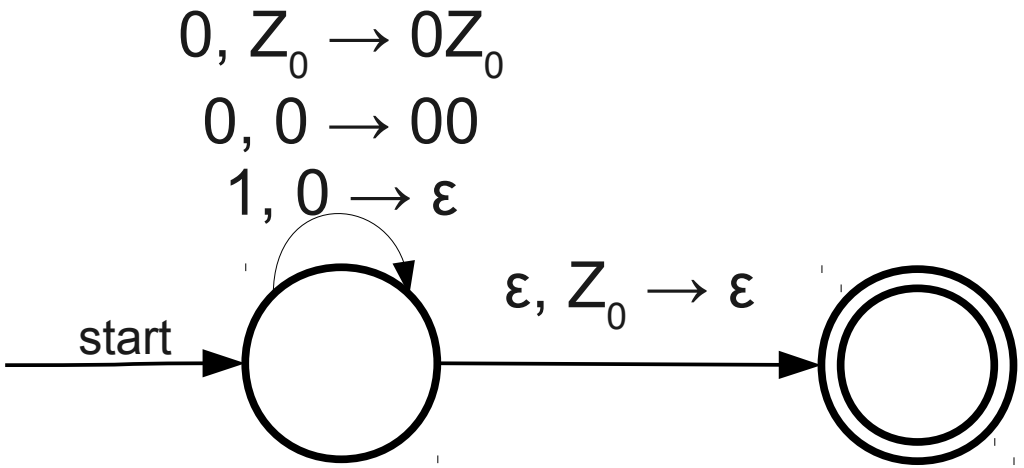

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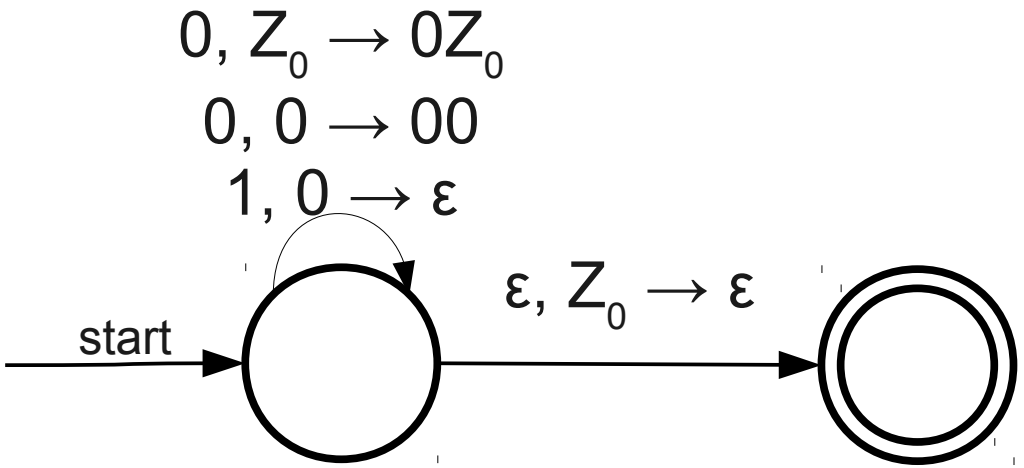

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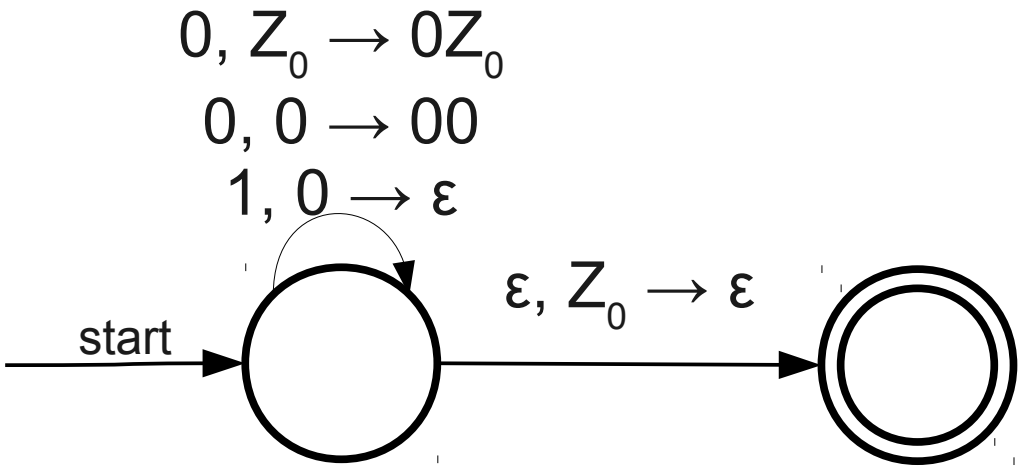

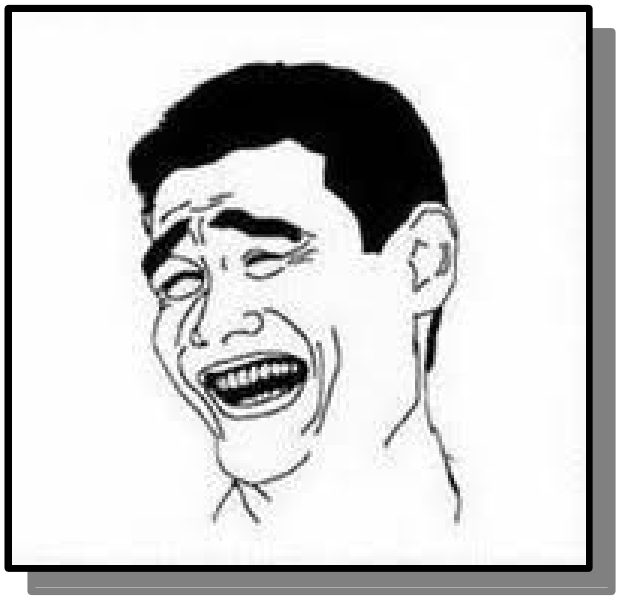

0 1 1 0 0 1

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The stack alphabet allows the PDA's stack to store extra information that can't otherwise be encoded by the input string.

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We only allow a finite set of choices to be made at each point.

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This ensures that there is a symbol on the stack that we can use to detect whether the stack has nothing else on it.

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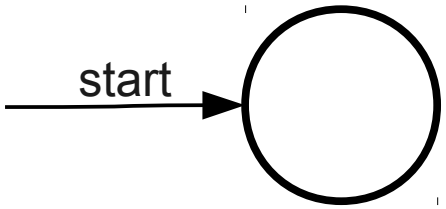
  - $Z_0 \in \Gamma$ is the **stack start symbol**, and
  - $F \subseteq Q$ is the set of **accepting states**.
- The automaton accepts if it ends in an accepting state with no input remaining.

A PDA for Palindromes

- Let Σ = {0, 1} and consider the language *PALINDROME* = { w | w is a palindrome }.
- How would we build a PDA for *PALINDROME*?
- **Idea**: Push the first half of the symbols on to the stack, then verify that the second half of the symbols match.
- **Nondeterministically** guess when we've read half of the symbols.
- This handles even-length strings; we'll see a cute trick to handle odd-length strings in a minute.

A PDA for Palindromes



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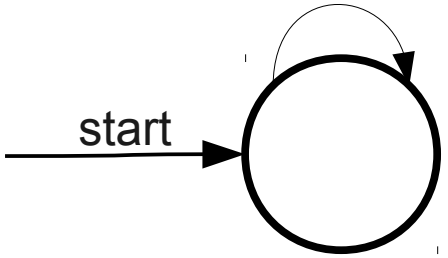
$0, Z_0 \rightarrow 0Z_0$



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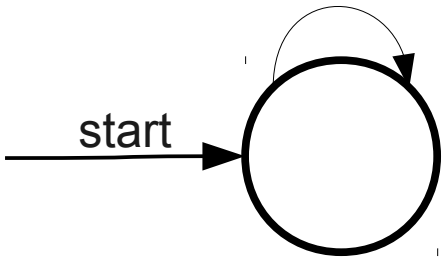


A PDA for Palindromes

$0, Z_0 \rightarrow 0Z_0$

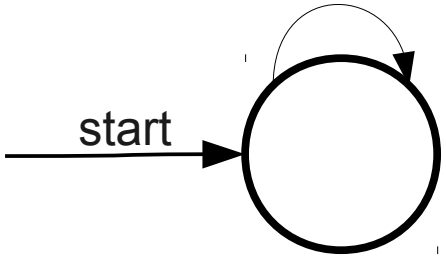
$0, 0 \rightarrow 00$

$0, 1 \rightarrow 01$

$1, Z_0 \rightarrow 1Z_0$

$1, 0 \rightarrow 10$

$1, 1 \rightarrow 11$



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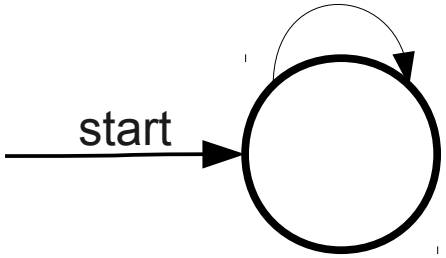

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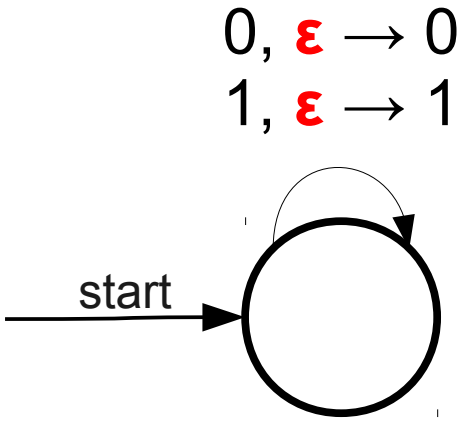

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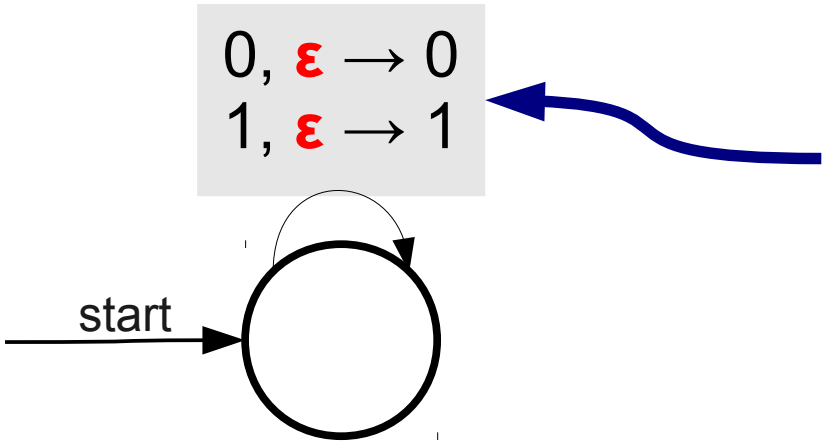

This transition indicates that the transition does not pop anything from the stack. It just pushes on a new symbol instead.

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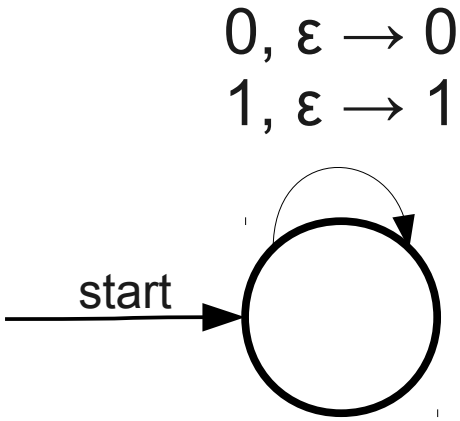

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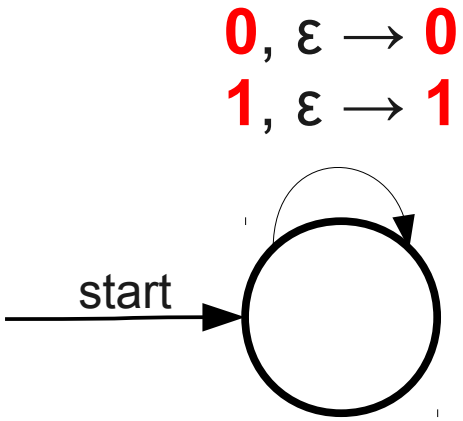

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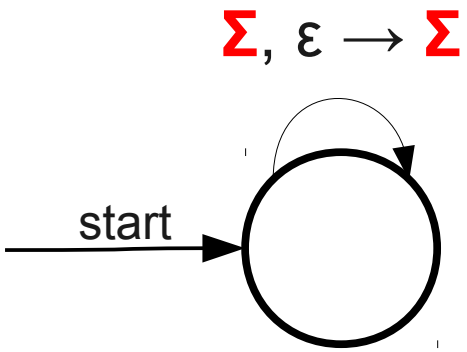

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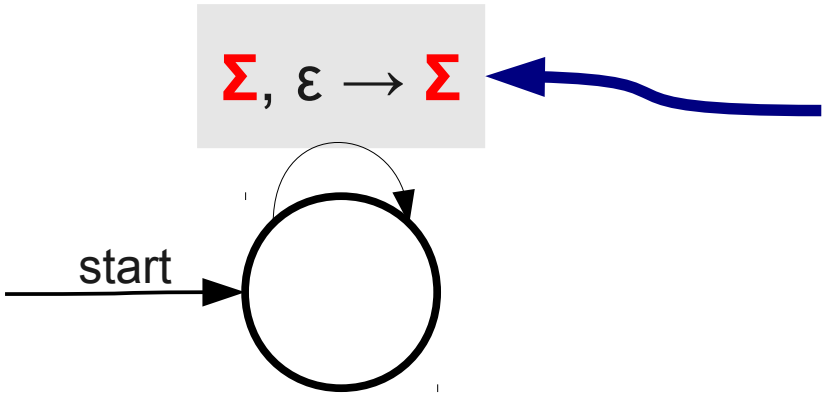

The Σ here refers to the same symbol in both contexts. It is a shorthand for "treat any symbol in Σ this way"

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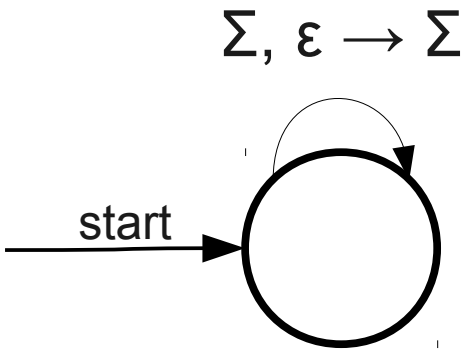

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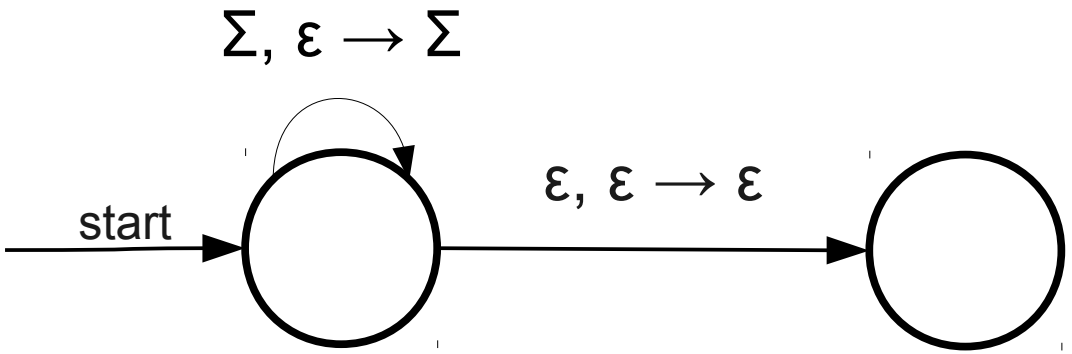

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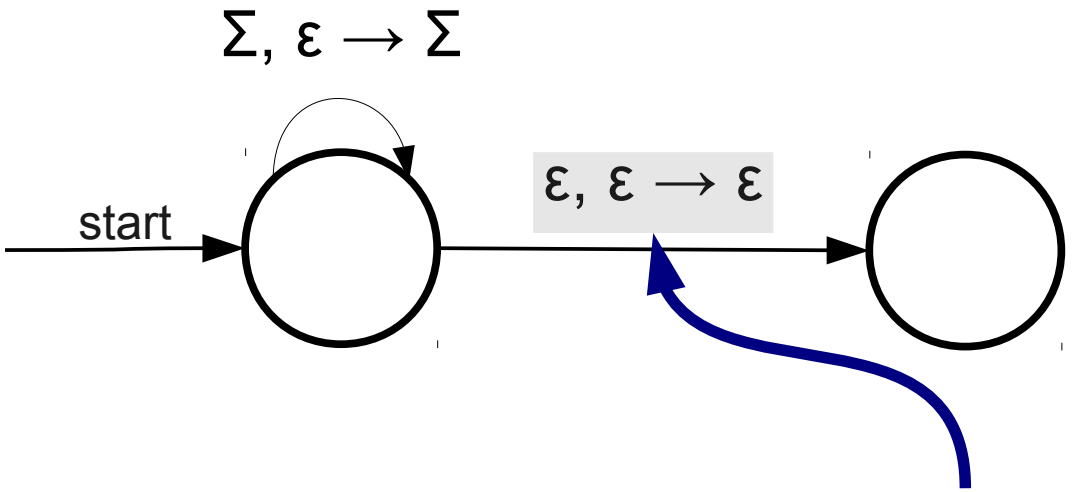

This transition means "don't consume any input, don't change the top of the stack, and don't add anything to a stack. It's the equivalent of an ε-transition in an NFA.

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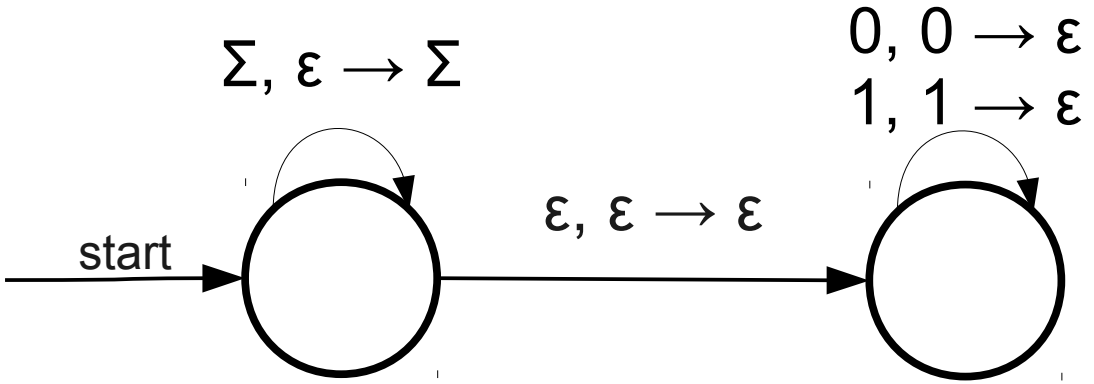

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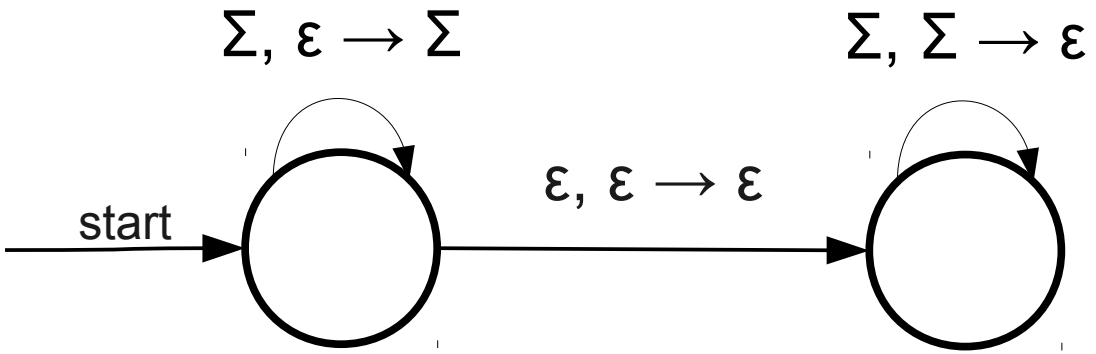

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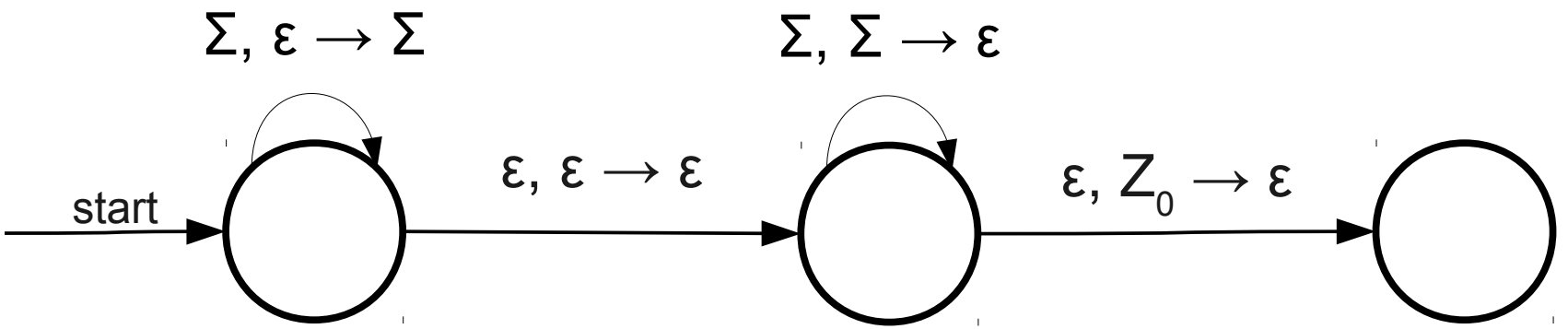

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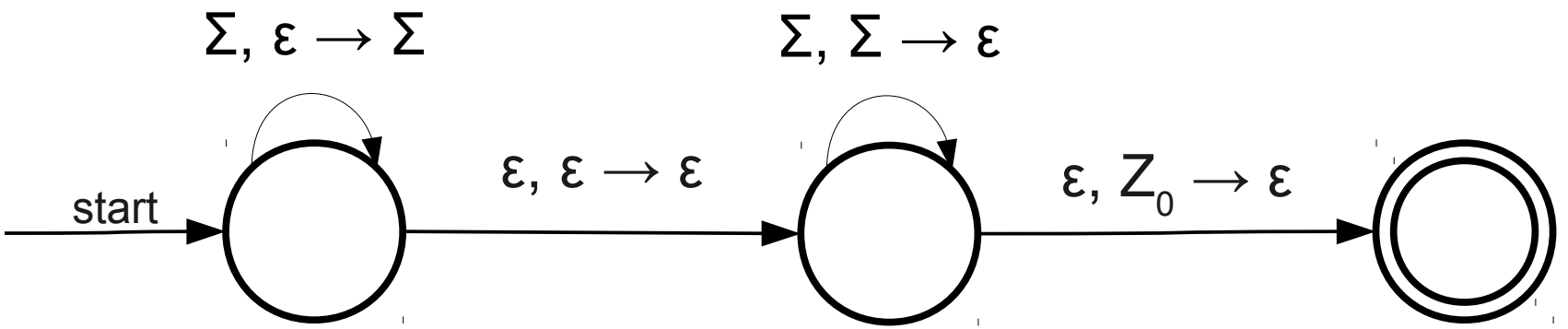

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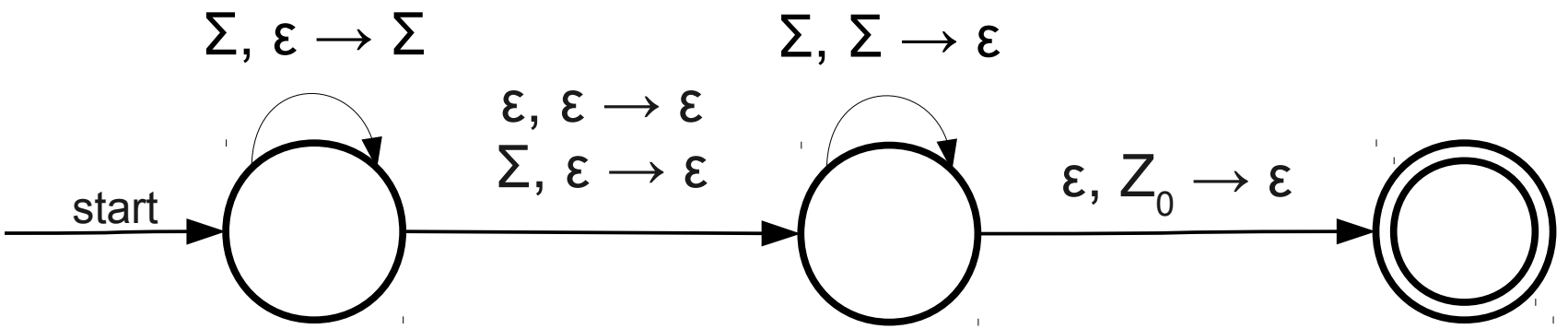
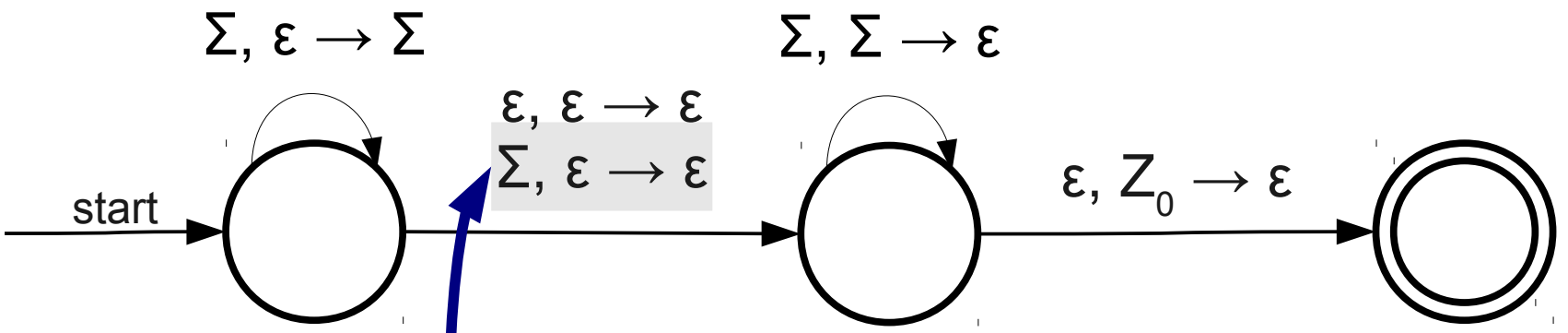
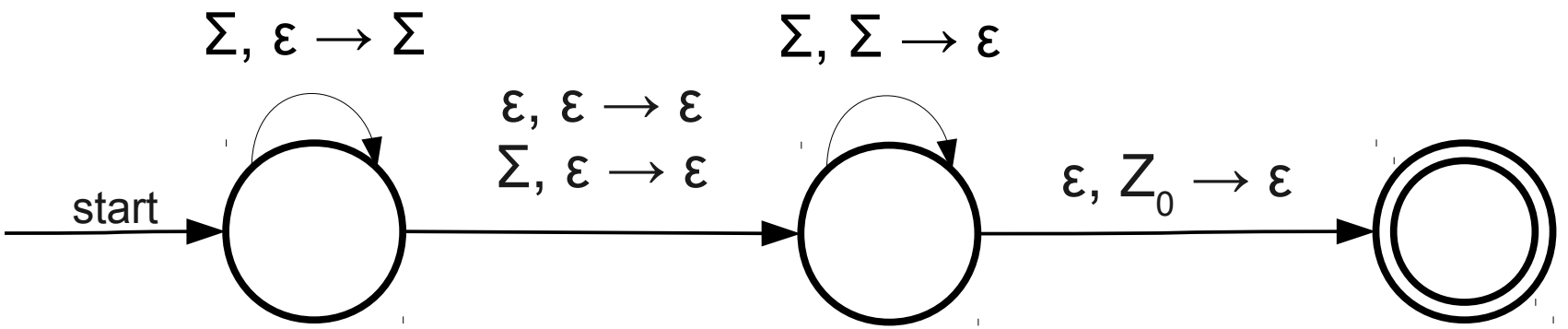


A PDA for Palindromes



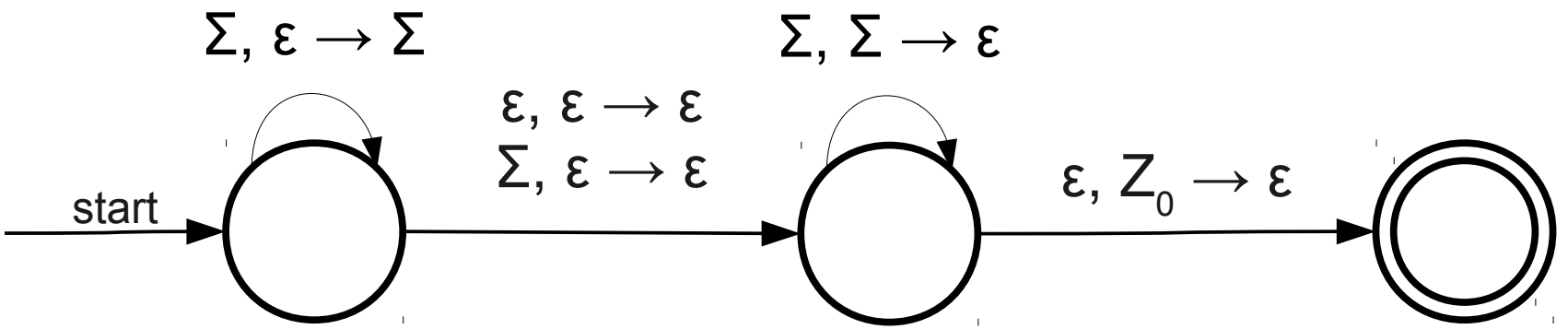
This transition lets us consume one character before we start matching what we just saw. This lets us match odd-length palindromes

A PDA for Palindromes



0 1 1 1 1 0

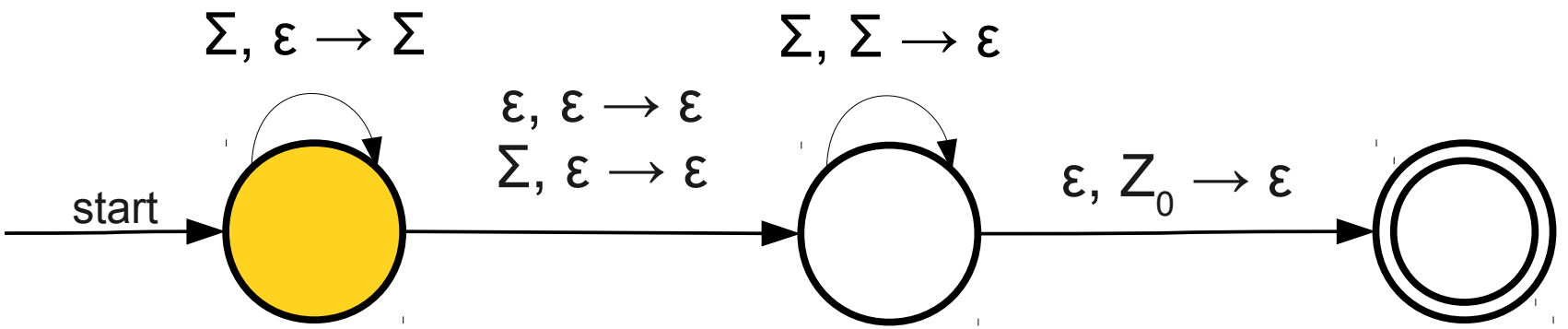
A PDA for Palindromes



0	1	1	1	1	0

Z_0

A PDA for Palindromes



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Z_0

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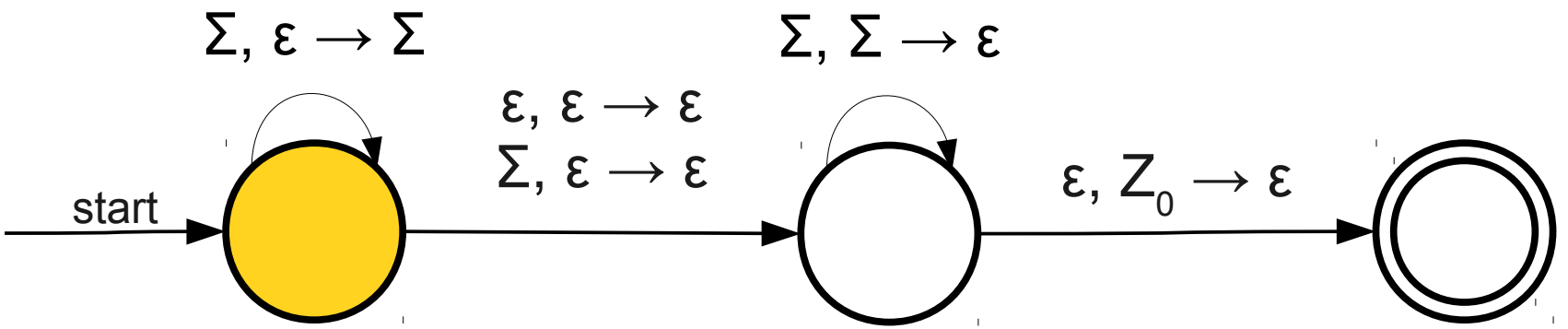

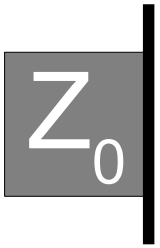

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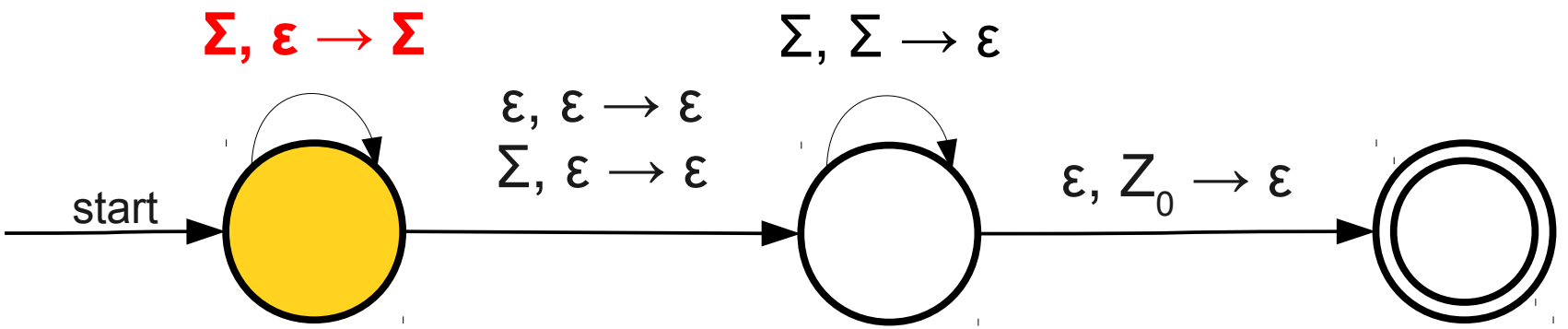

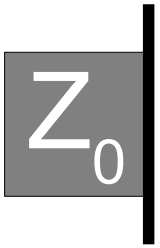

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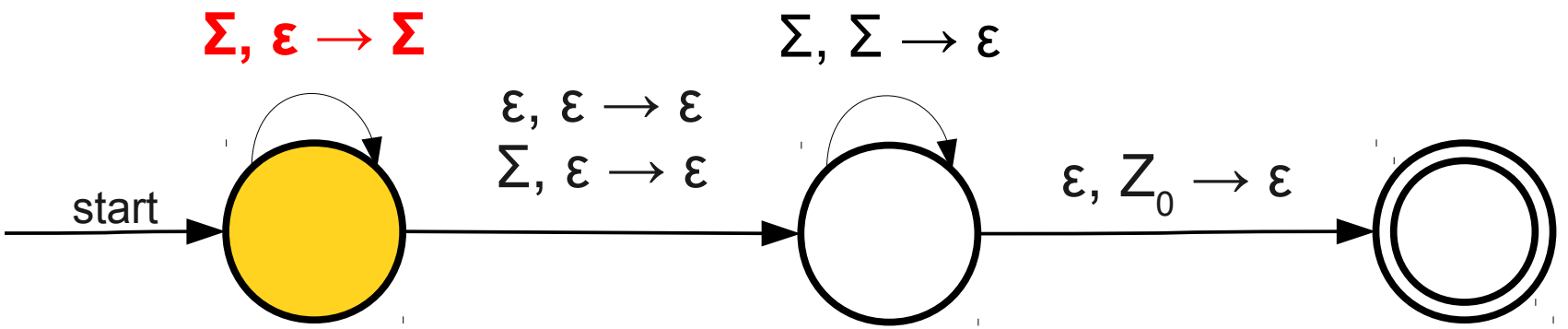

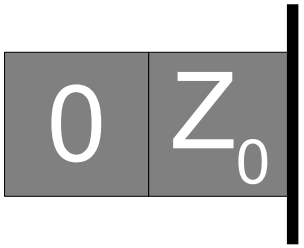

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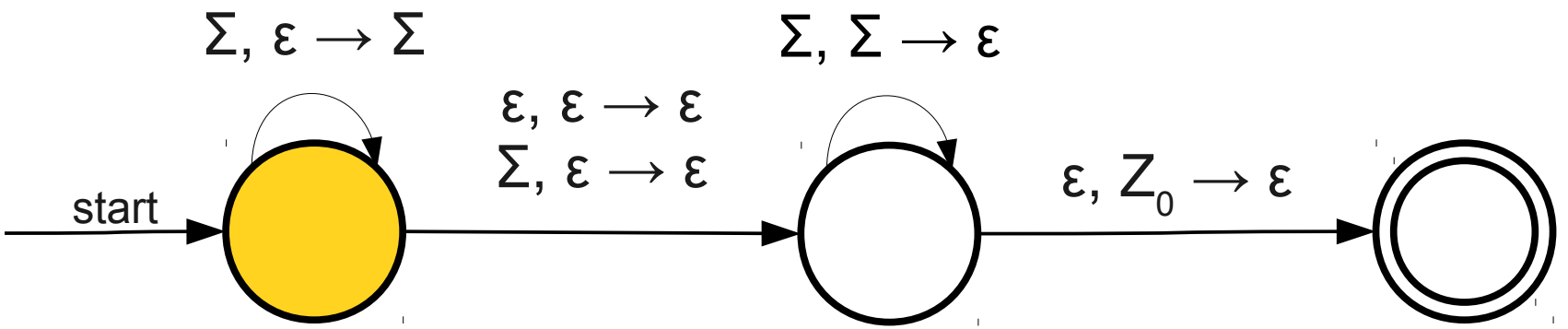

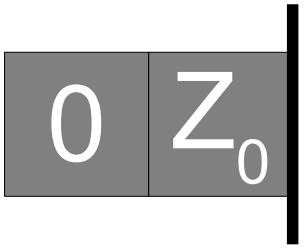

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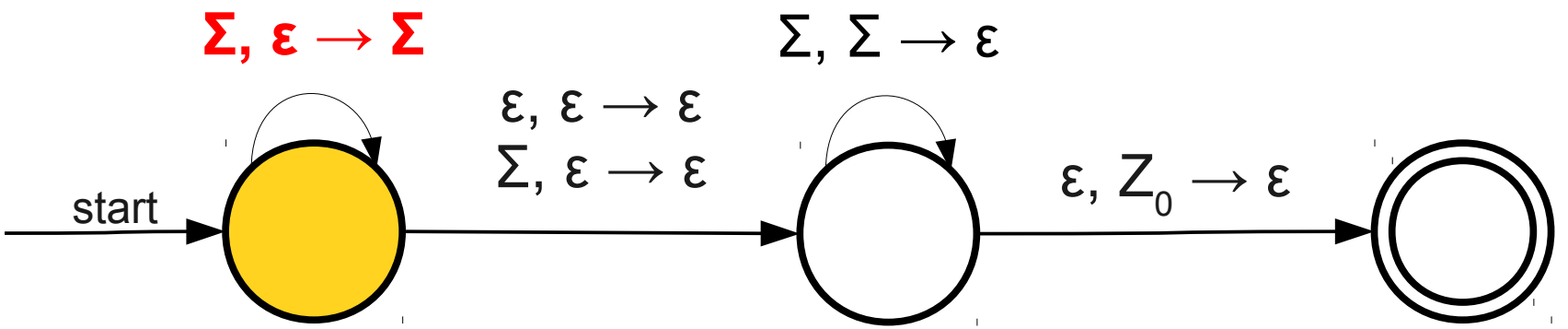

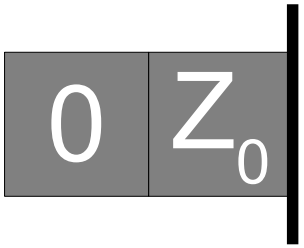

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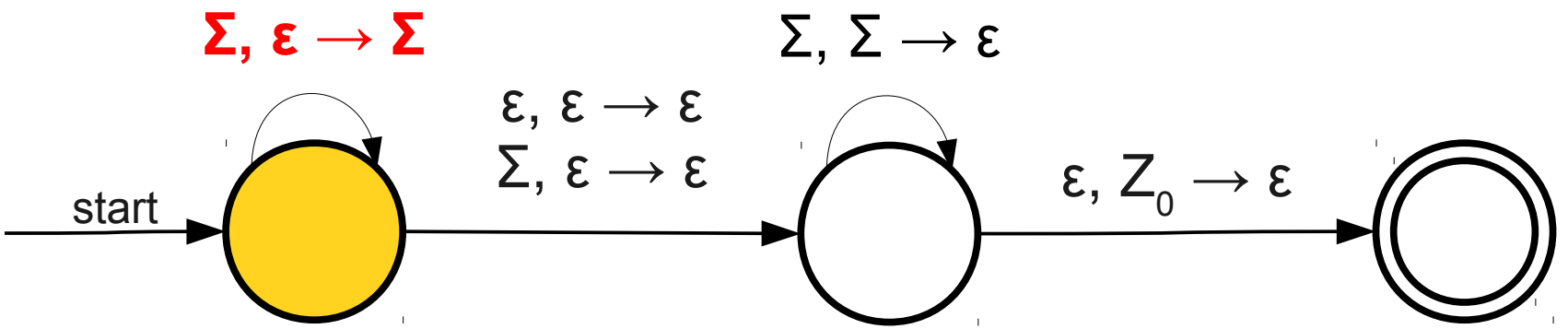

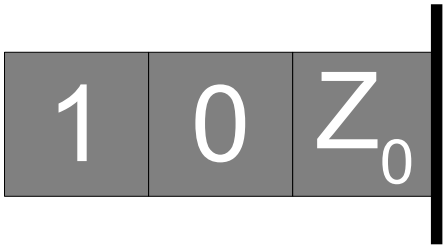

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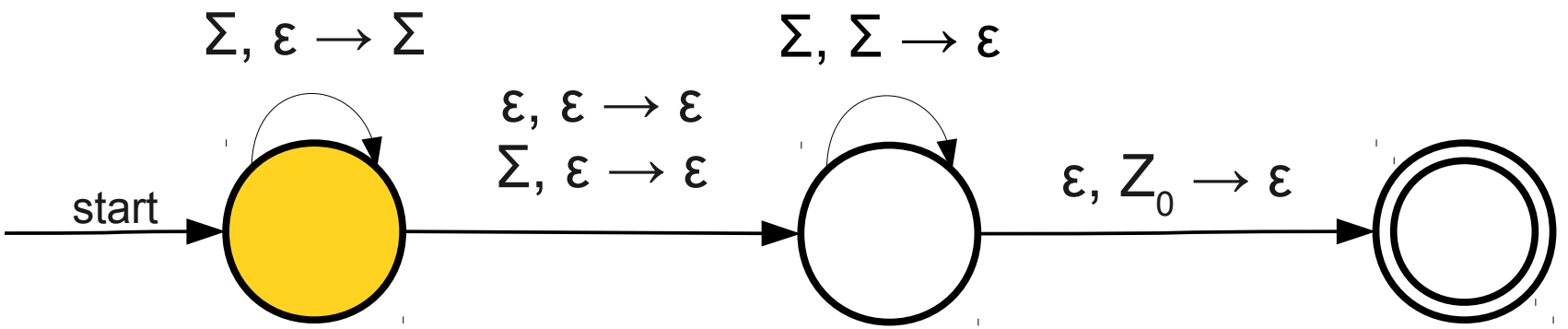

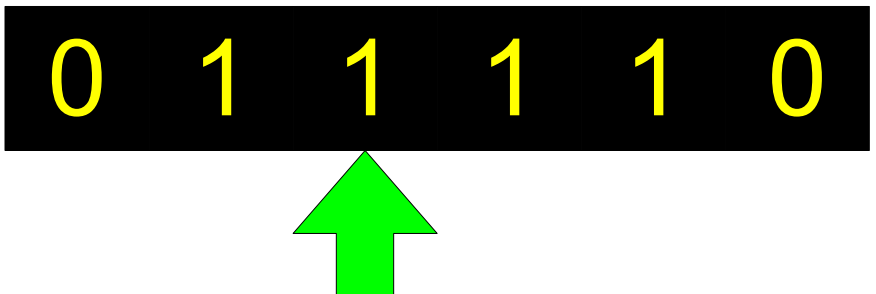

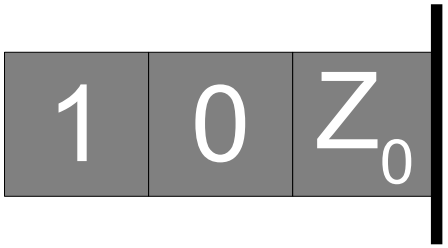

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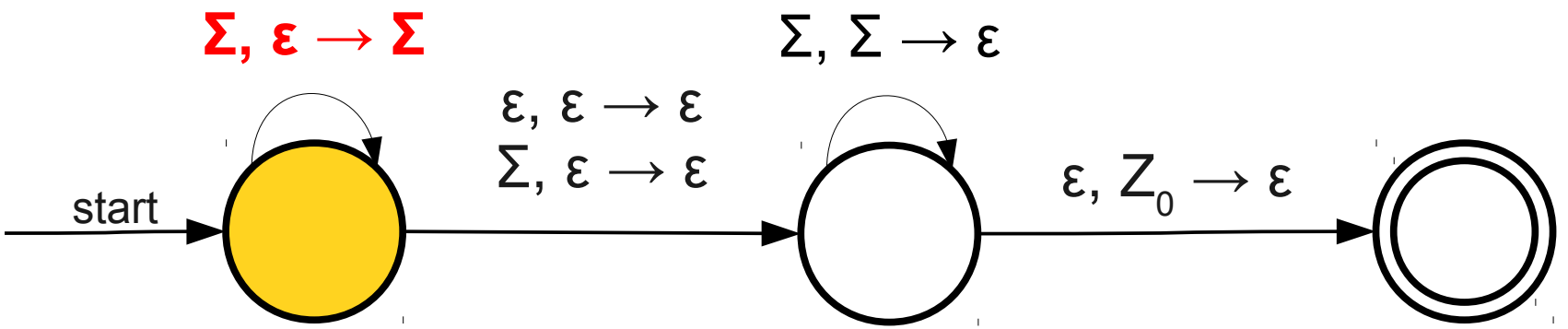

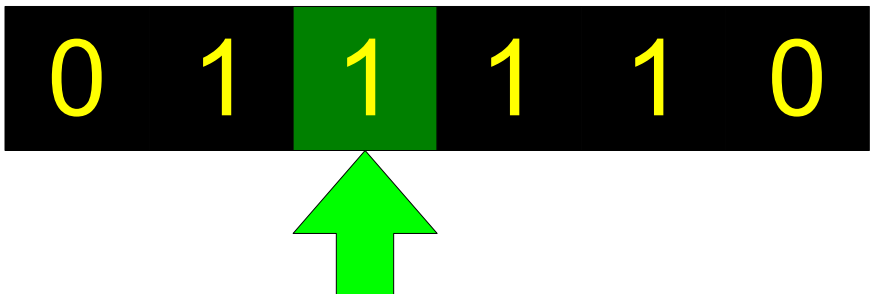

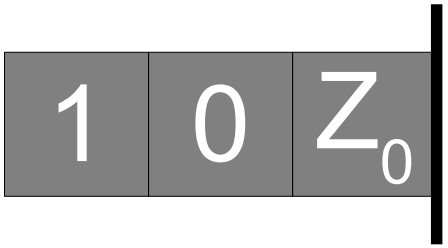

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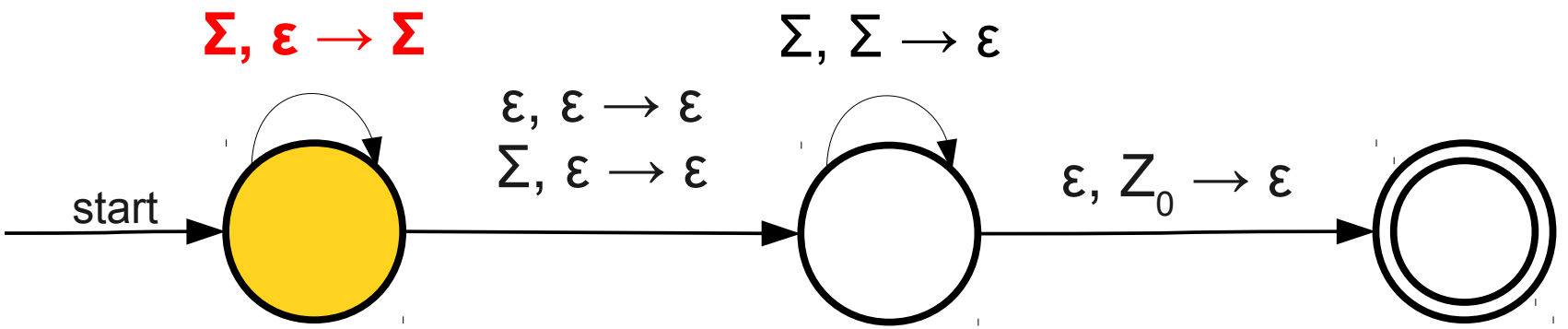

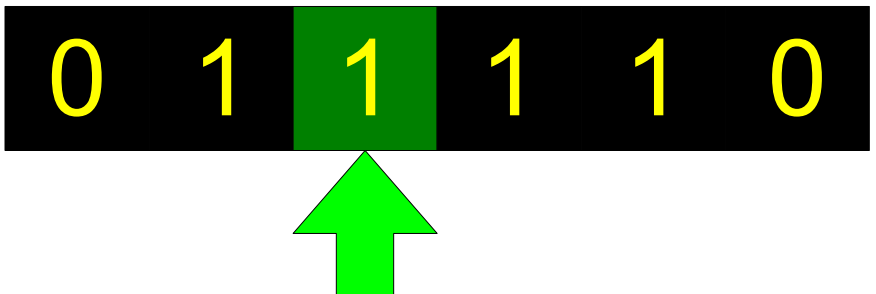

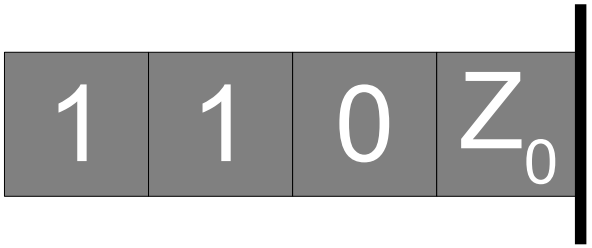

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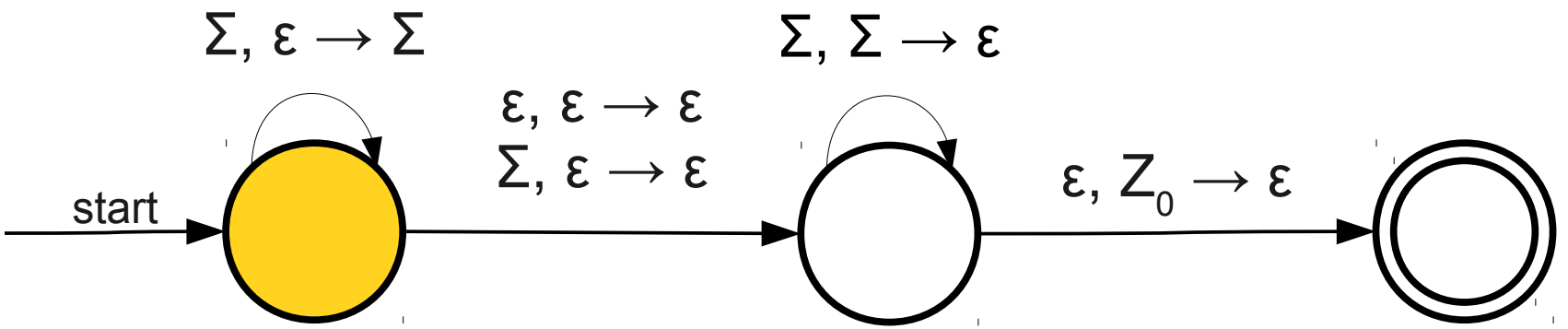

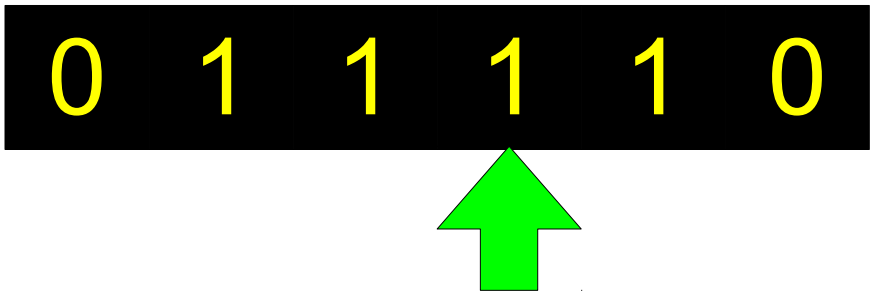

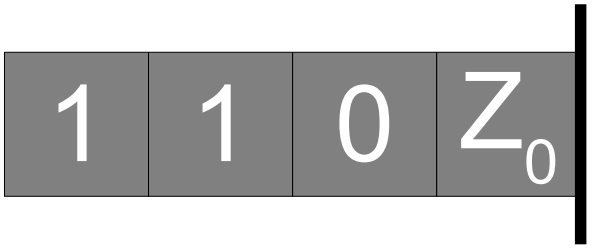

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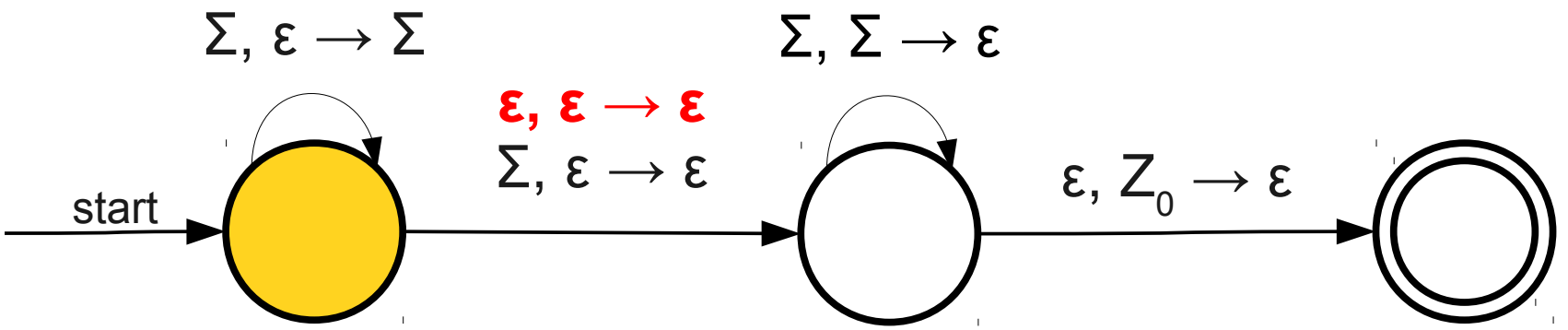

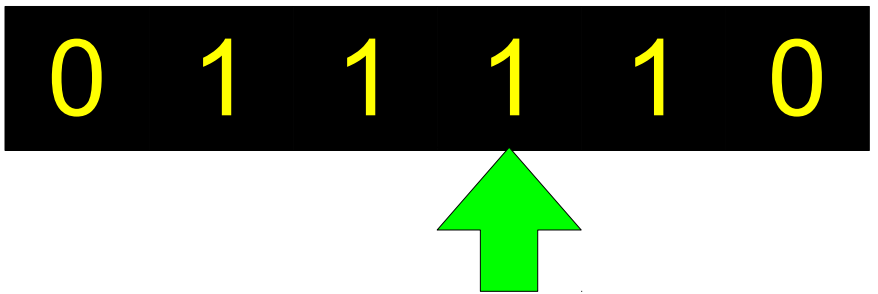

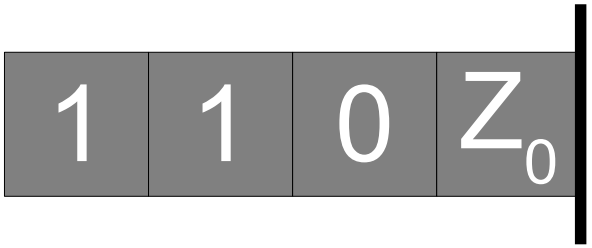

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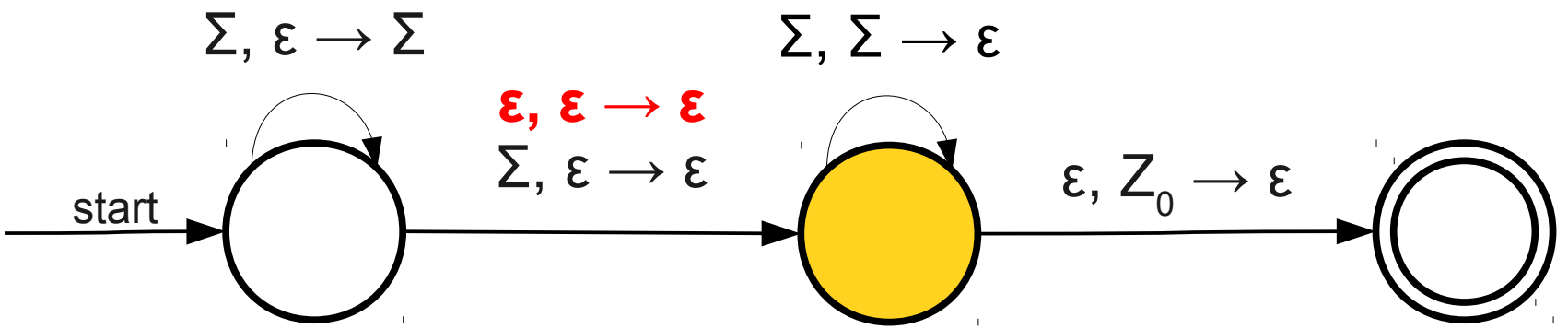

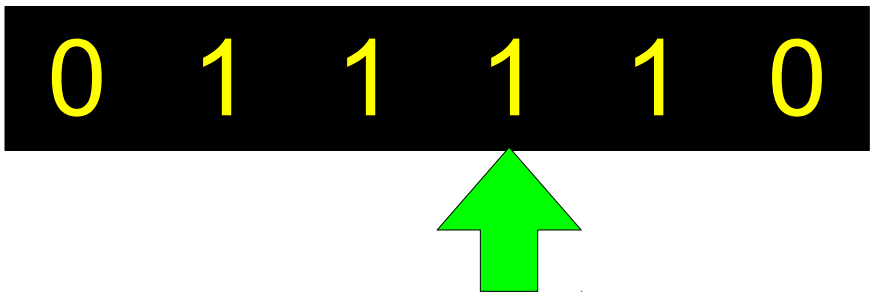

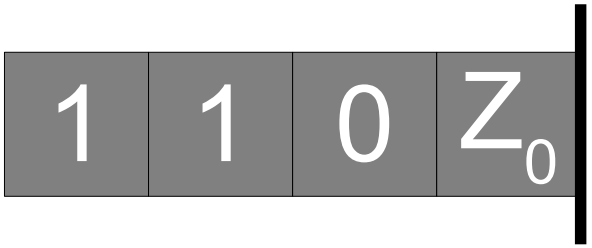

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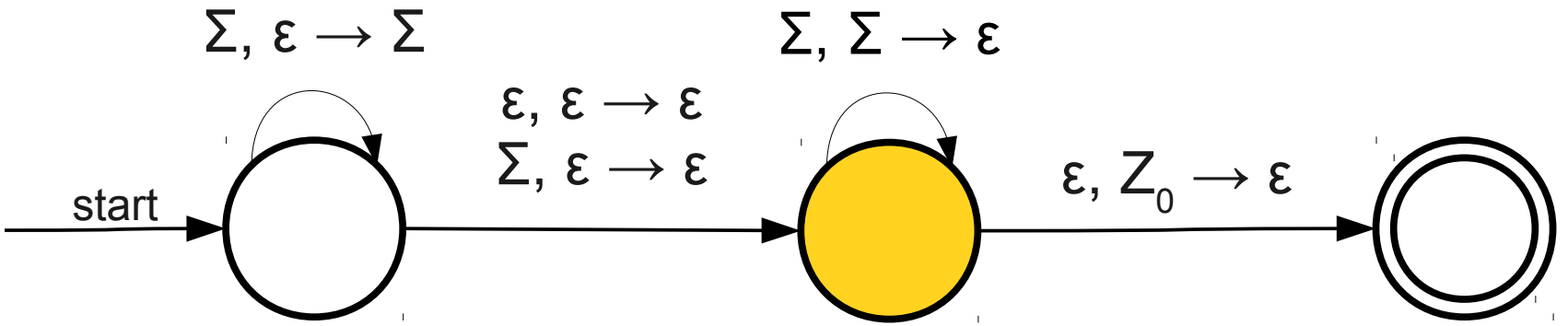

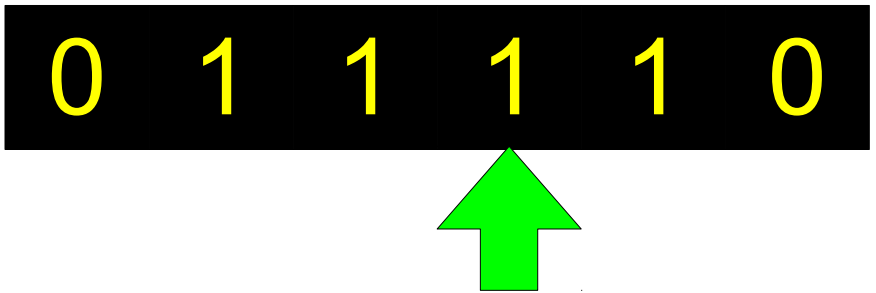

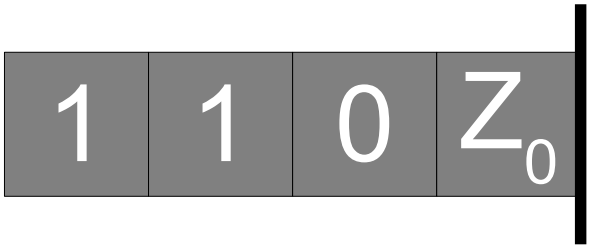

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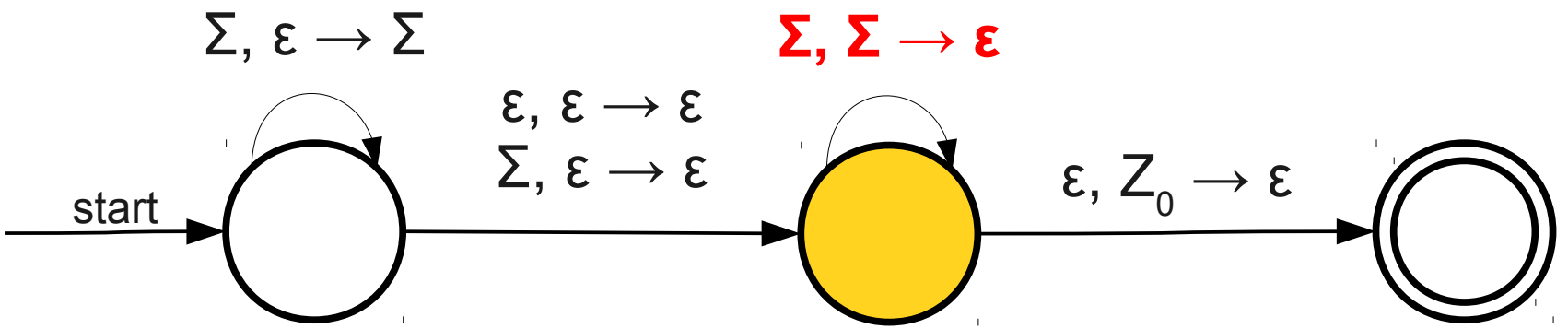

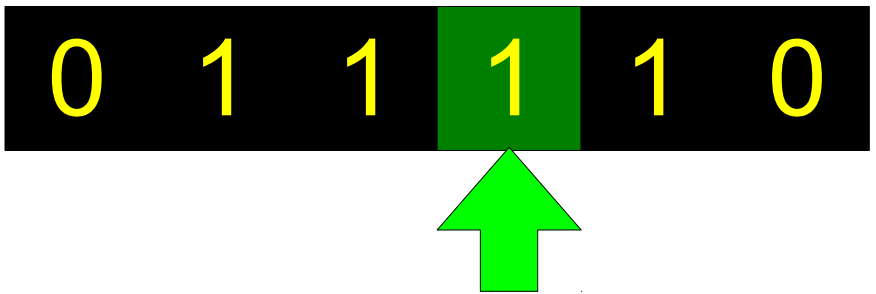

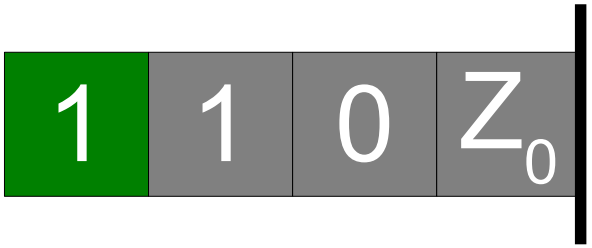

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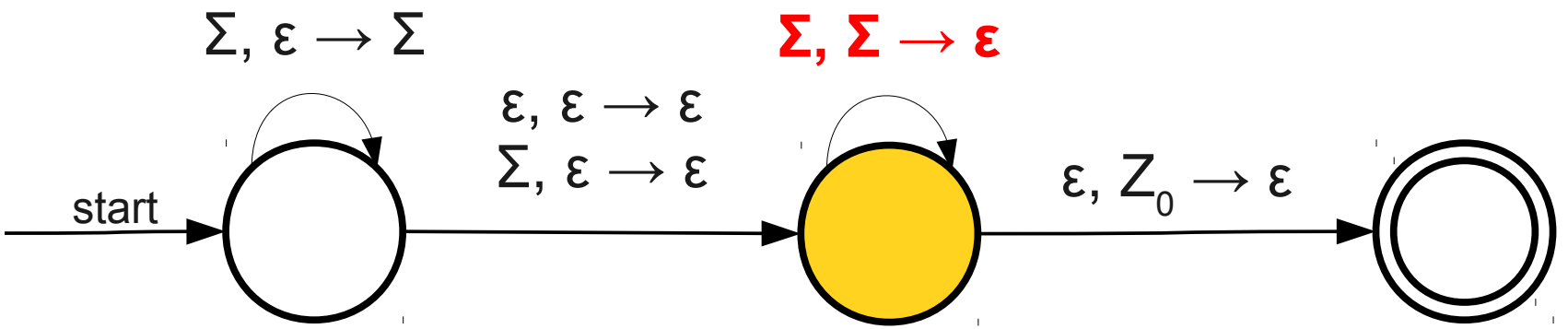

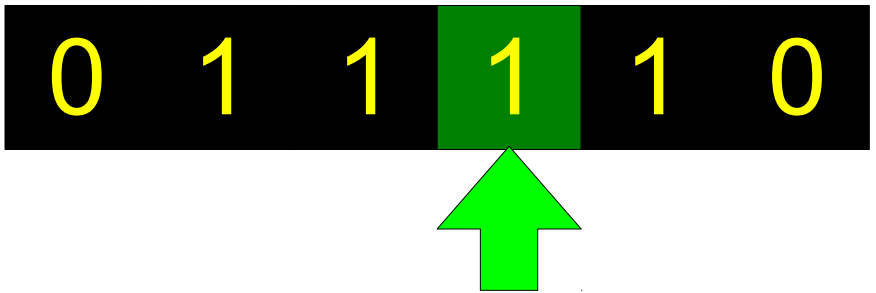

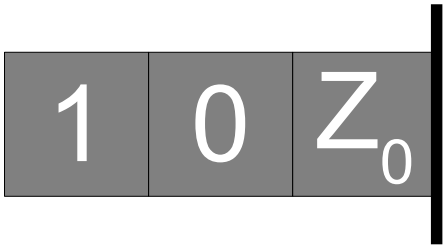

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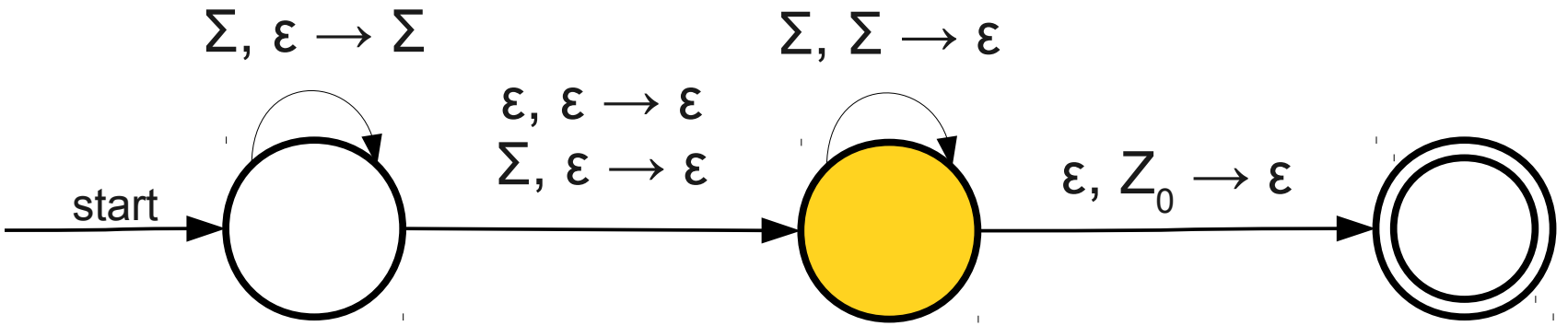

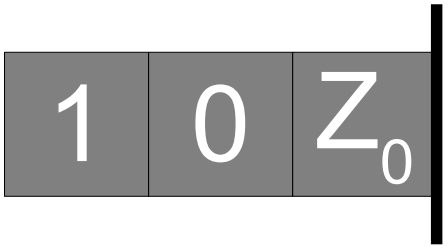

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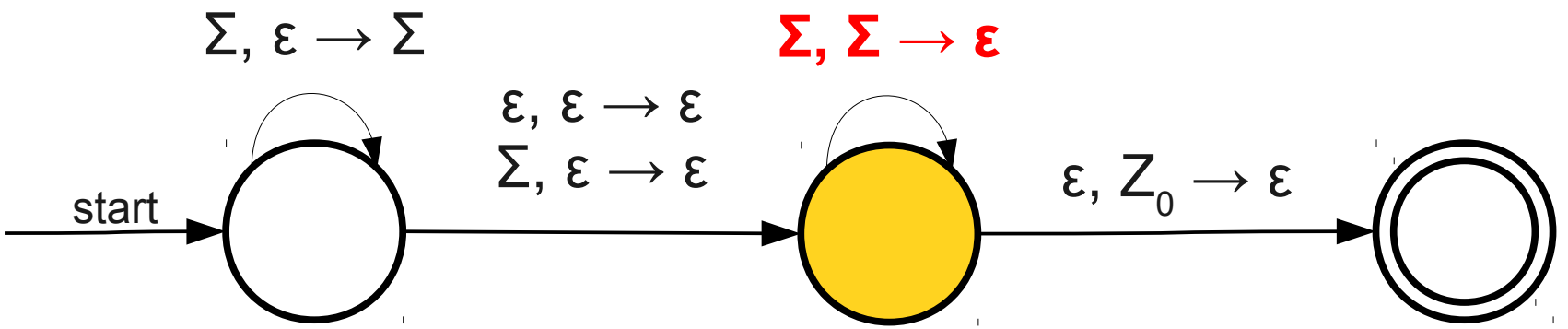

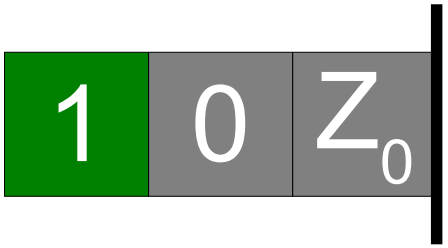

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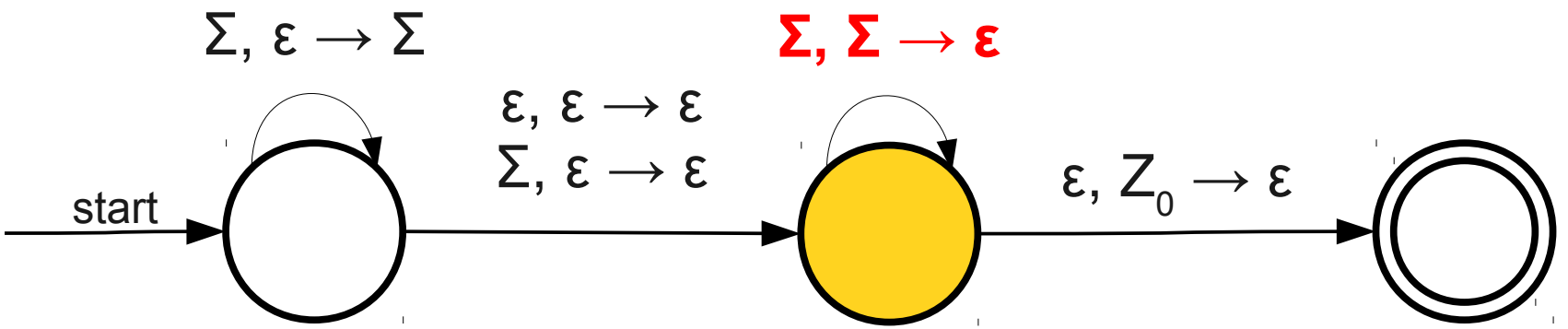

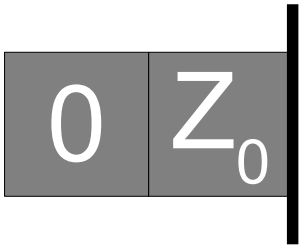

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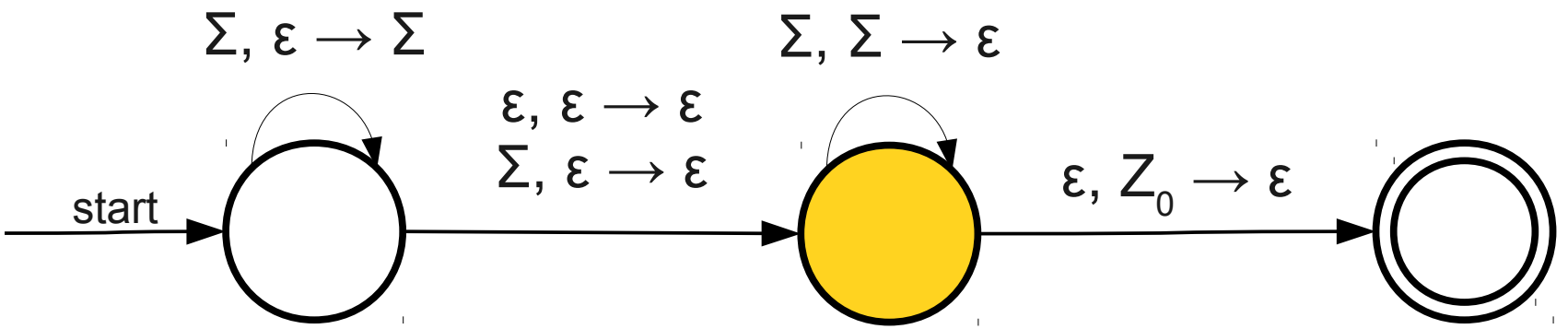

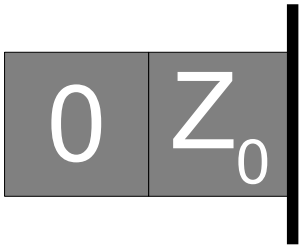

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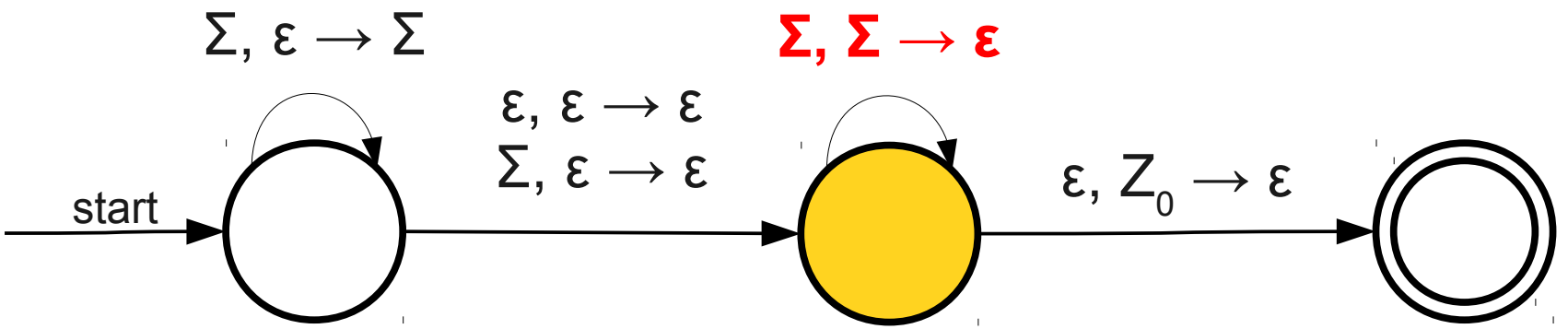
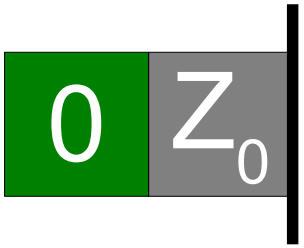
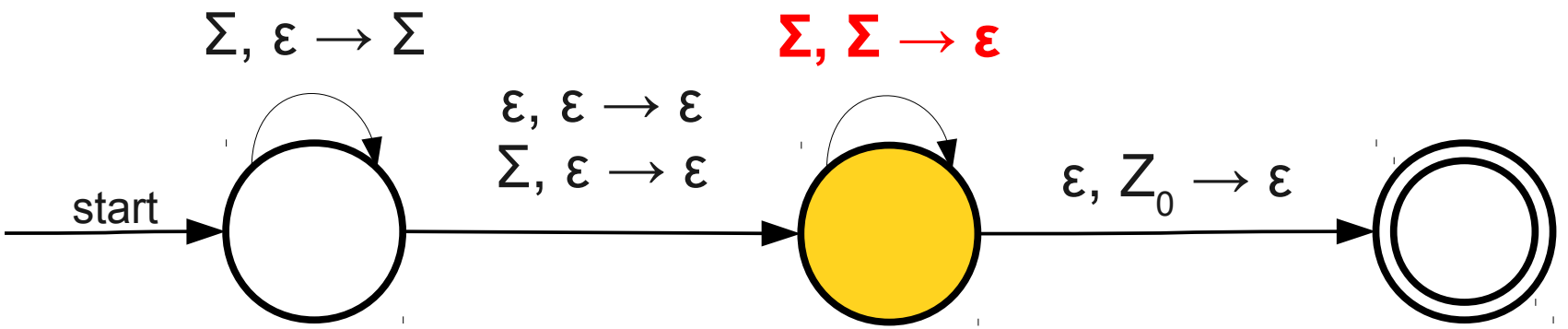
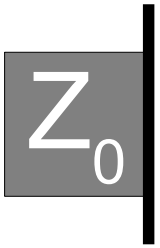




A PDA for Palindromes





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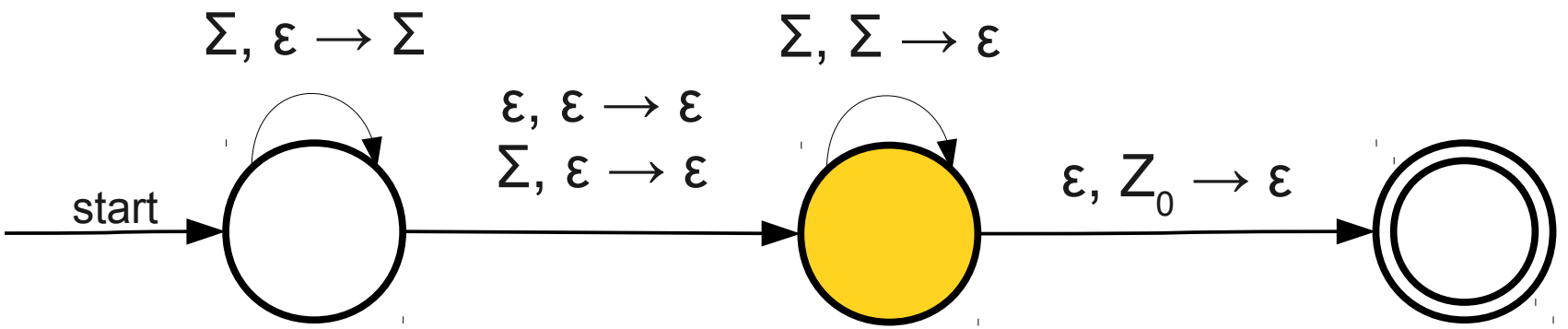

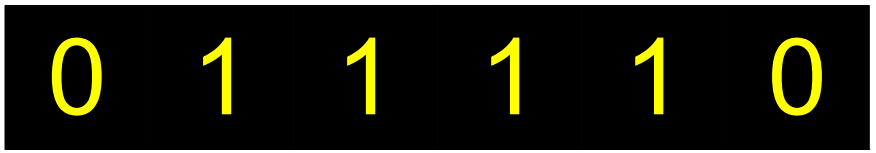

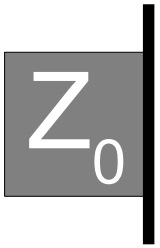

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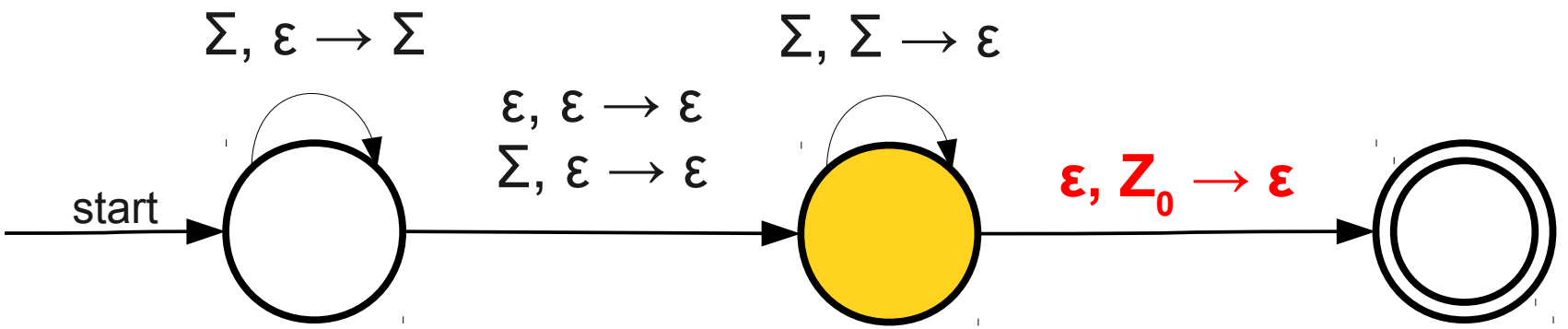

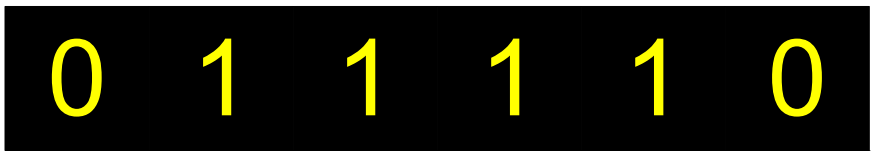

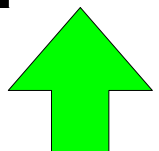

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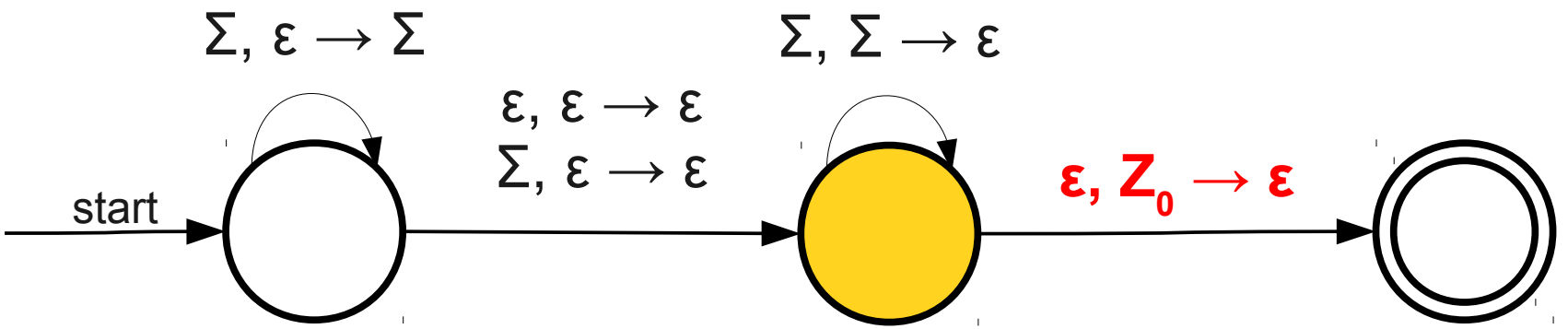

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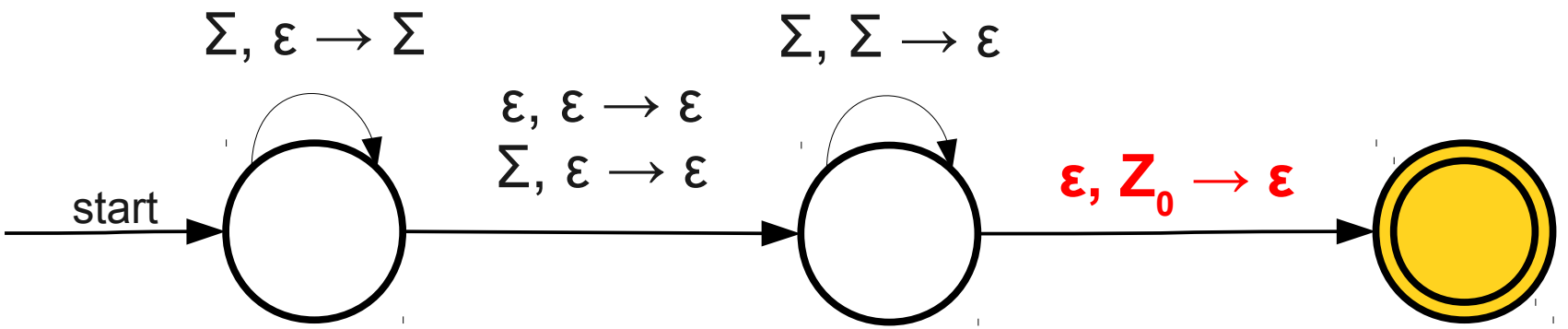

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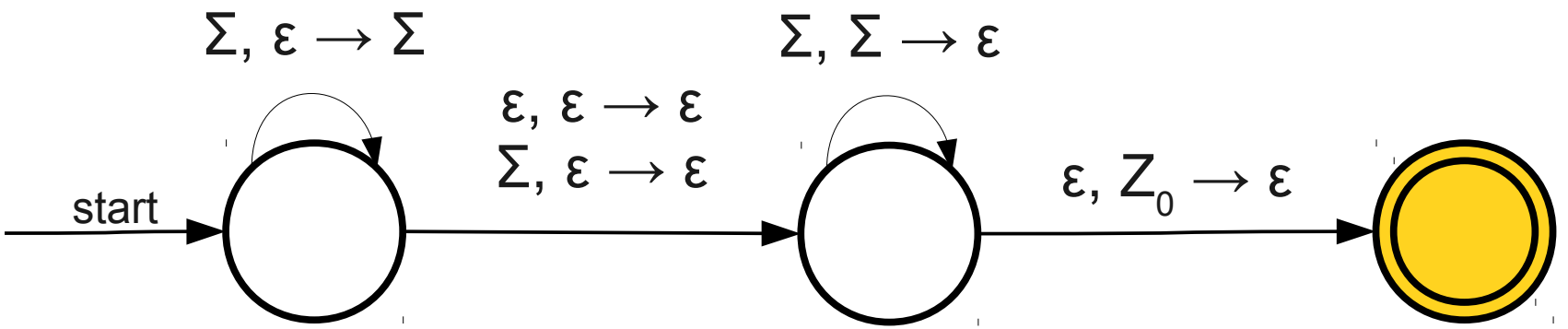

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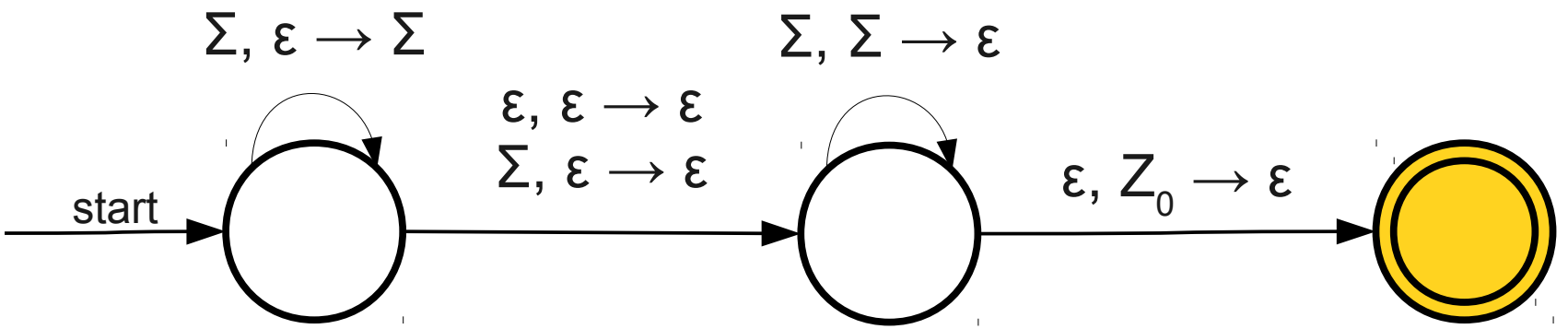

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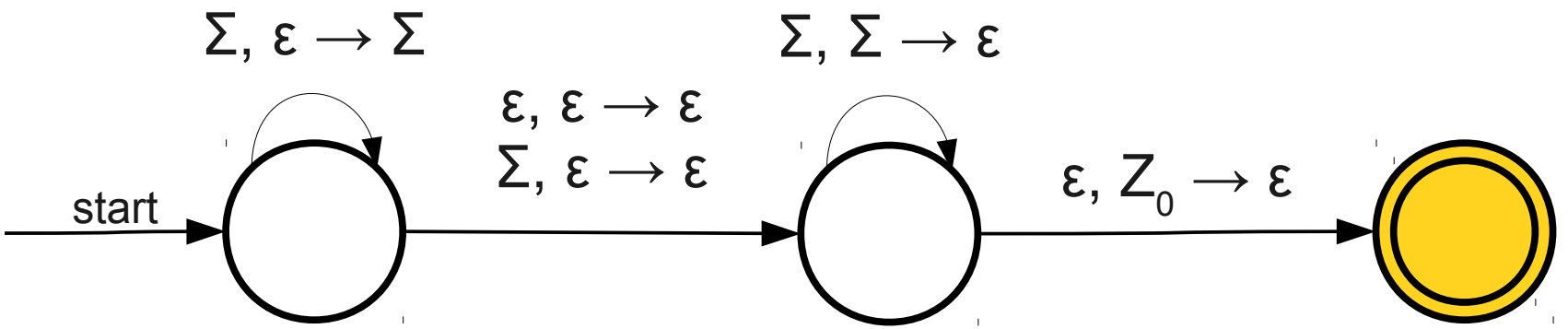

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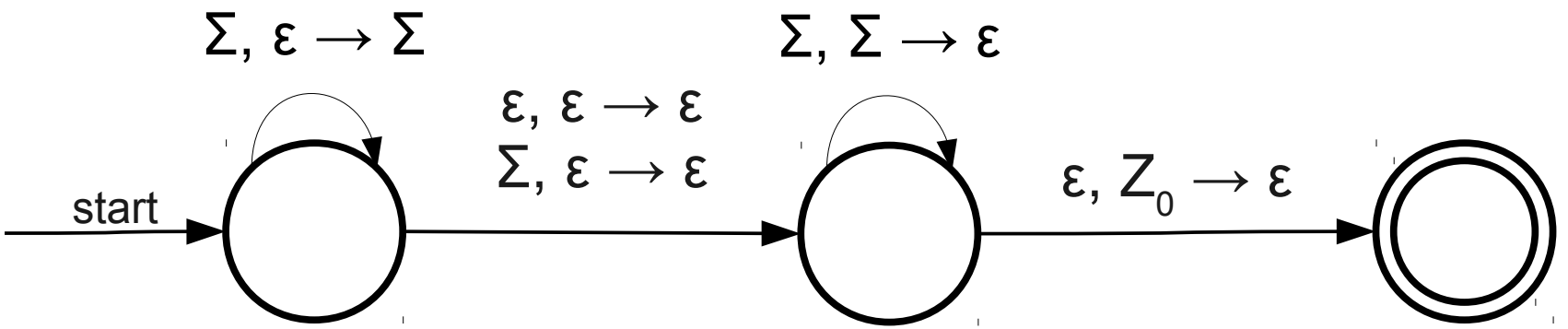

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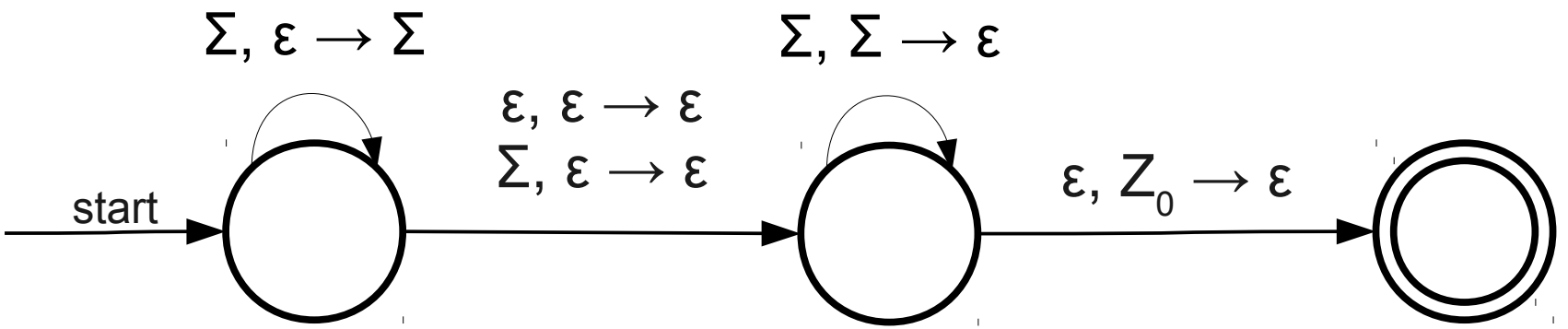

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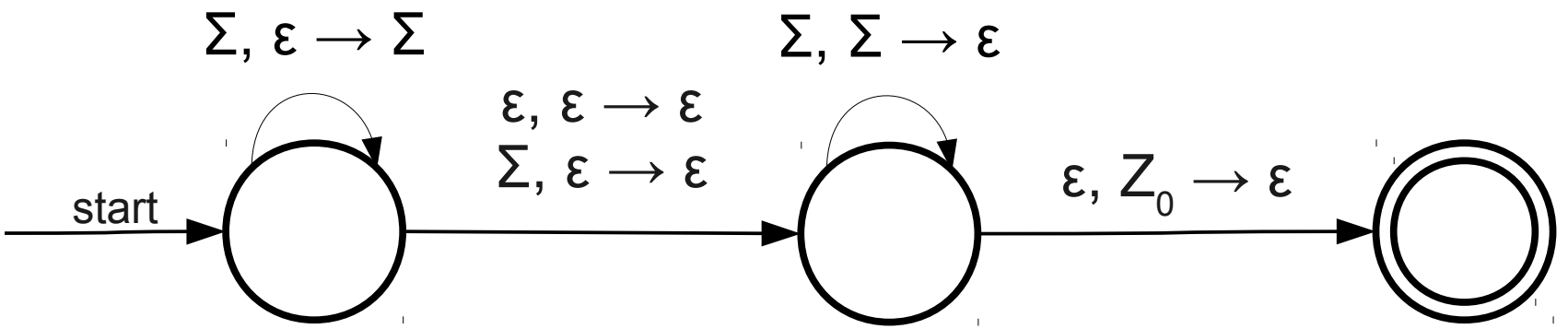

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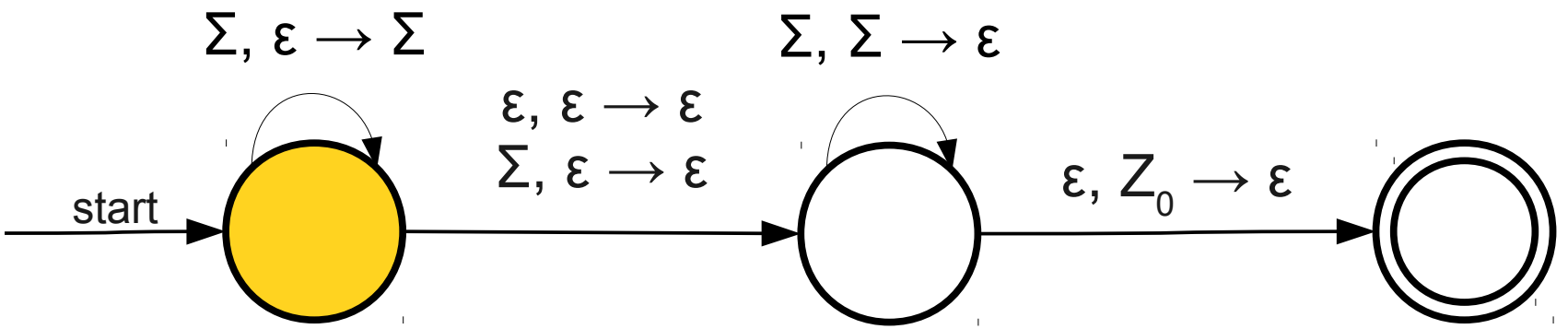

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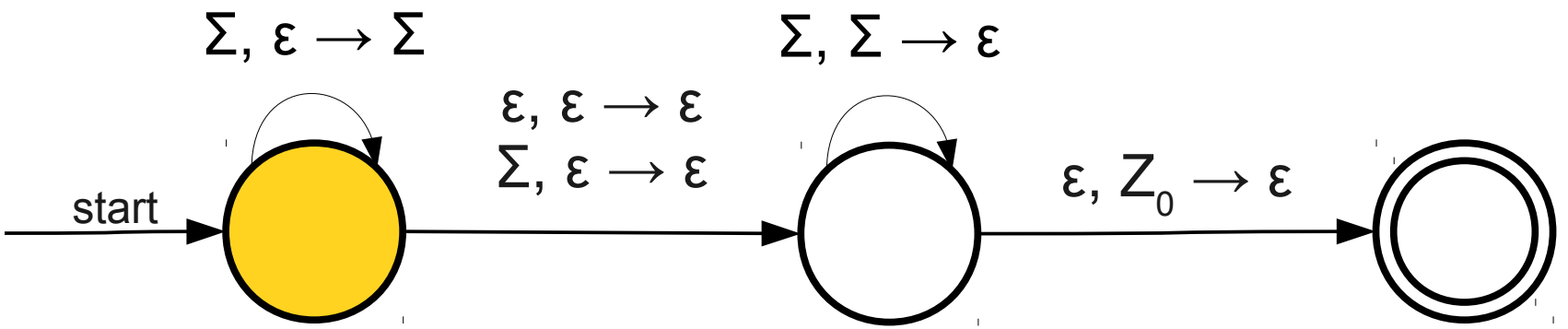

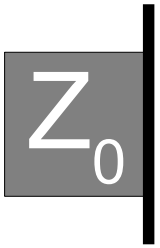

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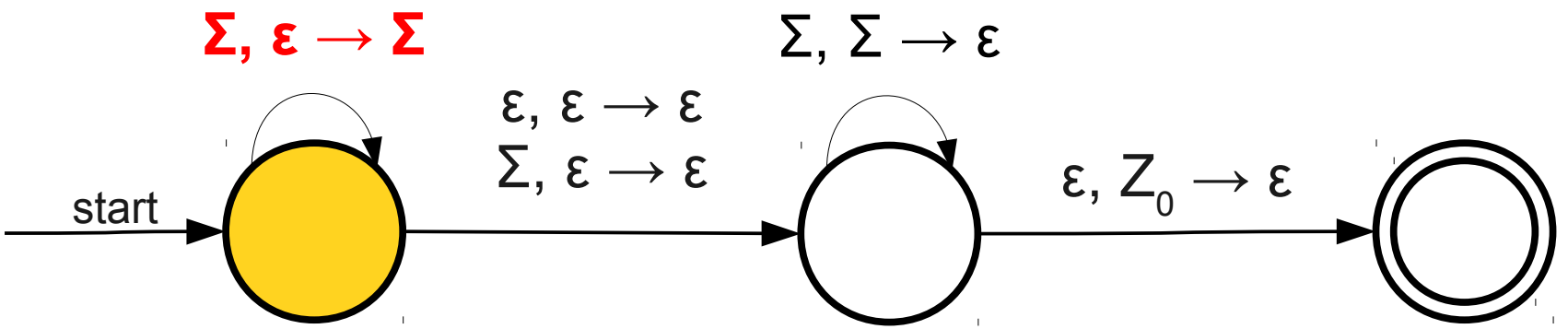

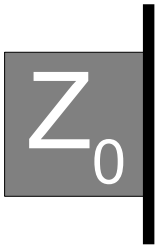

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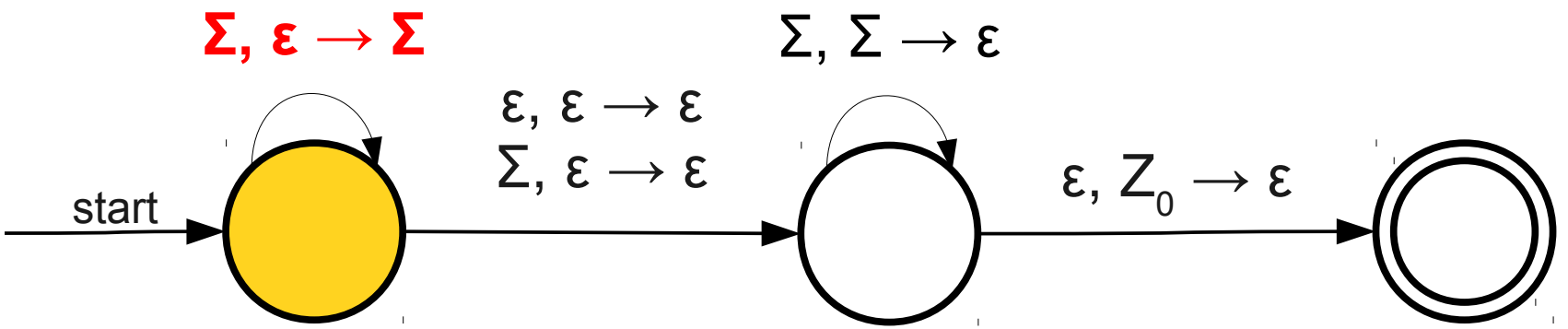

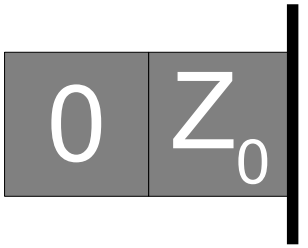

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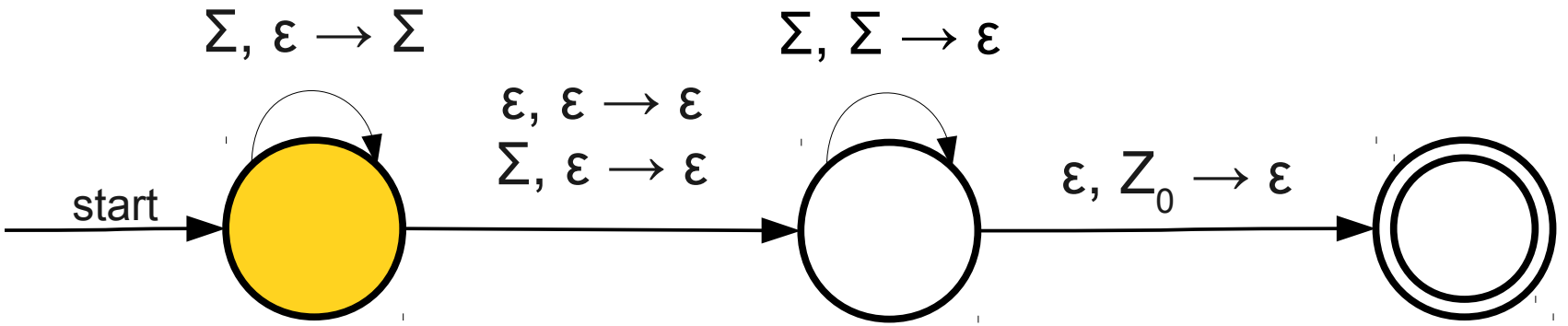

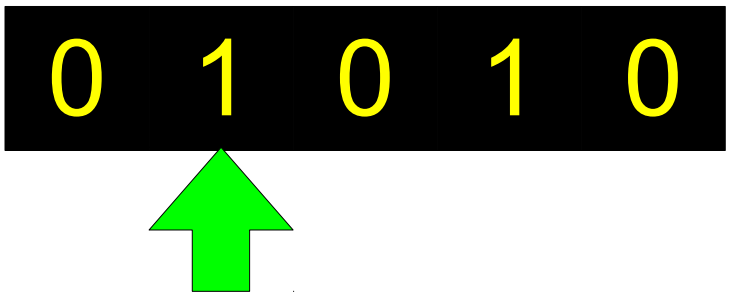

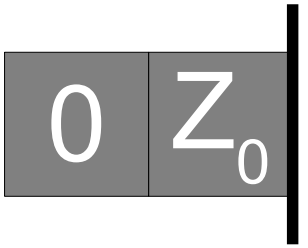

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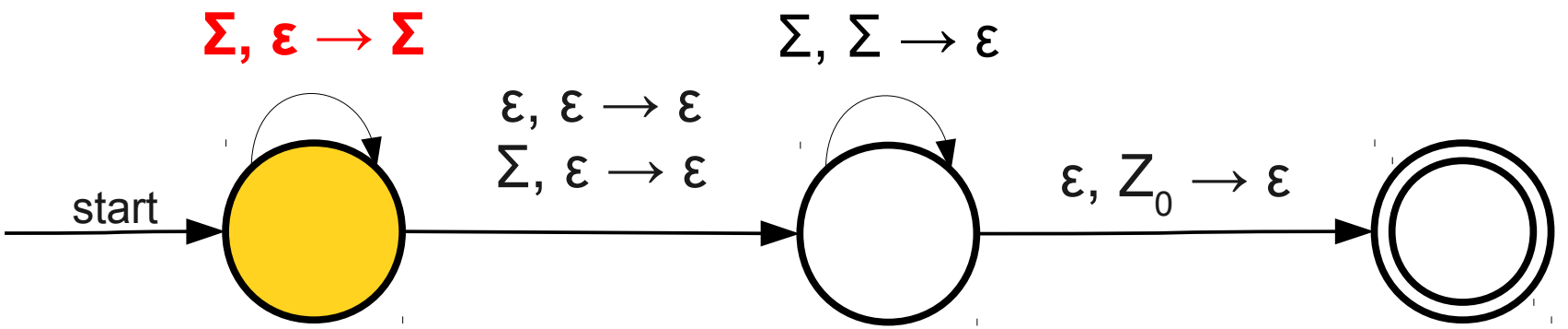

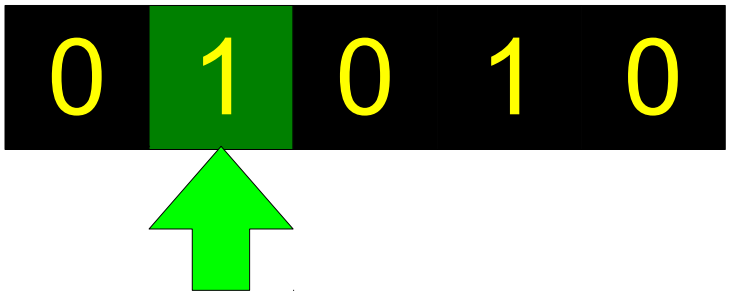

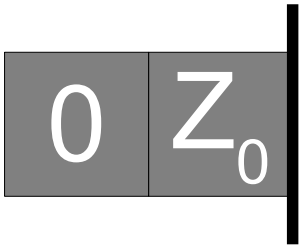

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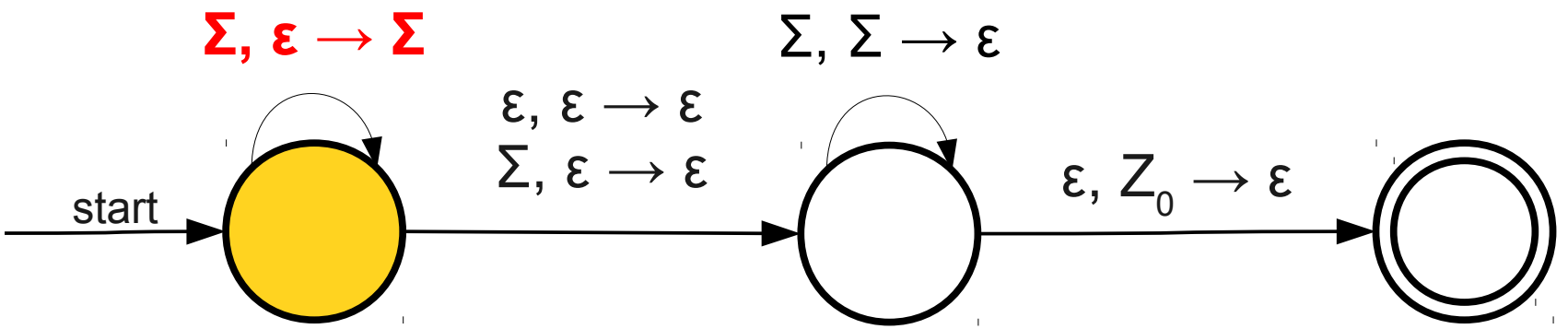

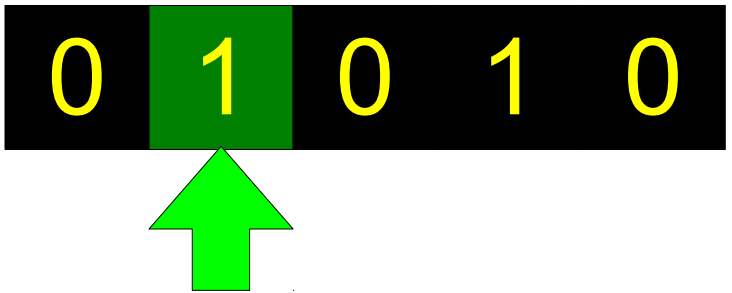

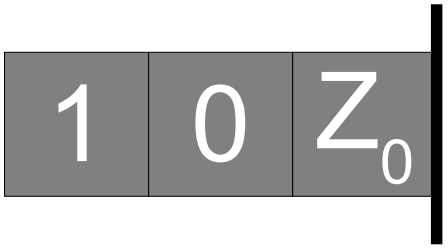

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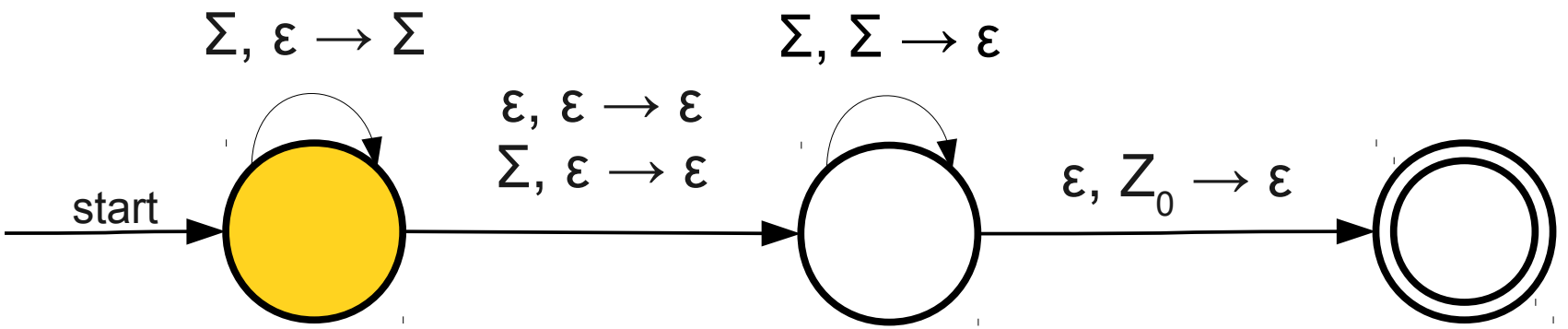

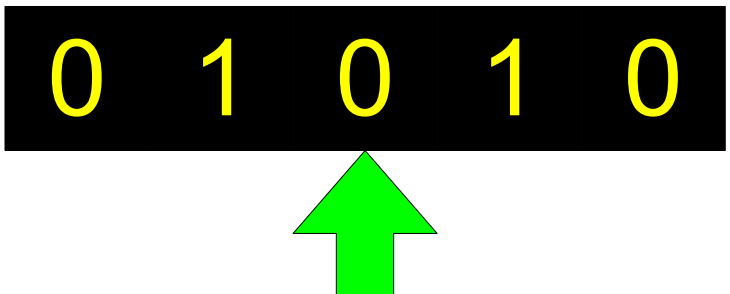

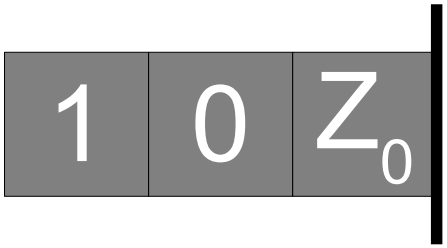

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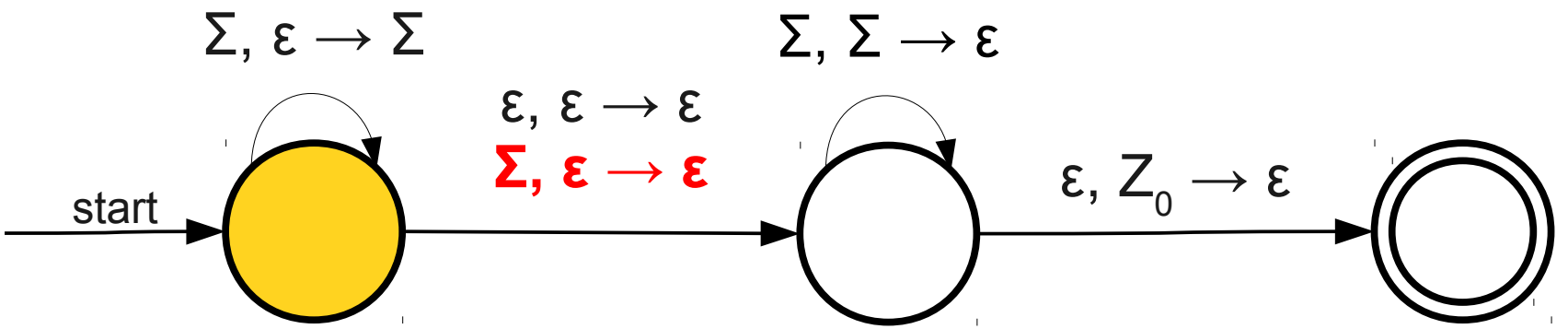

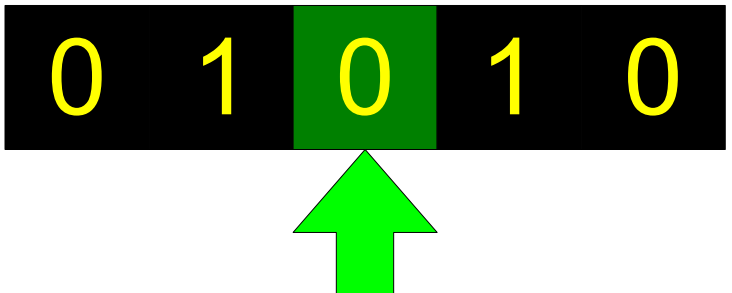

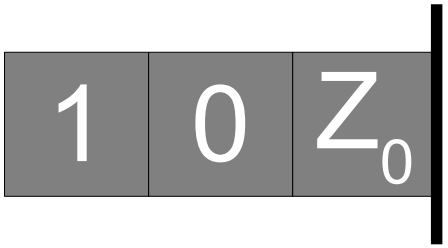

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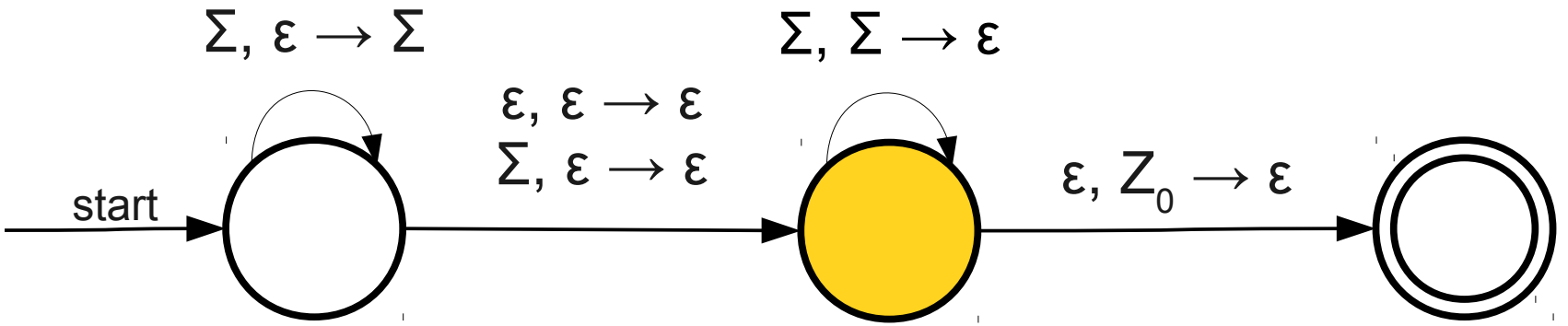

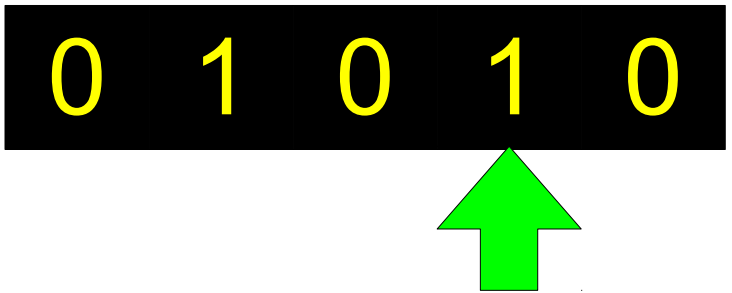

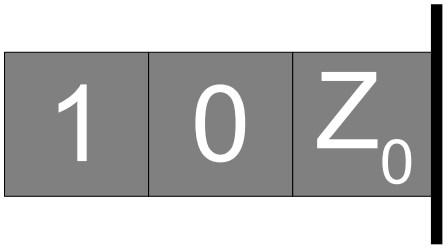

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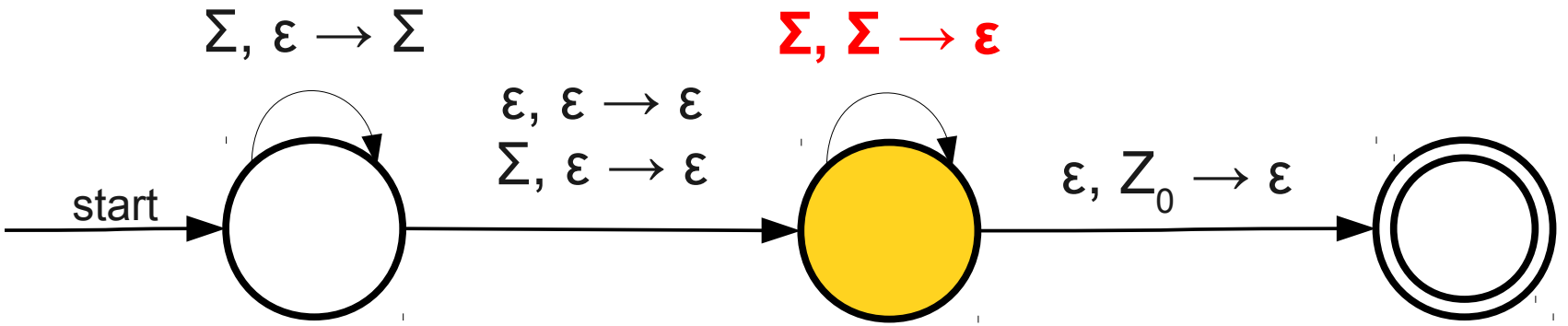

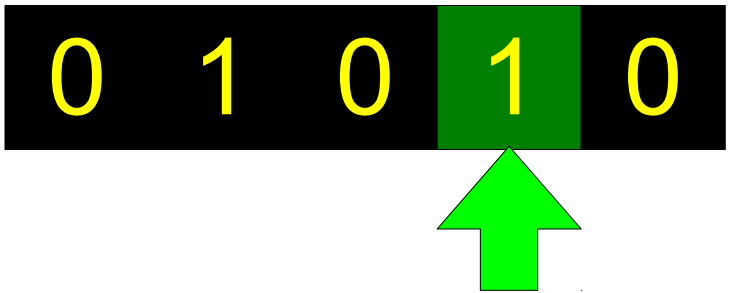

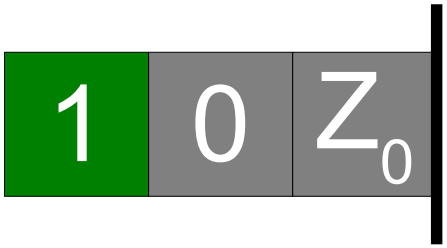

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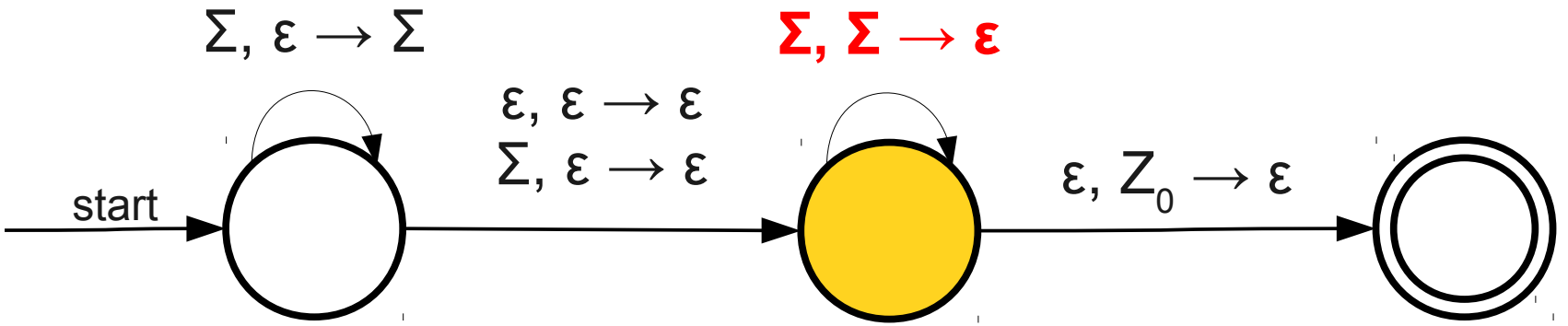

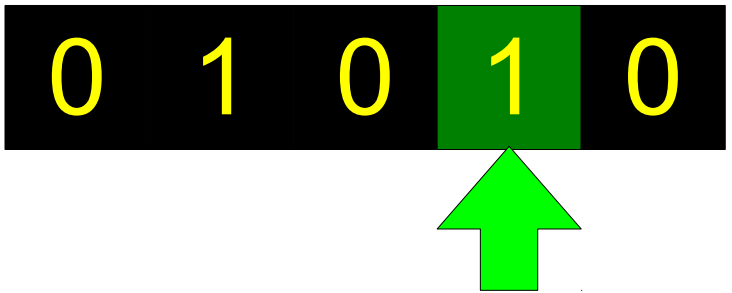

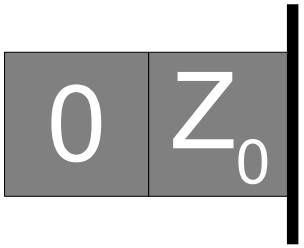

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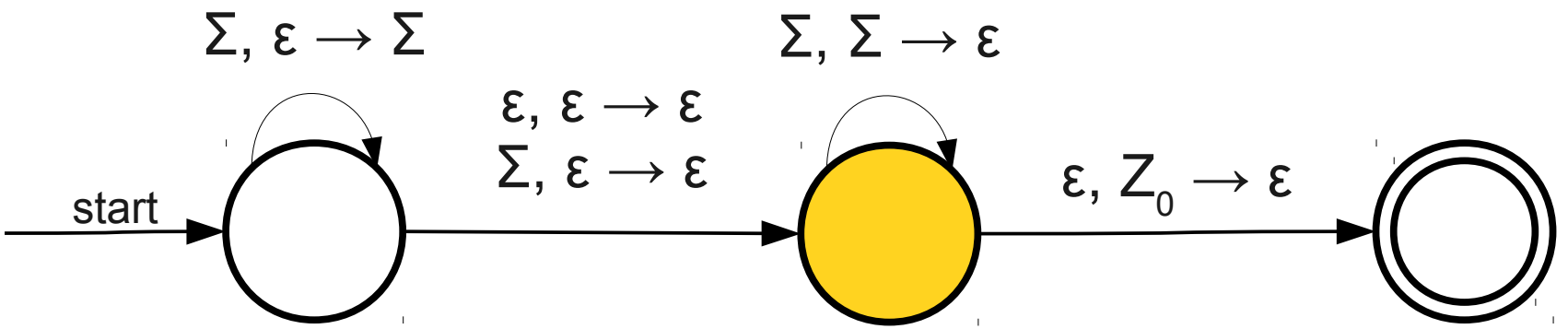

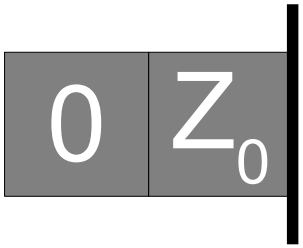

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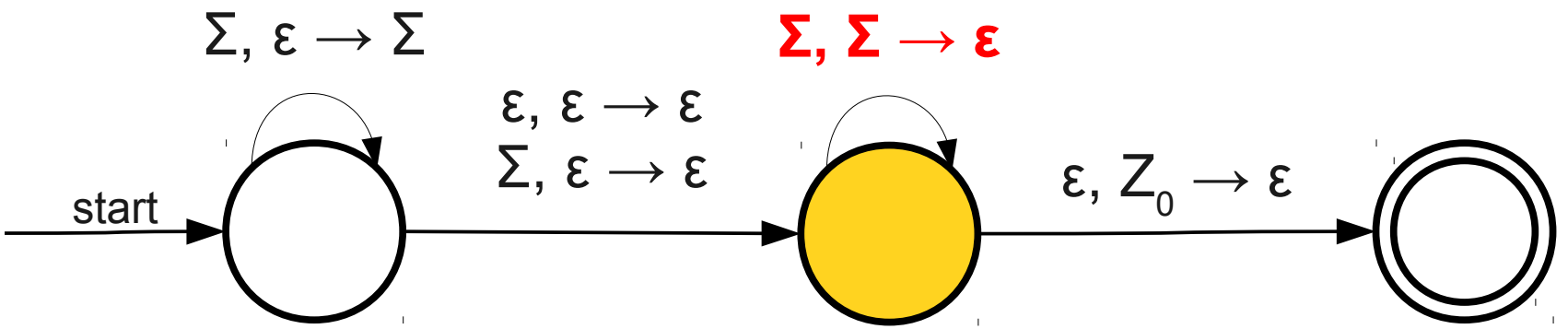

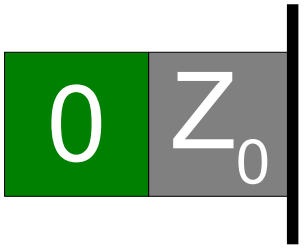

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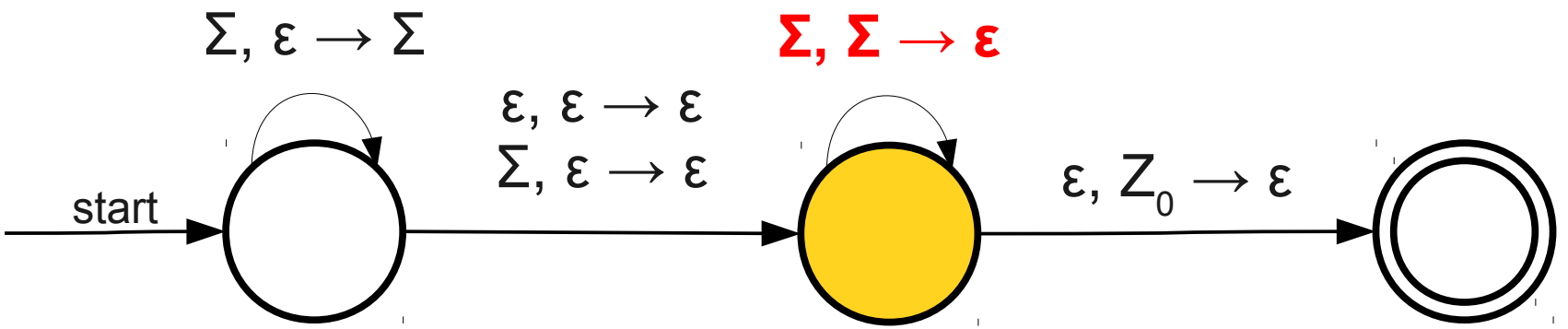
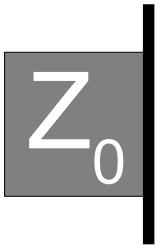
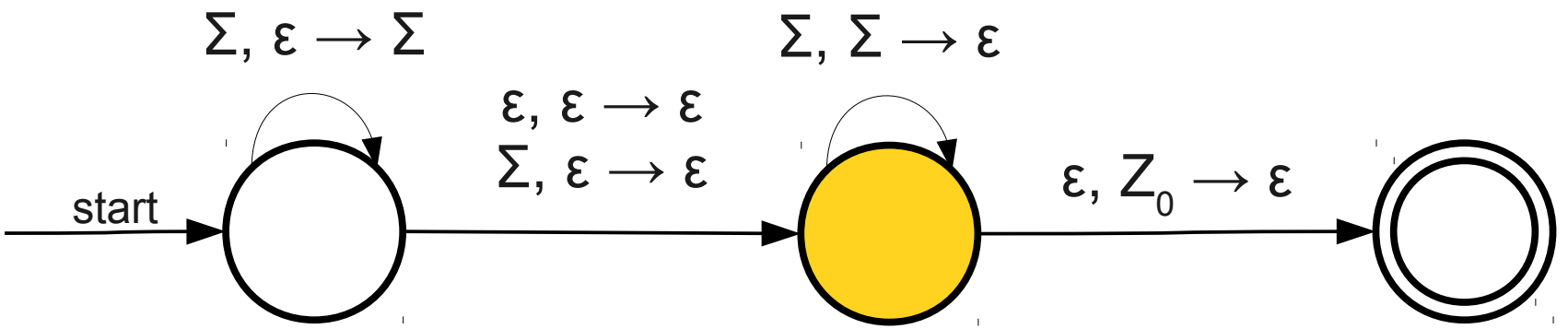
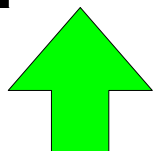


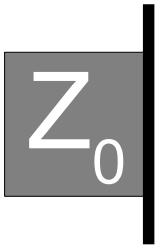


A PDA for Palindromes

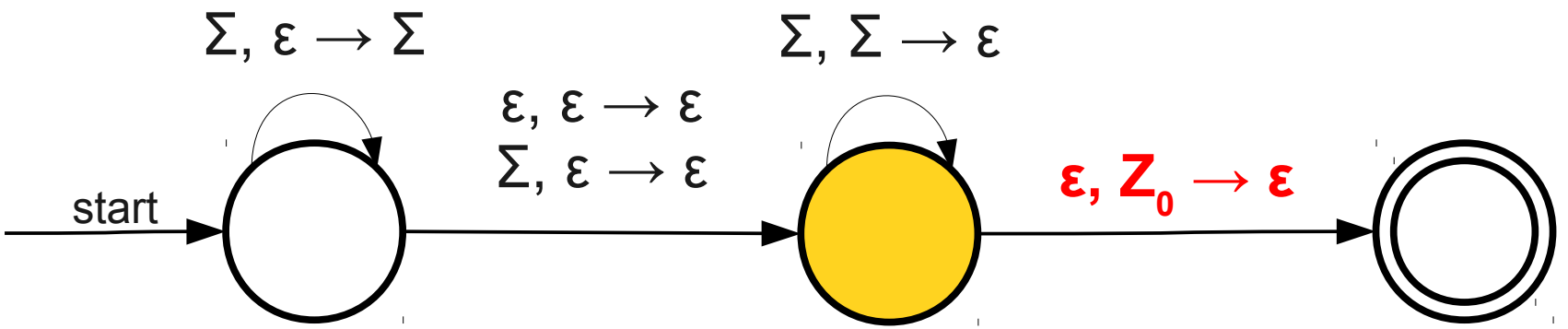


0 1 0 1 0





A PDA for Palindromes



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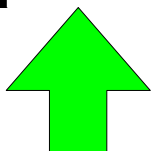
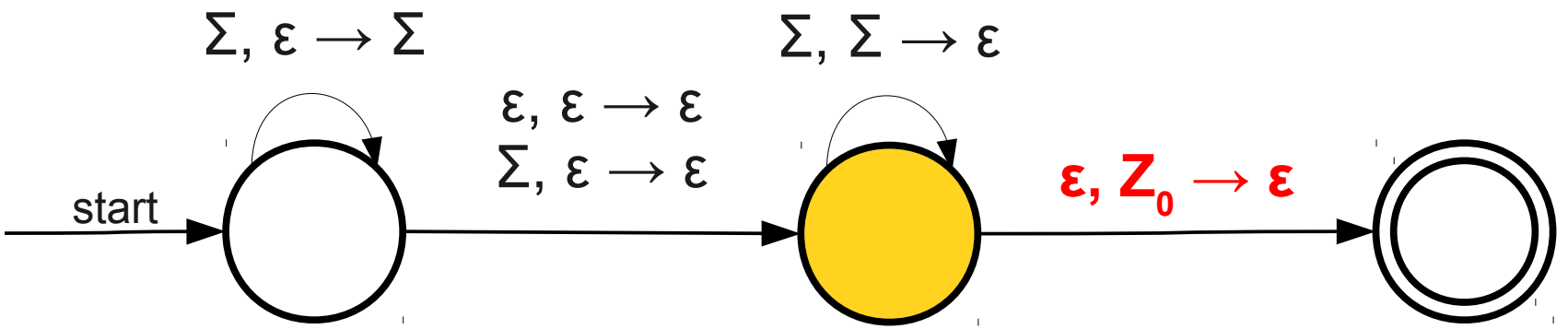


A PDA for Palindromes



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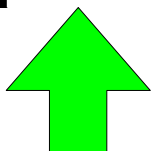
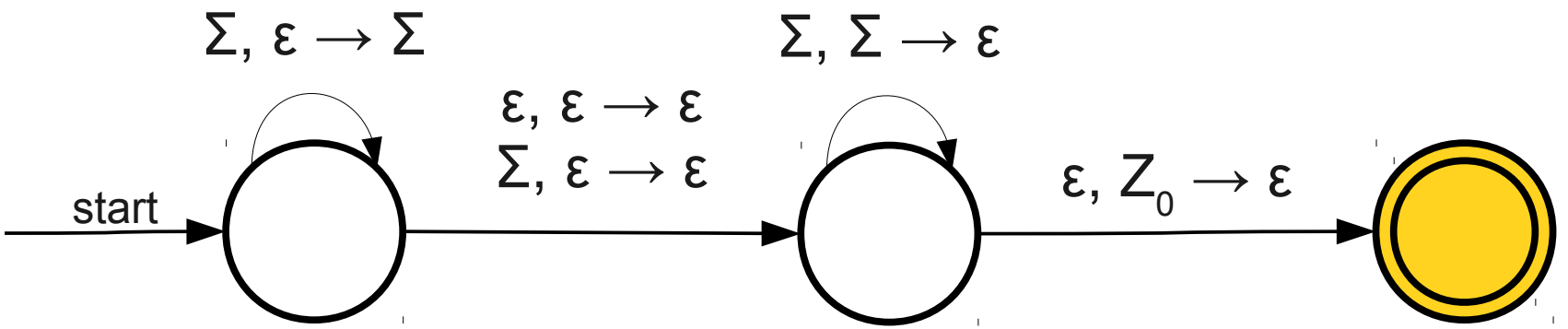


A PDA for Palindromes



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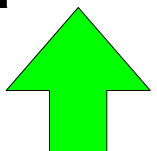
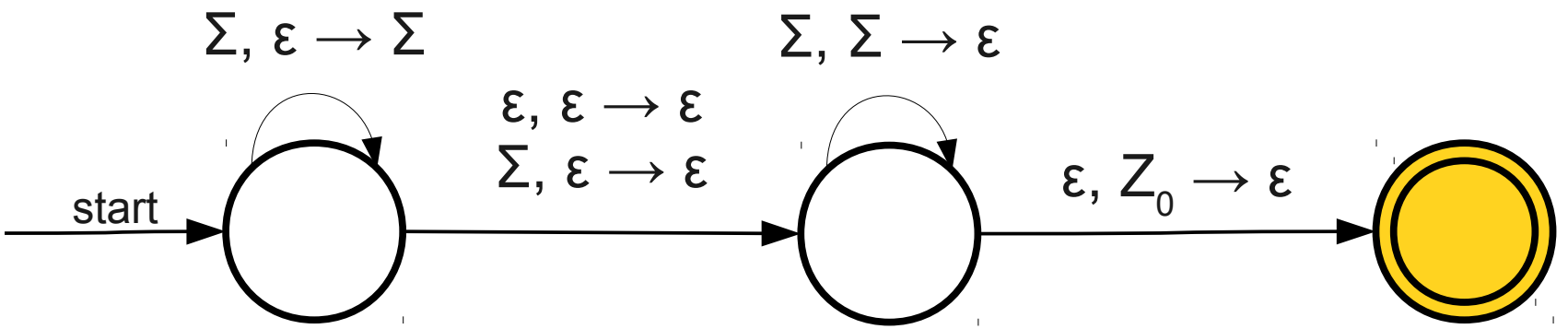
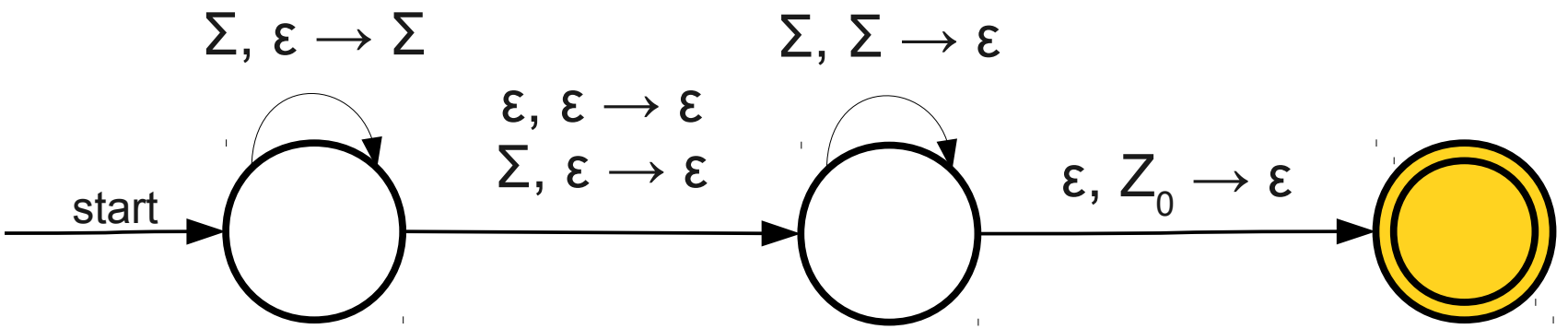


A PDA for Palindromes



0 1 0 1 0

A PDA for Palindromes



0 1 0 1 0

Why PDAs Matter

- Recall: A language is context-free iff there is some CFG that generates it.
- **Important, non-obvious theorem:** A language is context-free iff there is some PDA that recognizes it.
- Need to prove two directions:
  - If L is context-free, then there is a PDA for it.
  - If there is a PDA for L, then L is context-free.
- Part (1) is absolutely beautiful and we'll see it in a second.
- Part (2) is brilliant, but a bit too involved for lecture (you should read this in Sipser).

From CFGs to PDAs

- **Theorem:** If G is a CFG for a language L, then there exists a PDA for L as well.
- **Idea:** Build a PDA that simulates expanding out the CFG from the start symbol to some particular string.
- Stack holds the part of the string we haven't matched yet.

From CFGs to PDAs

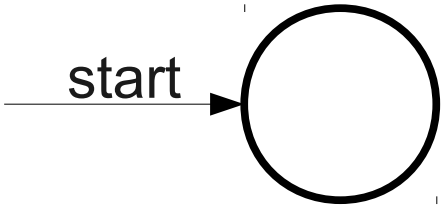
- Example: Let $\Sigma = \{ 1, \geq \}$ and let $GE = \{ 1^m \geq 1^n \mid m, n \in \mathbb{N} \wedge m \geq n \}$
  - $111 \geq 11 \in GE$
  - $11 \geq 11 \in GE$
  - $1111 \geq 11 \in GE$
  - $\geq \in GE$
- One CFG for *GE* is (from the problem session)
  - $S \rightarrow 1S1 \mid 1S \mid \geq$
- How would we build a PDA for *GE*?

From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

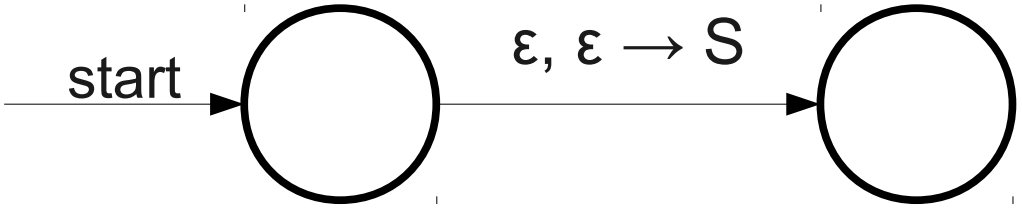
From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



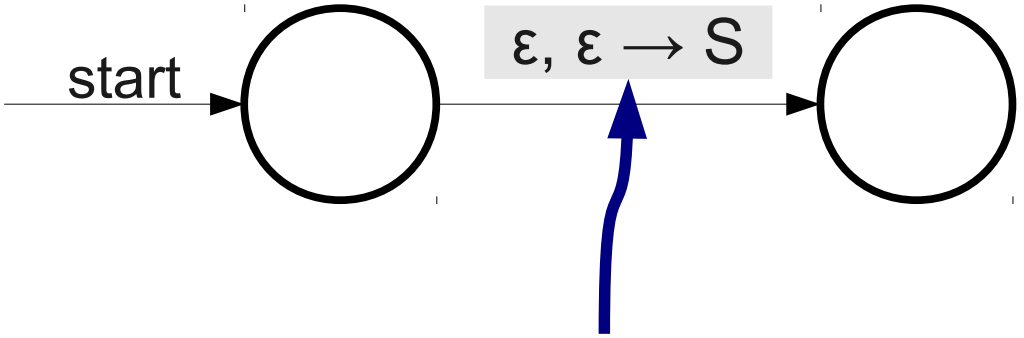
From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

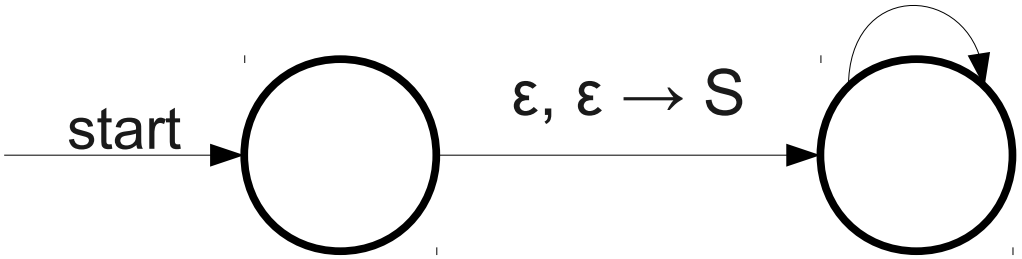


We begin by putting the start symbol of the grammar onto the stack so that we can begin applying productions.

From CFGs to PDAs

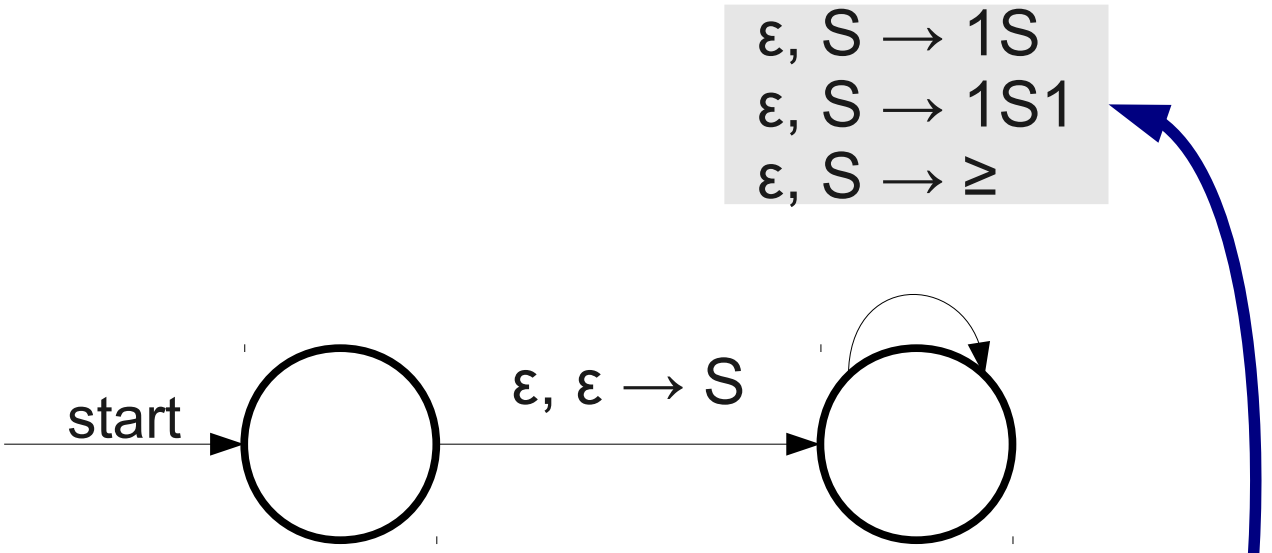
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

$\varepsilon, S \rightarrow 1S$
$\varepsilon, S \rightarrow 1S1$
$\varepsilon, S \rightarrow \geq$



From CFGs to PDAs

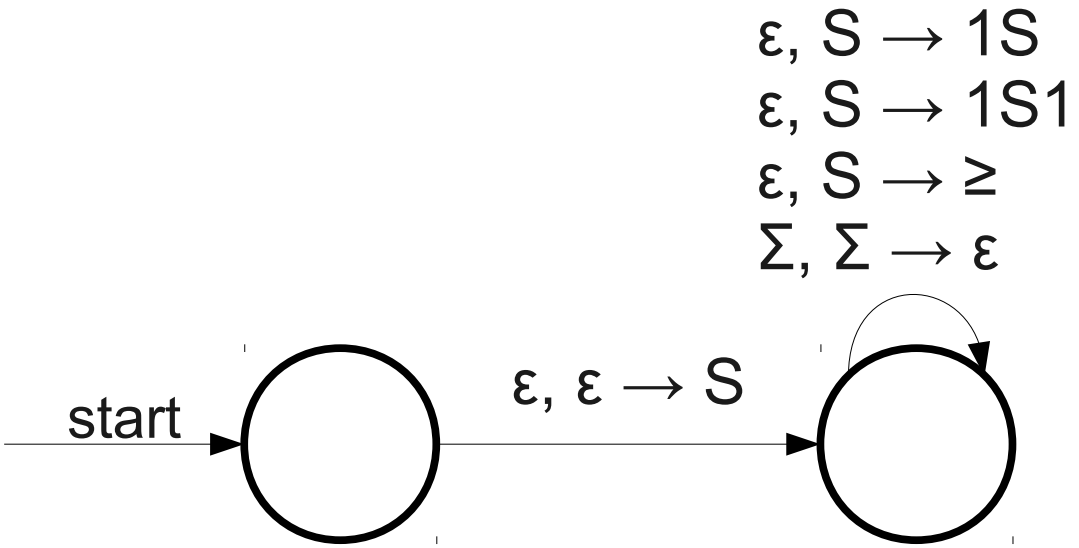
$S \to 1S1$
$S \to 1S$
$S \to \varepsilon$



These transitions allow us to nondeterministically guess which production to use when the top of the stack is a nonterminal.

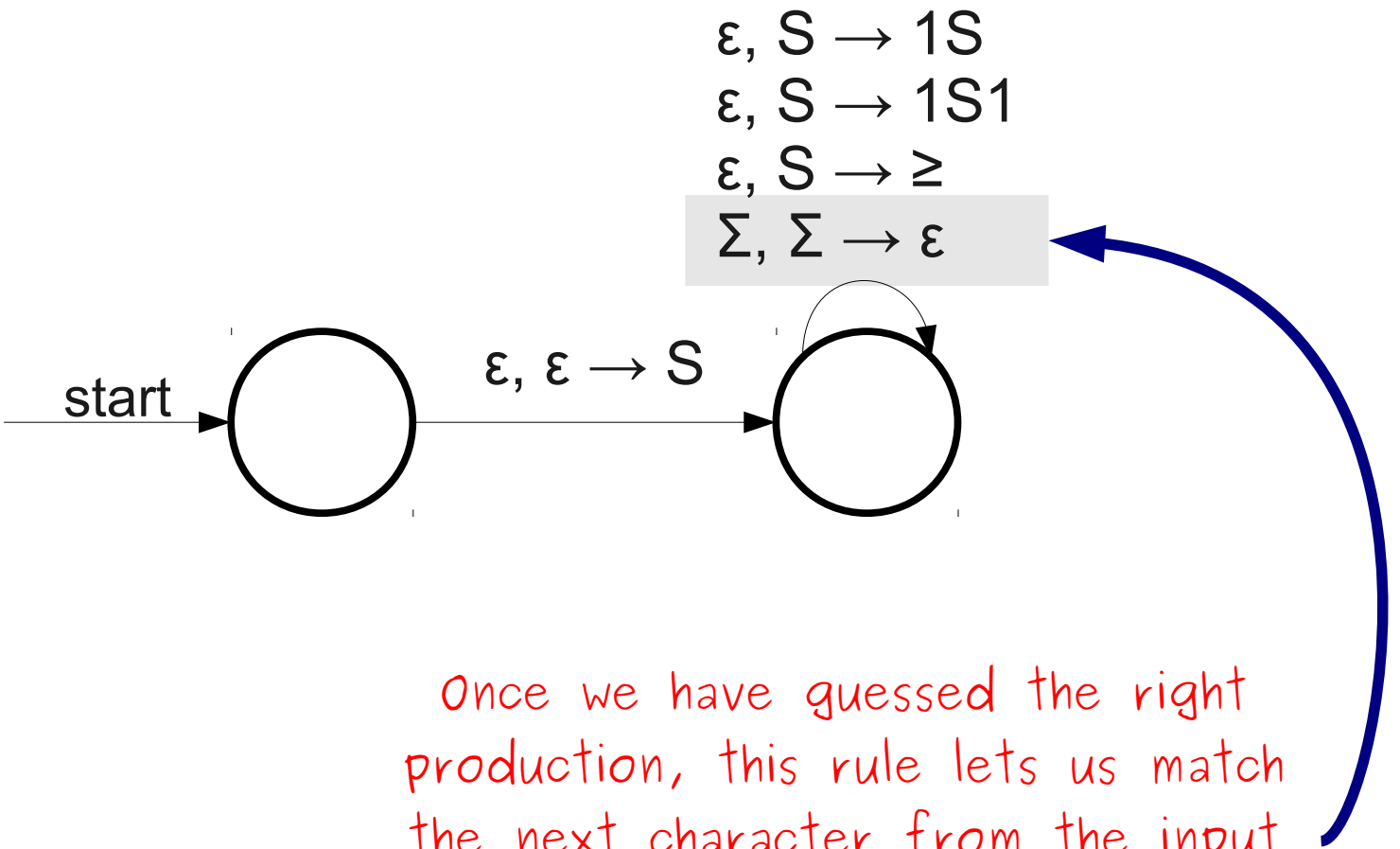
From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



From CFGs to PDAs

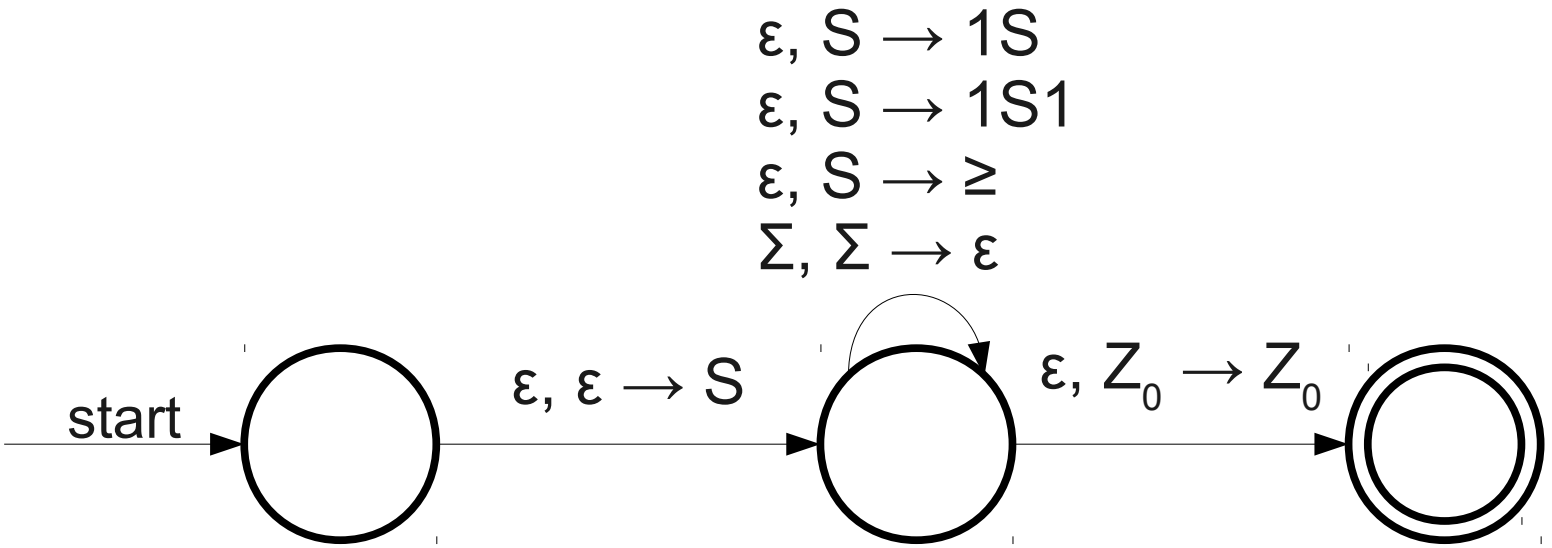
$S \to 1S1$
$S \to 1S$
$S \to \varepsilon$



Once we have guessed the right production, this rule lets us match the next character from the input with the next terminal we produced.

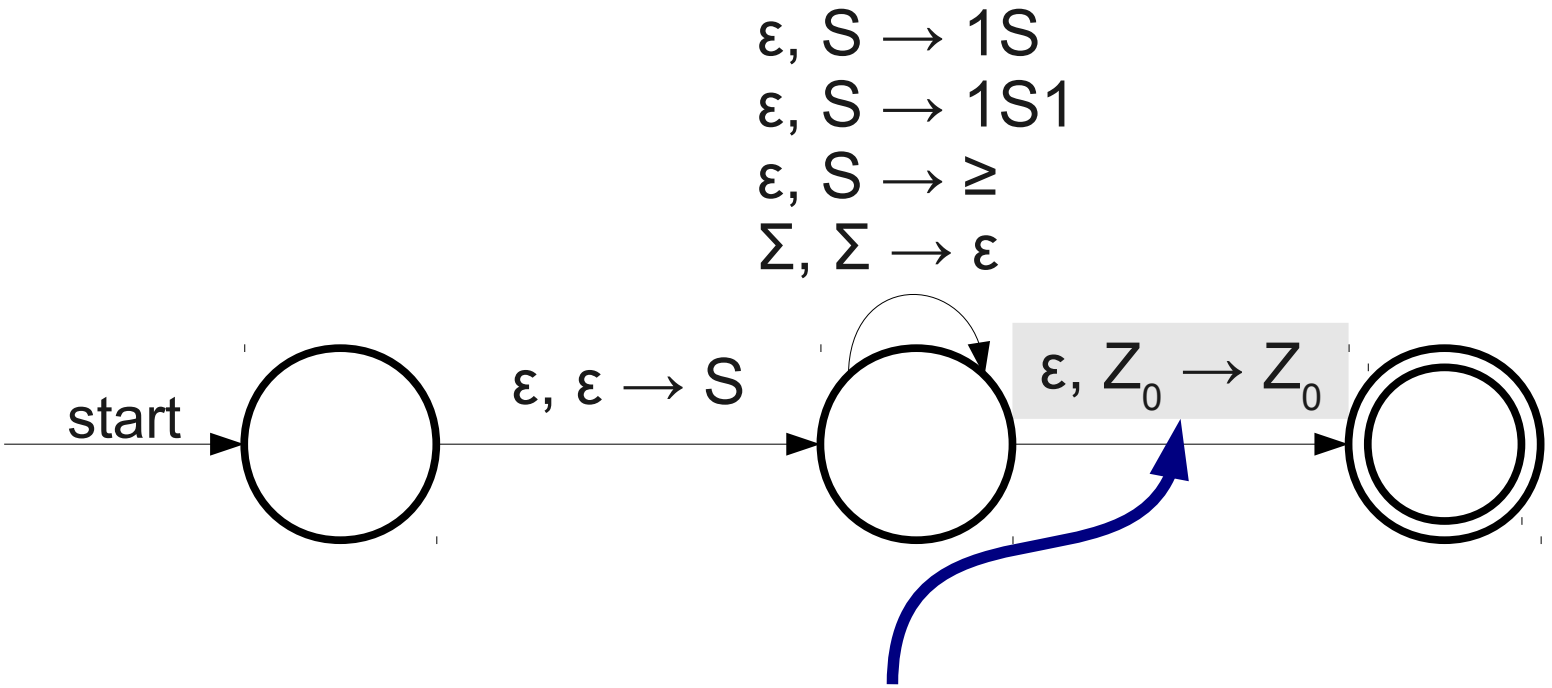
From CFGs to PDAs

$S \to 1S1$
$S \to 1S$
$S \to \geq$



From CFGs to PDAs

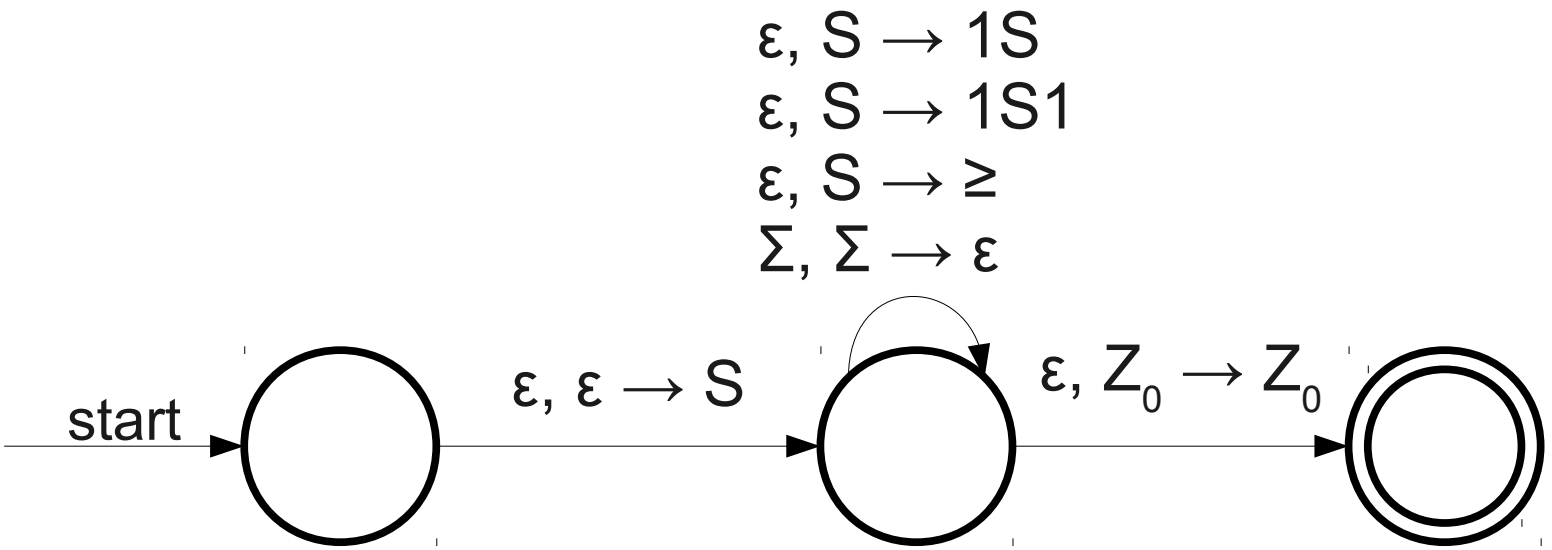
$S \to 1S1$
$S \to 1S$
$S \to \geq$



Once we have fully expanded out all nonterminals and matched all the terminals on the stack, we can transition into the accepting state.

From CFGs to PDAs

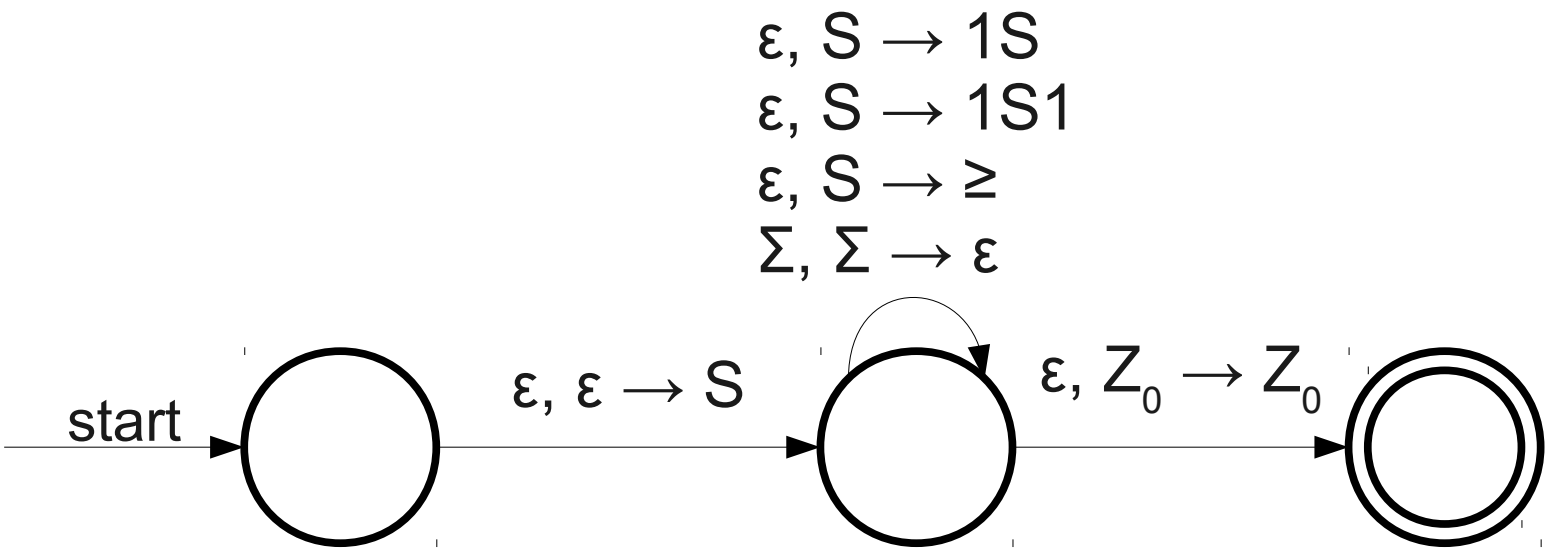
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



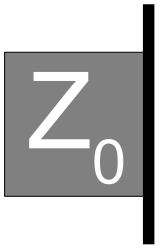
1 1 1 ≥ 1 1

From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

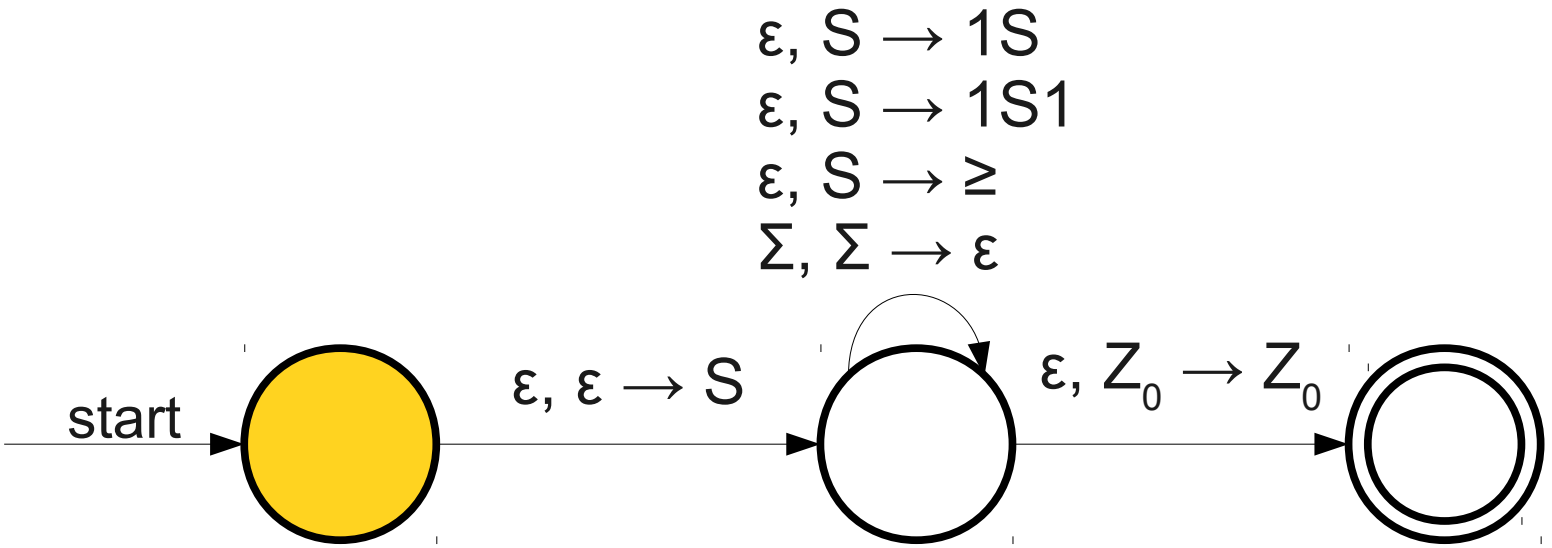


1 1 1 ≥ 1 1

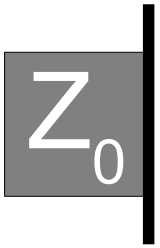


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

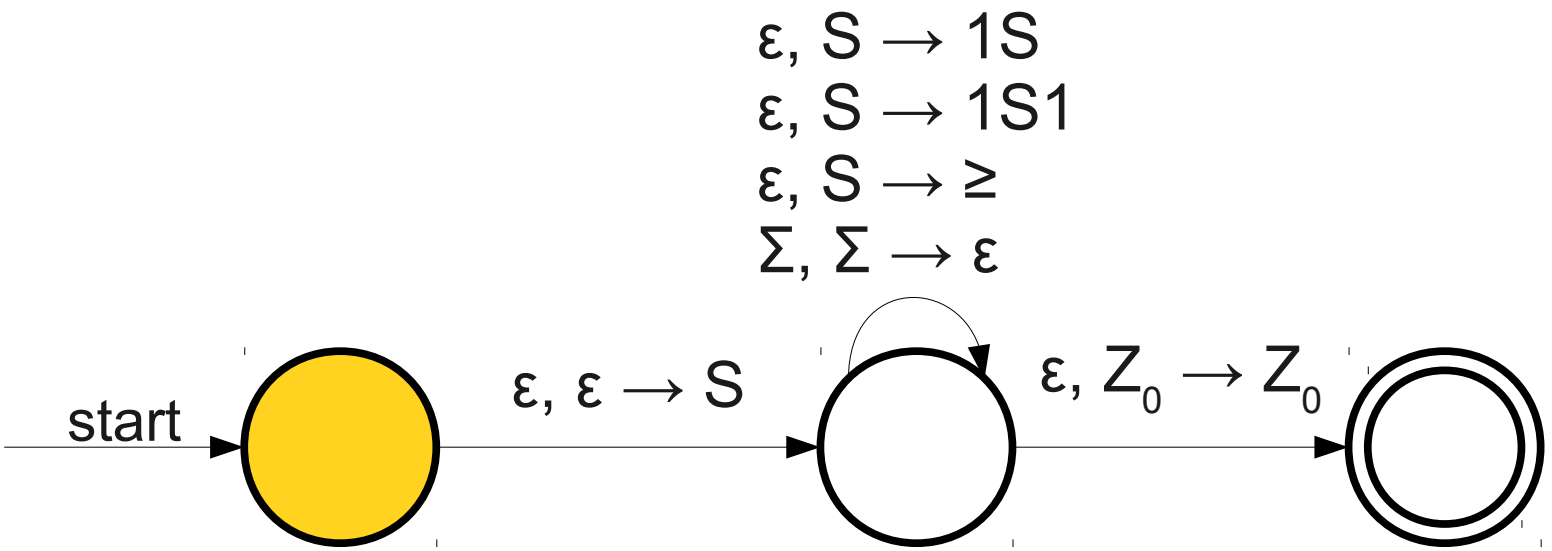


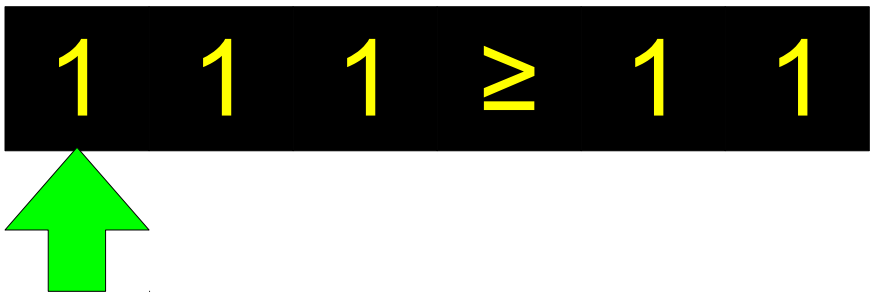
1 1 1 ≥ 1 1

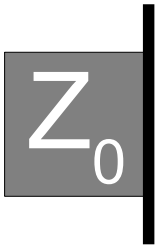


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

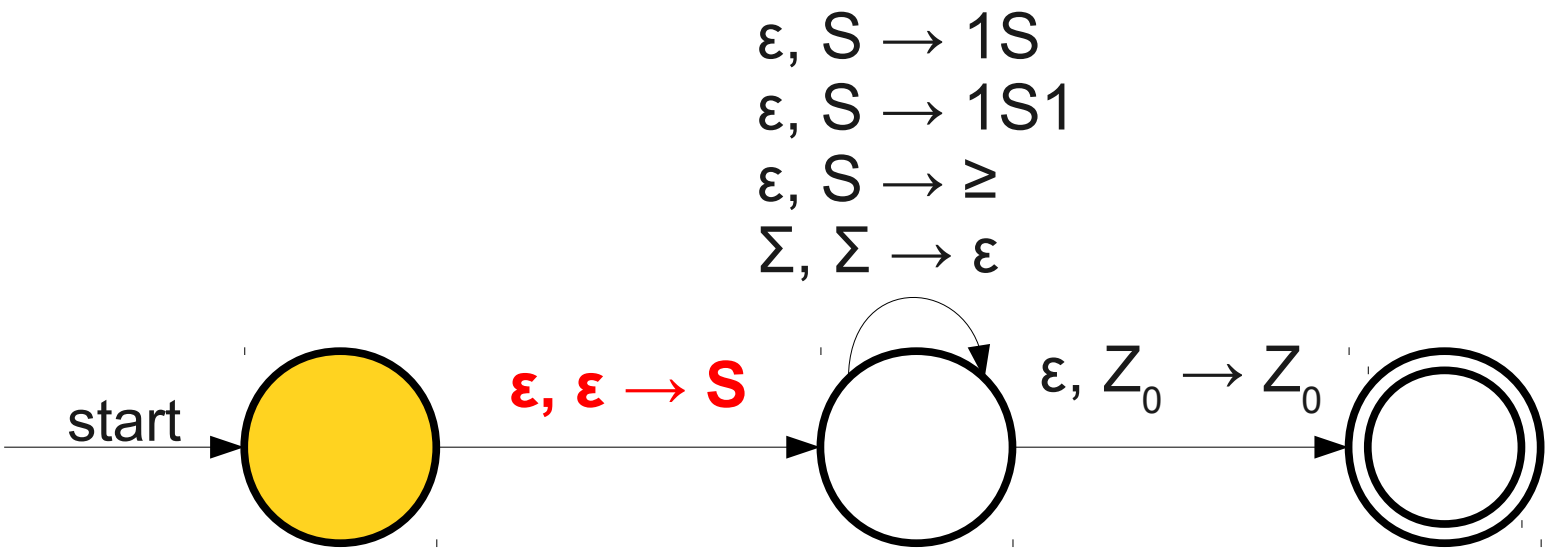


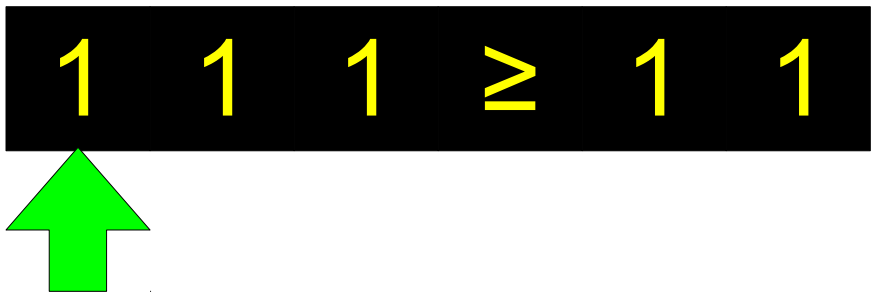


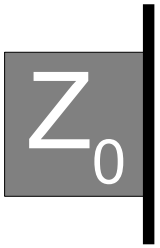


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

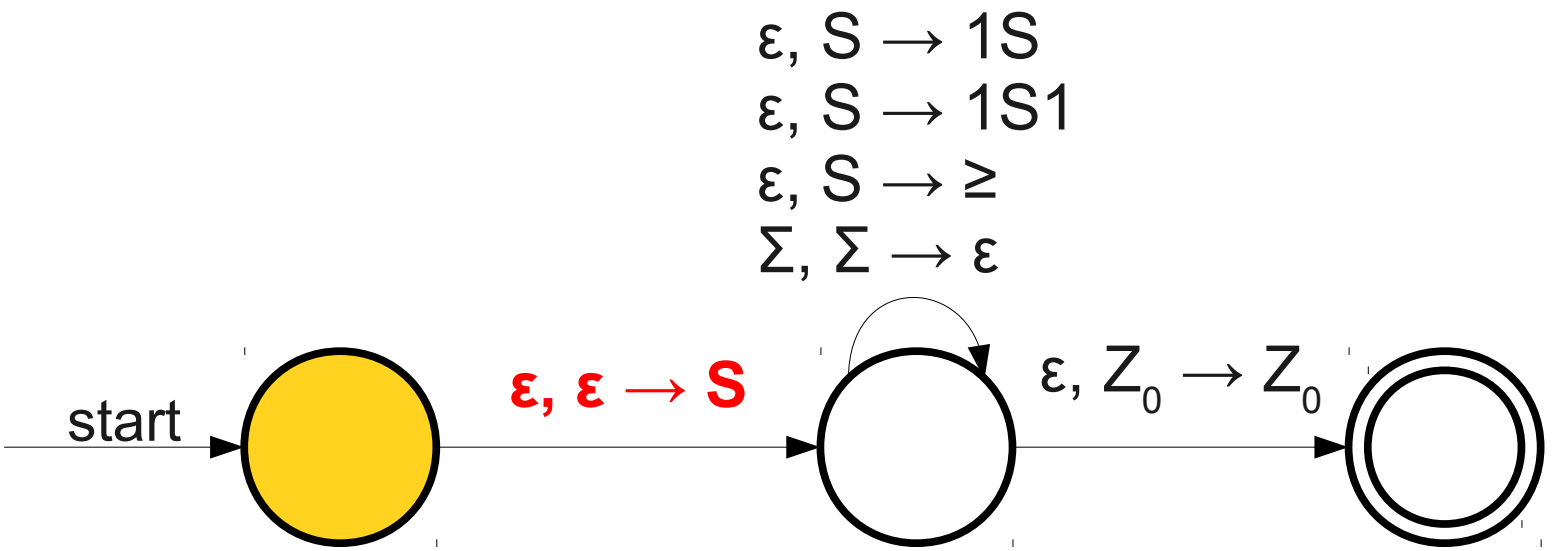


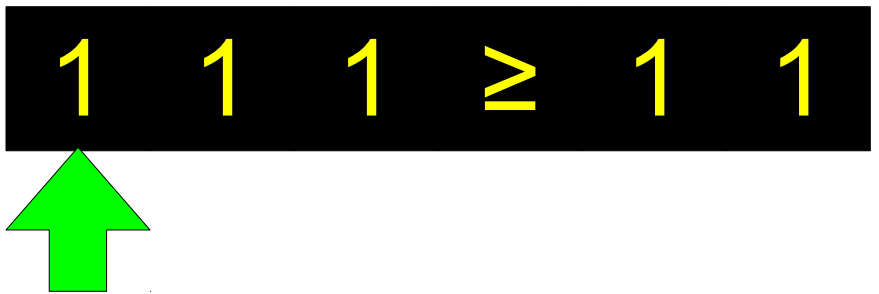


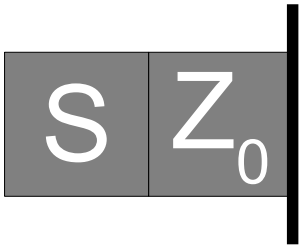


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

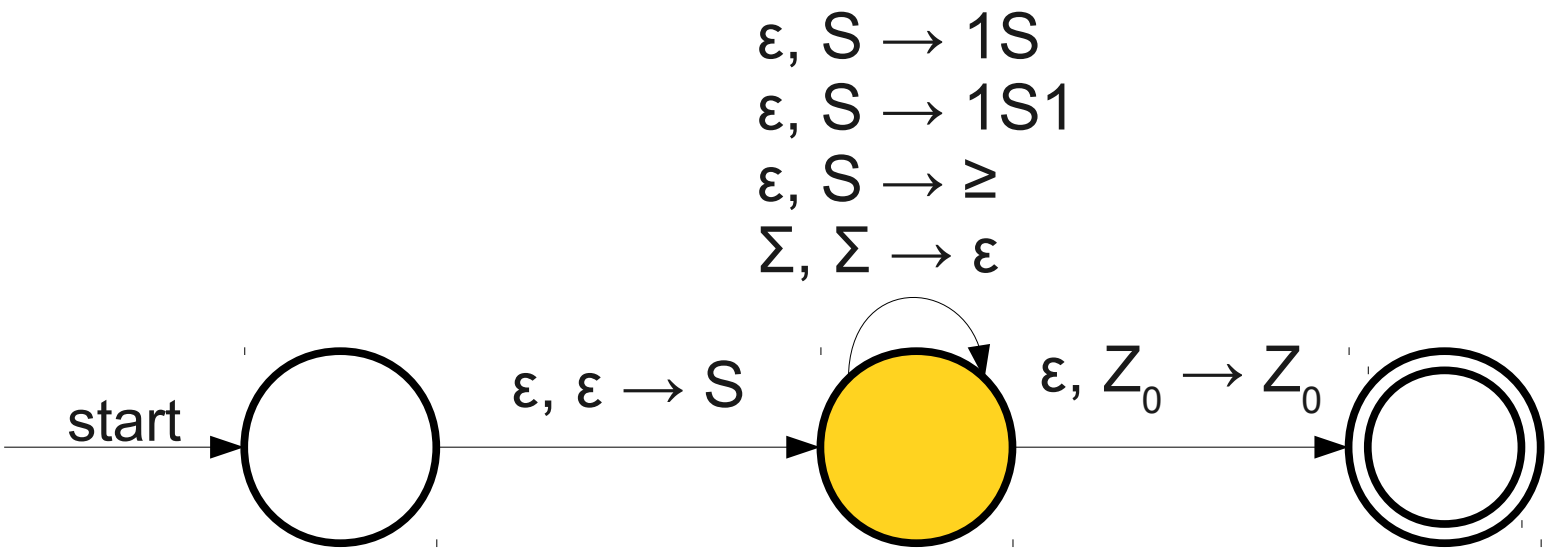


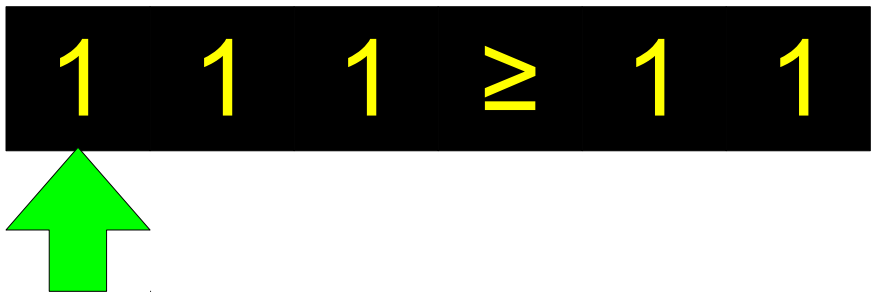


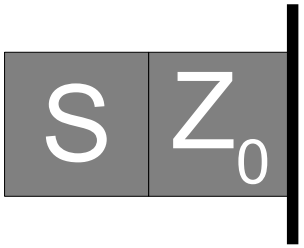


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

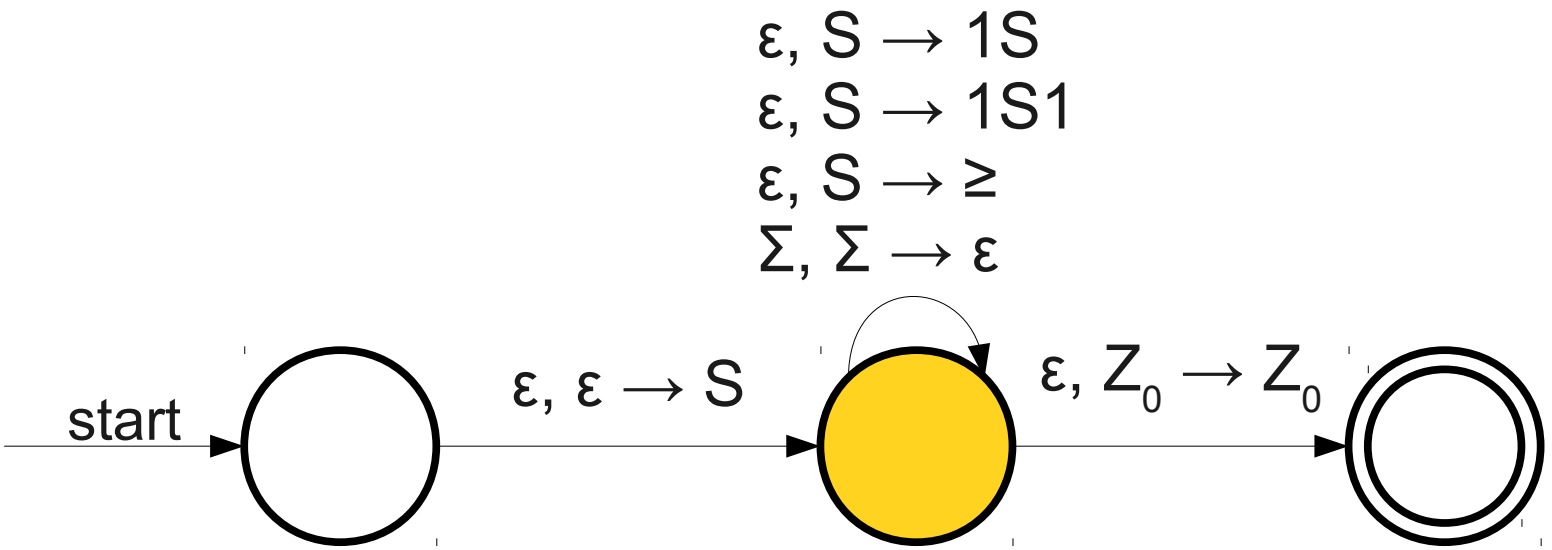




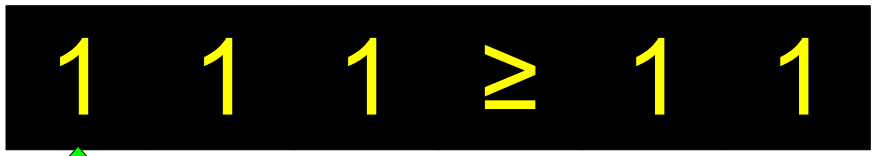


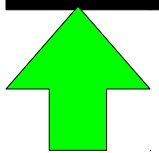
From CFGs to PDAs

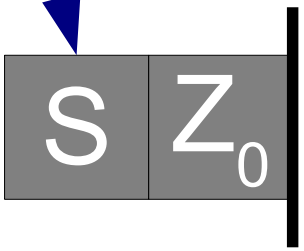
$S \to 1S1$
$S \to 1S$
$S \to \geq$



Now that the stack top is a nonterminal, we guess which production to use.

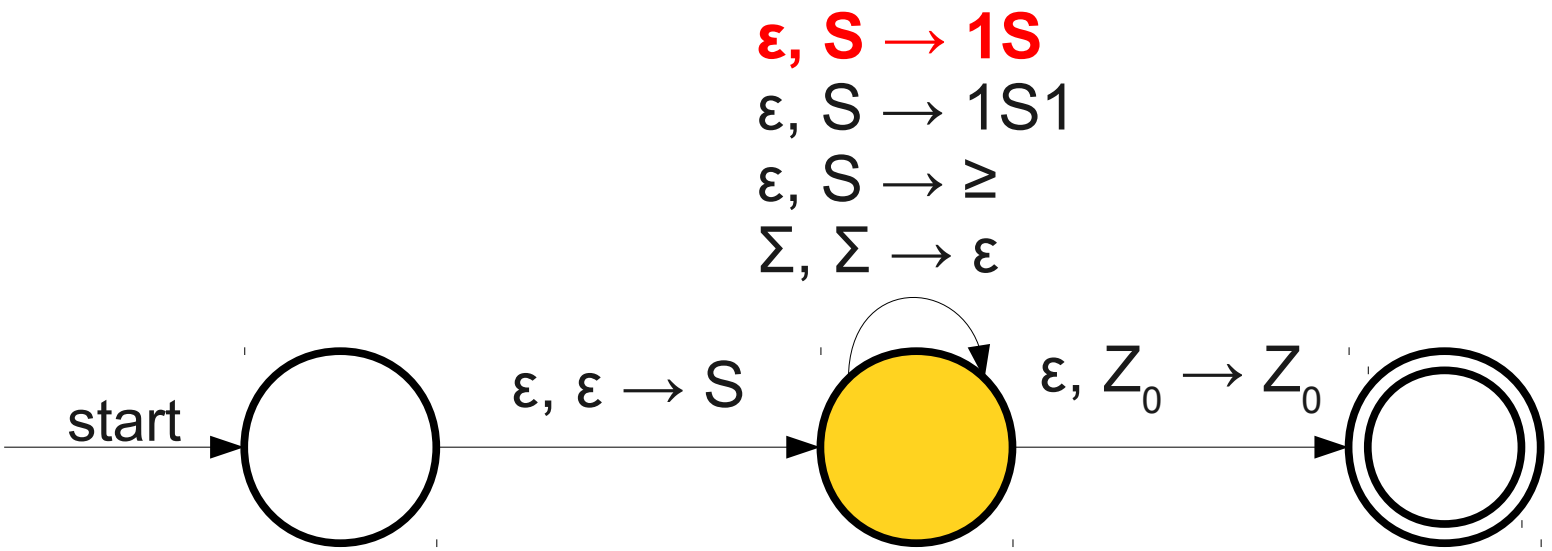


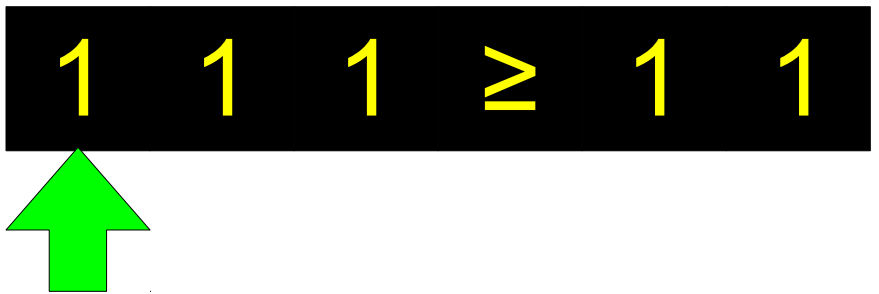


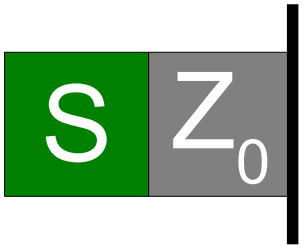


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

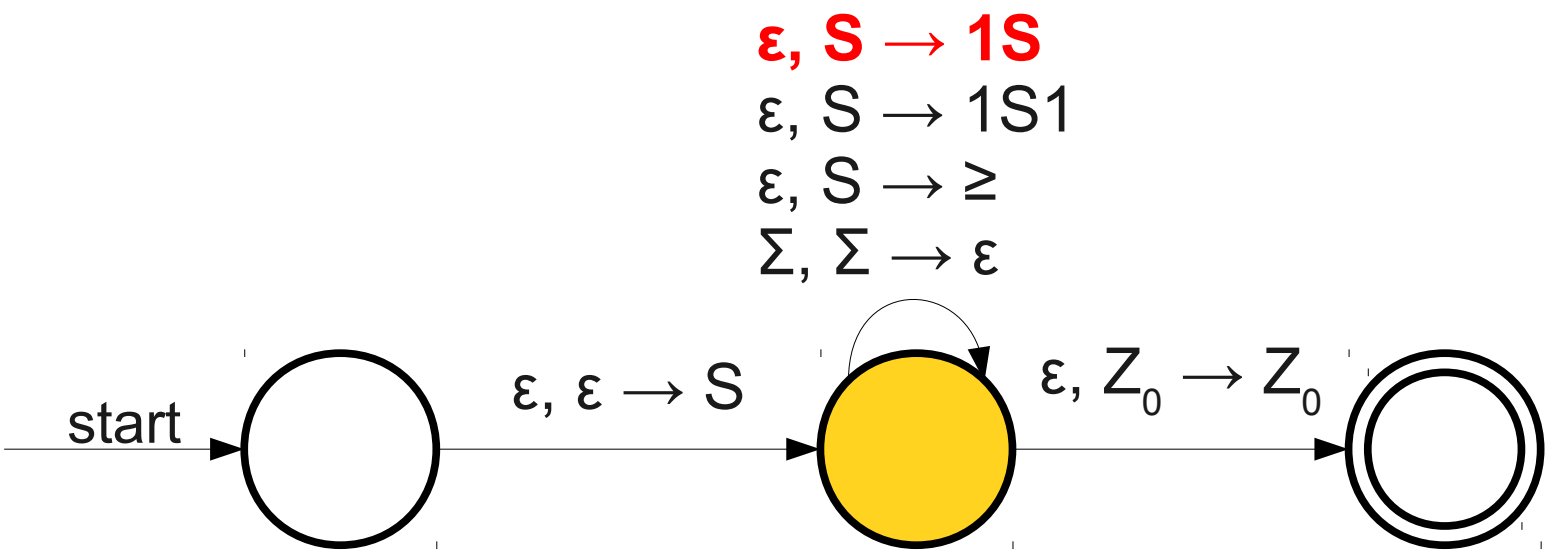


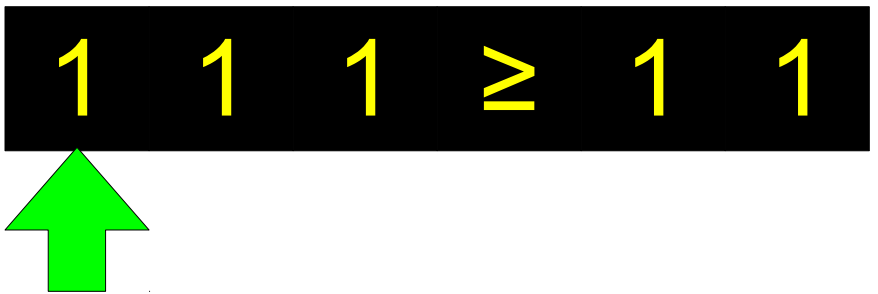


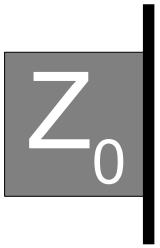


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

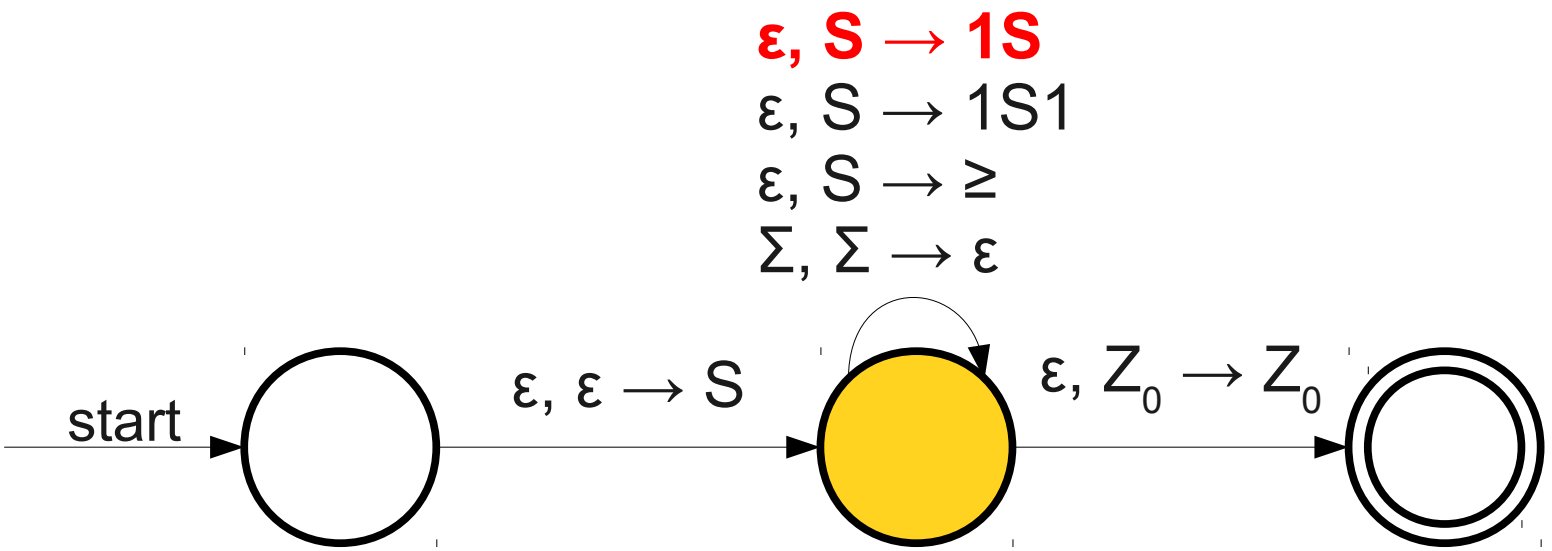


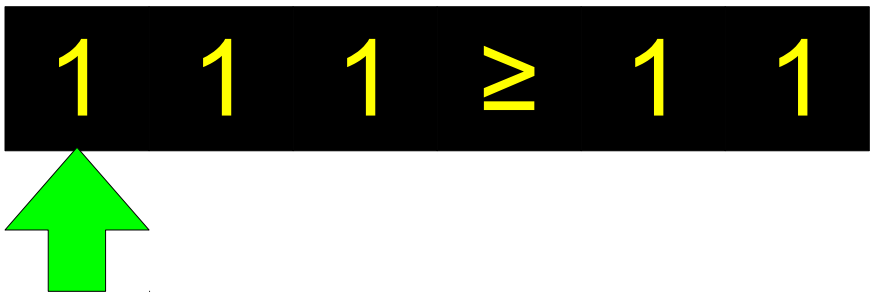




From CFGs to PDAs

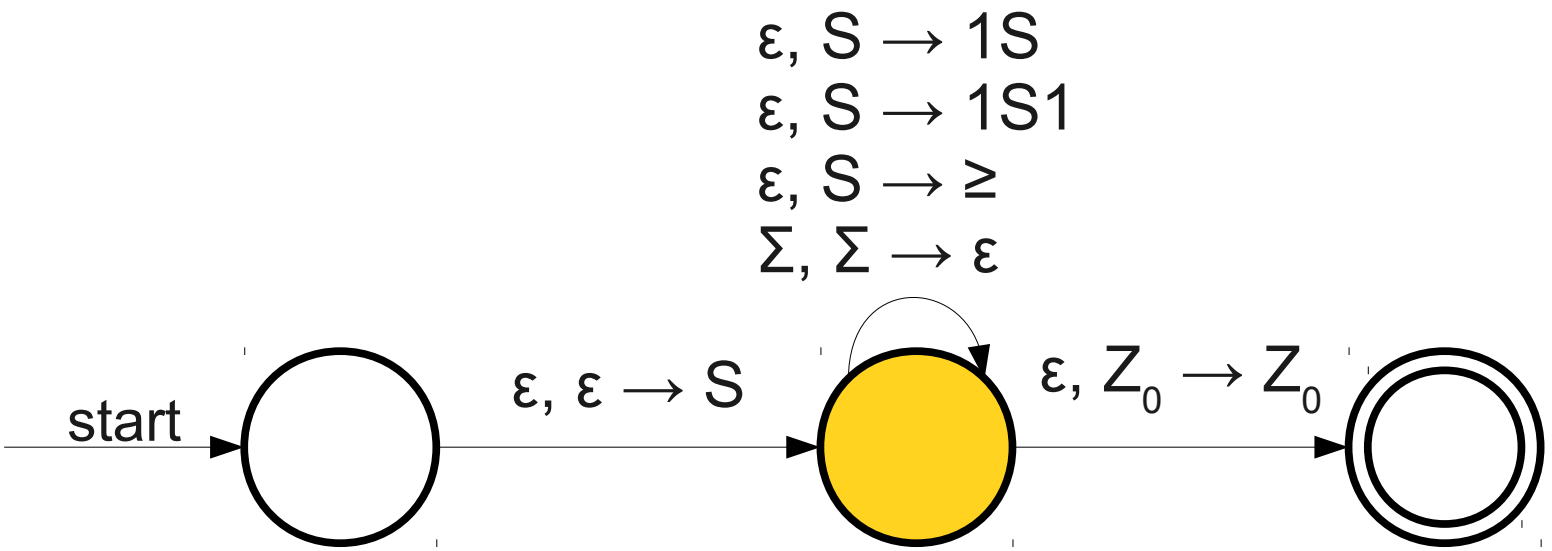
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

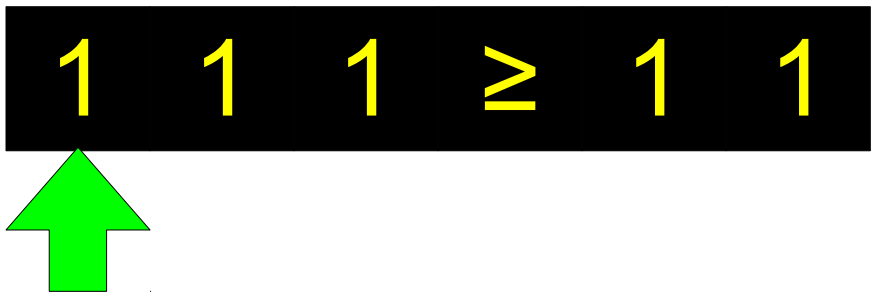




From CFGs to PDAs

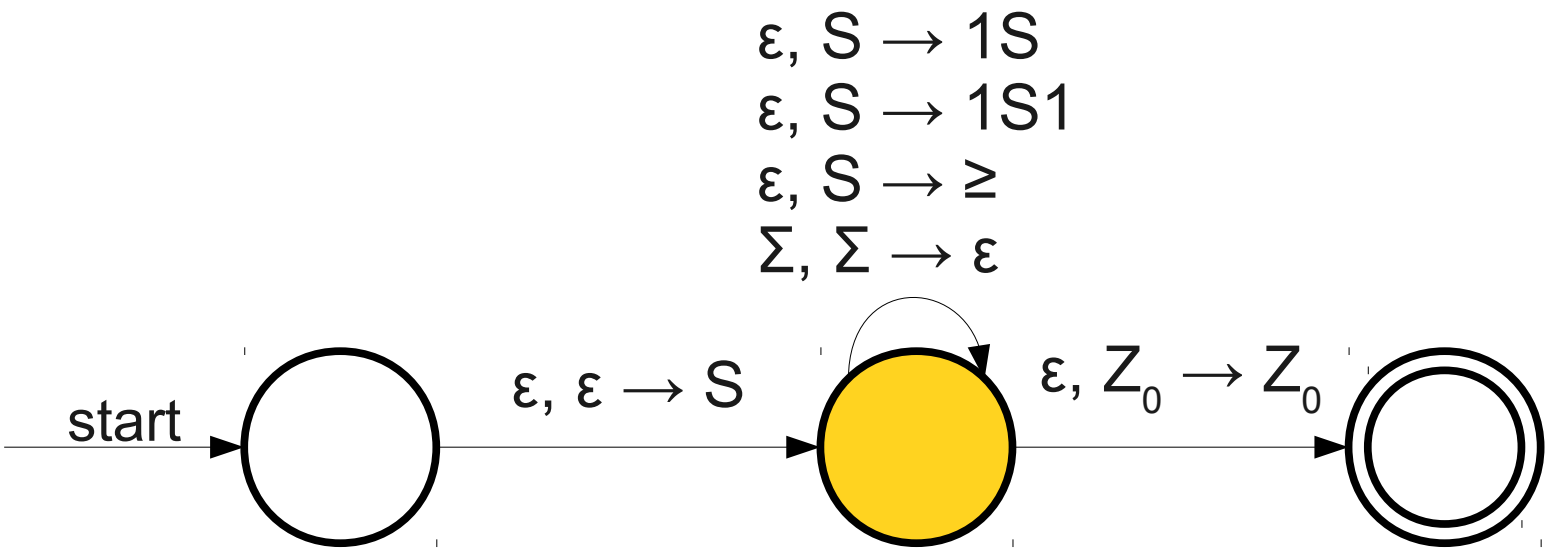
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



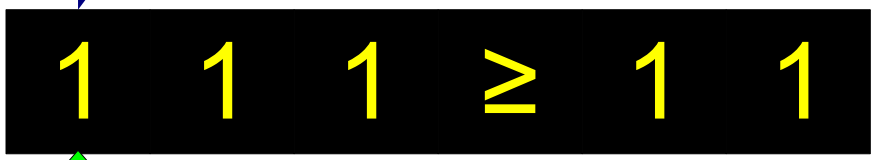


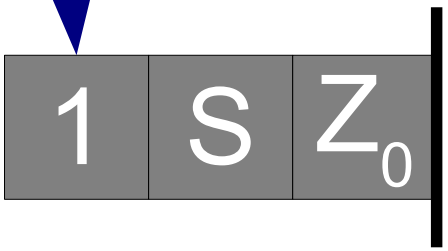
From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



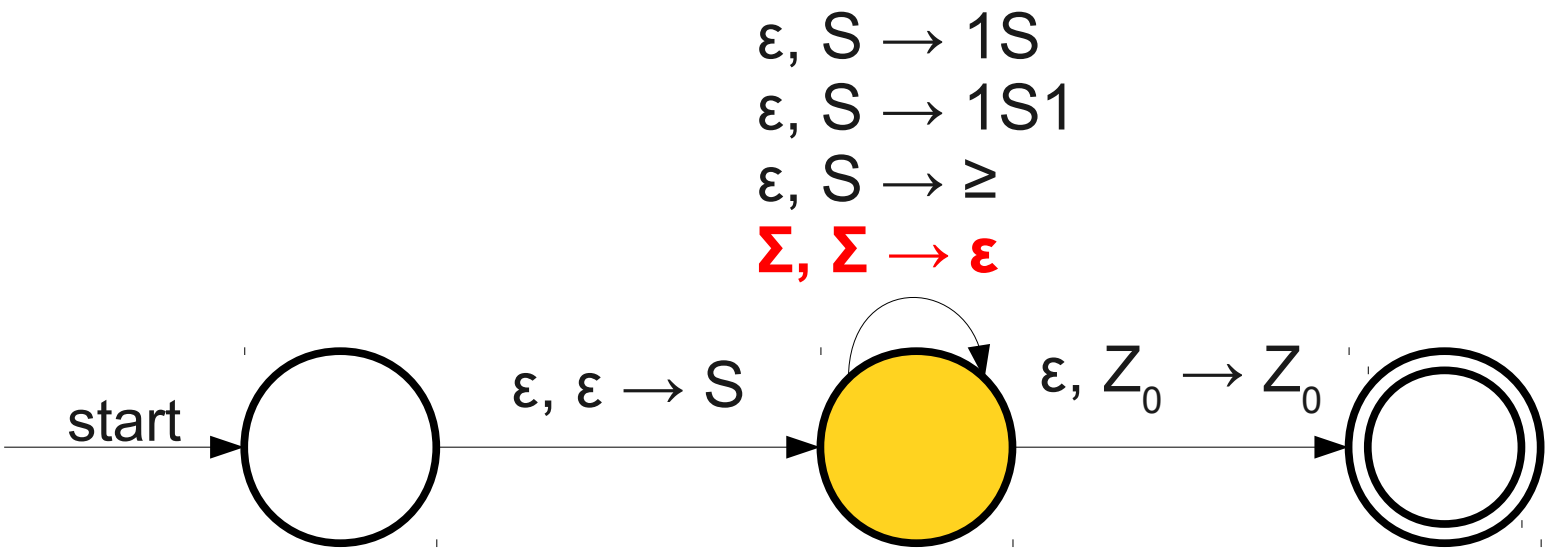
Since the top of the stack is a terminal, we can match it with the next input symbol.

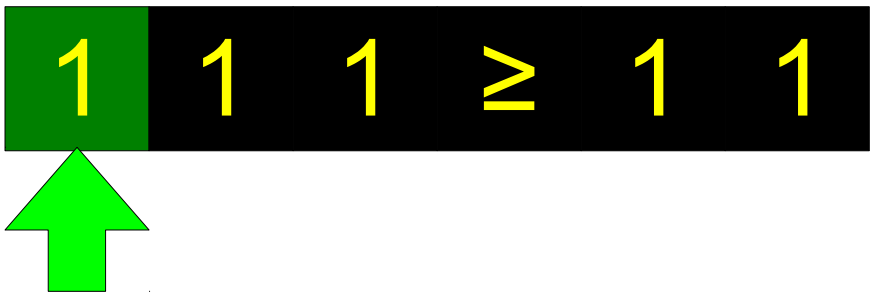


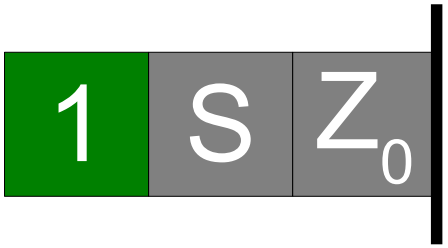


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

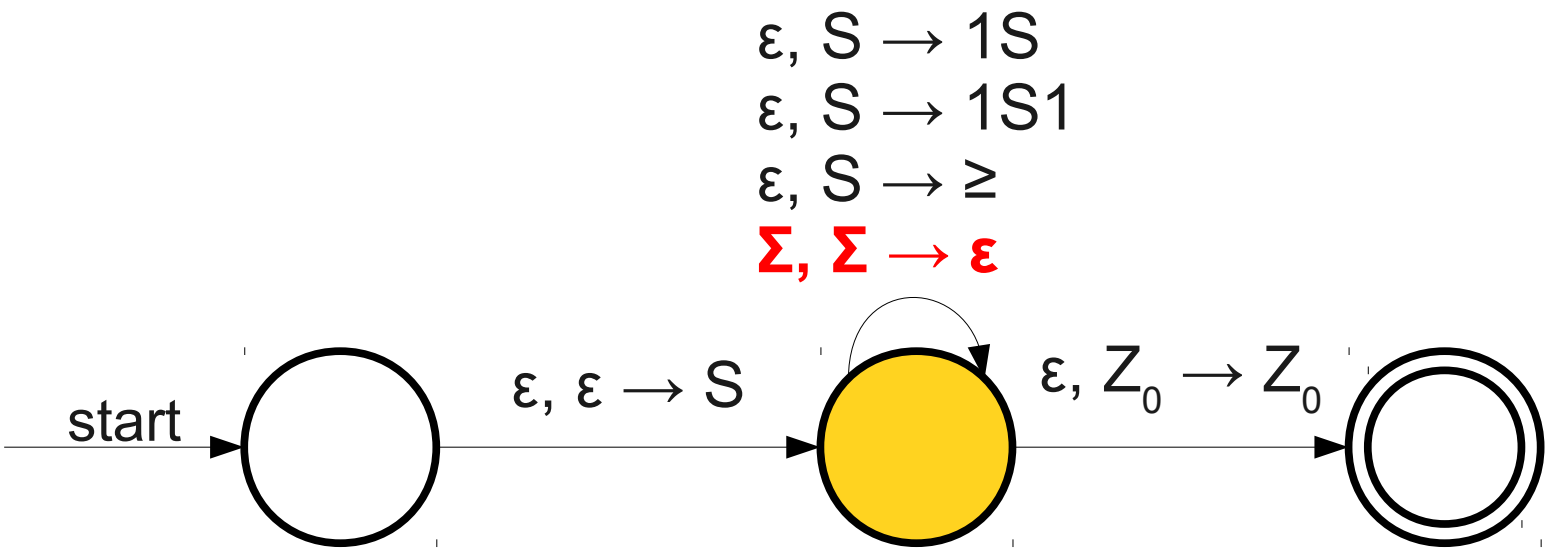


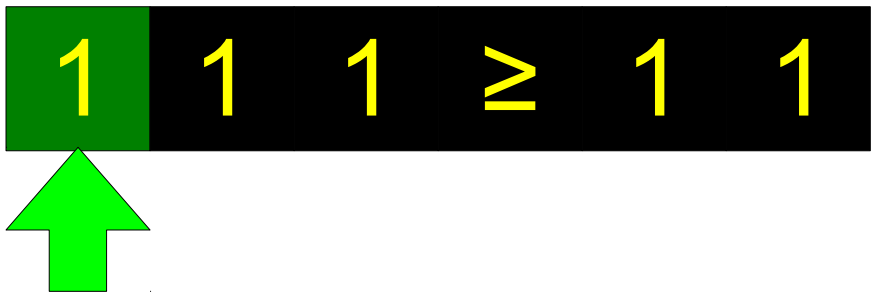


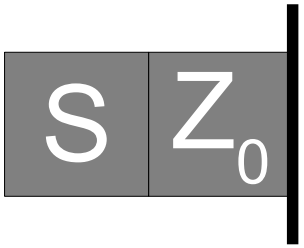


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

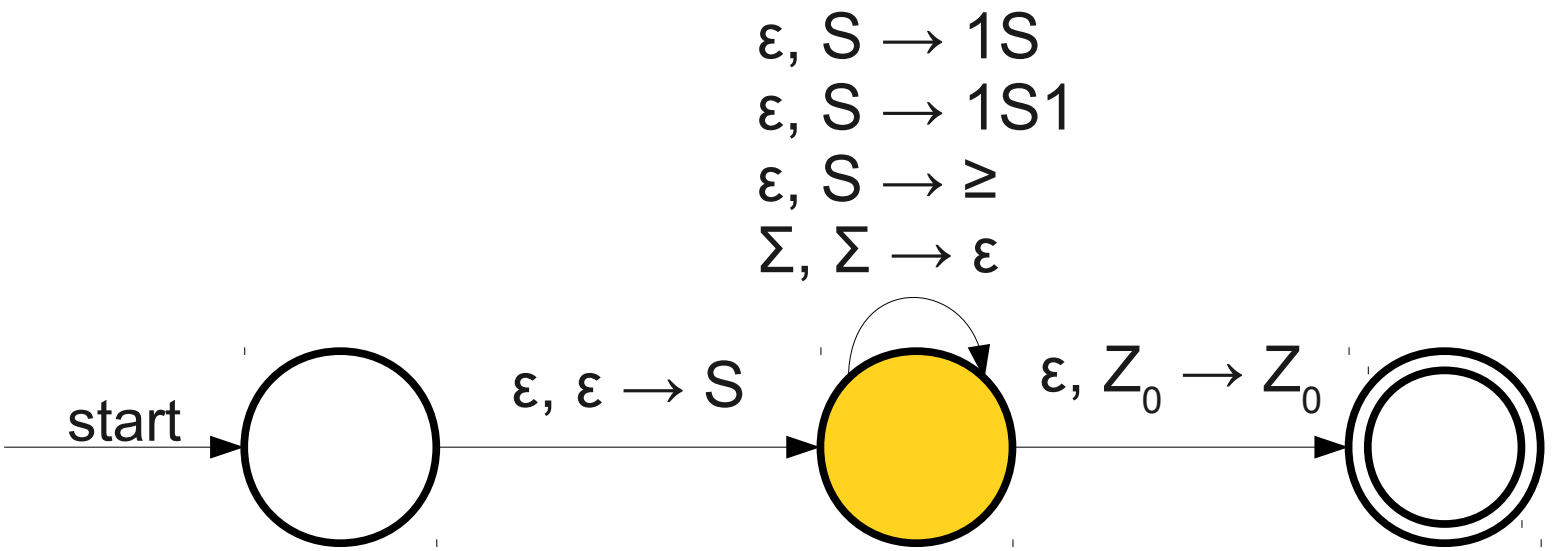


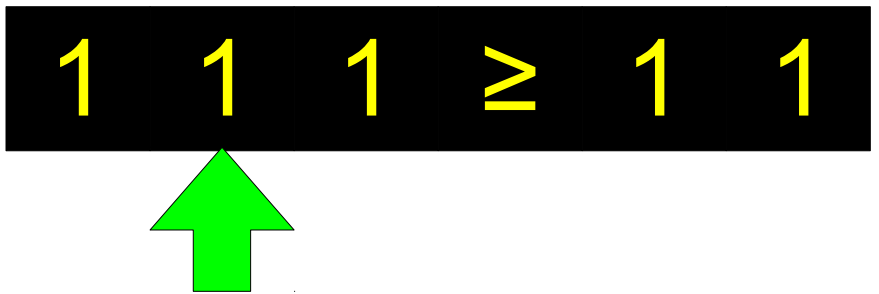


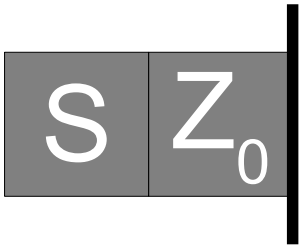


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

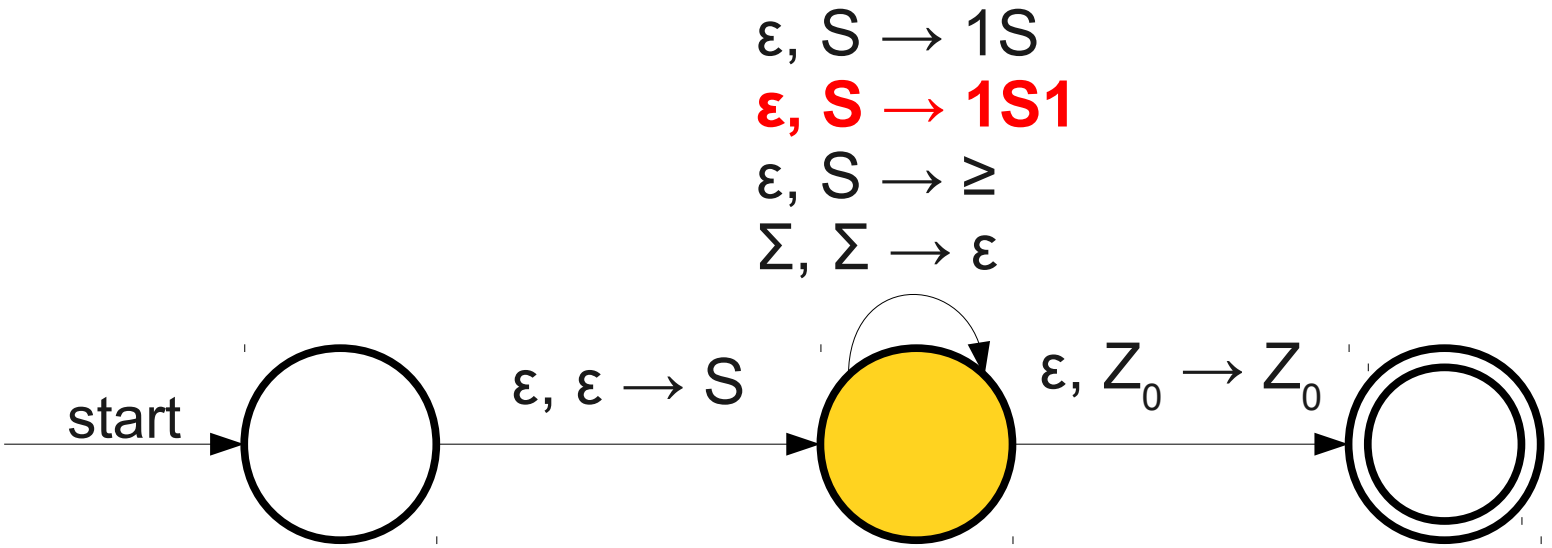


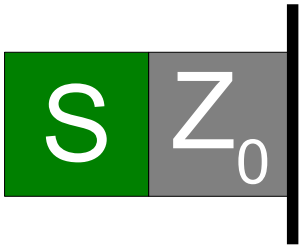




From CFGs to PDAs

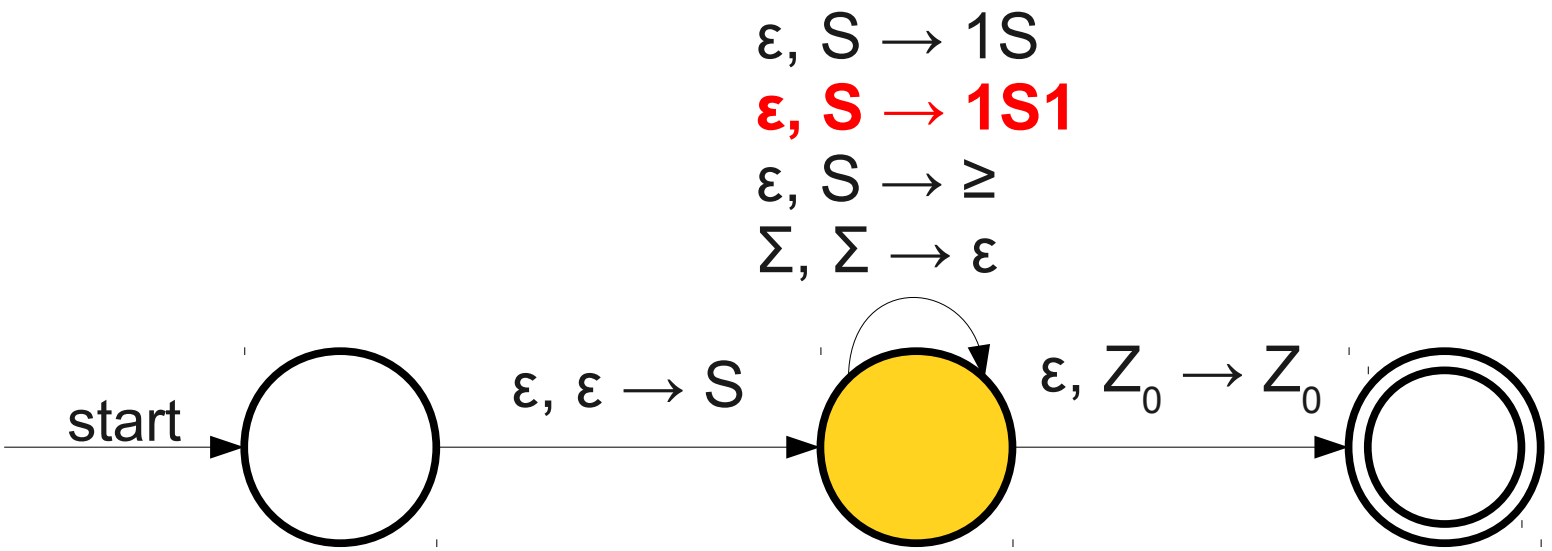
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

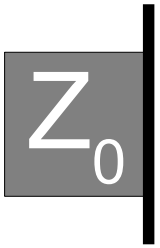




From CFGs to PDAs

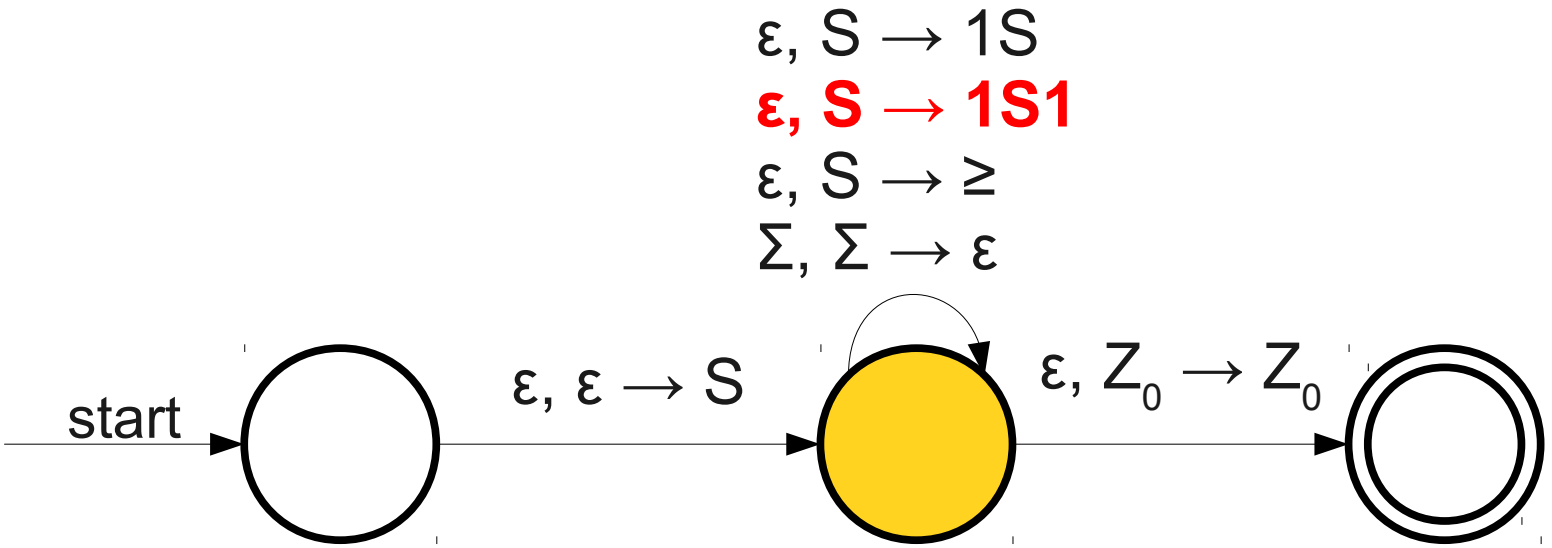
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

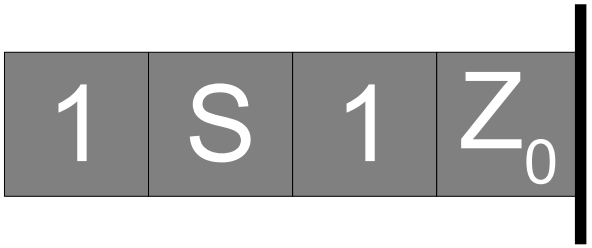




From CFGs to PDAs

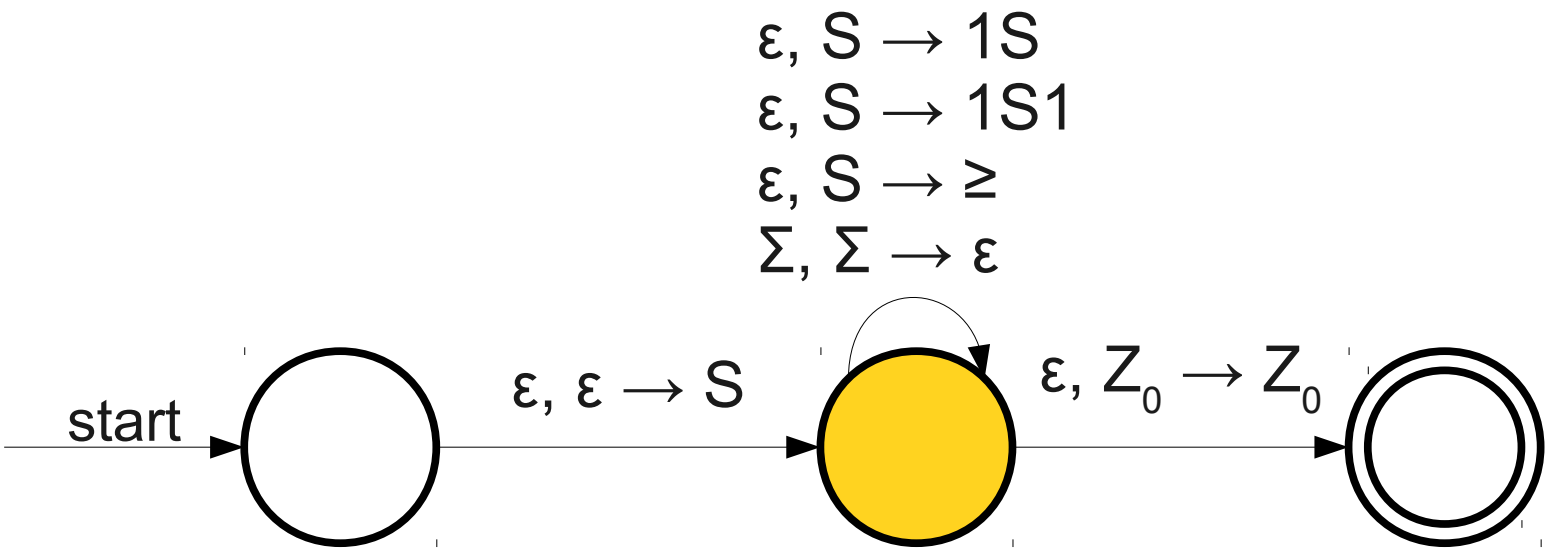
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

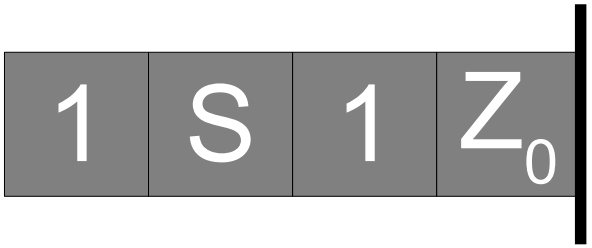




From CFGs to PDAs

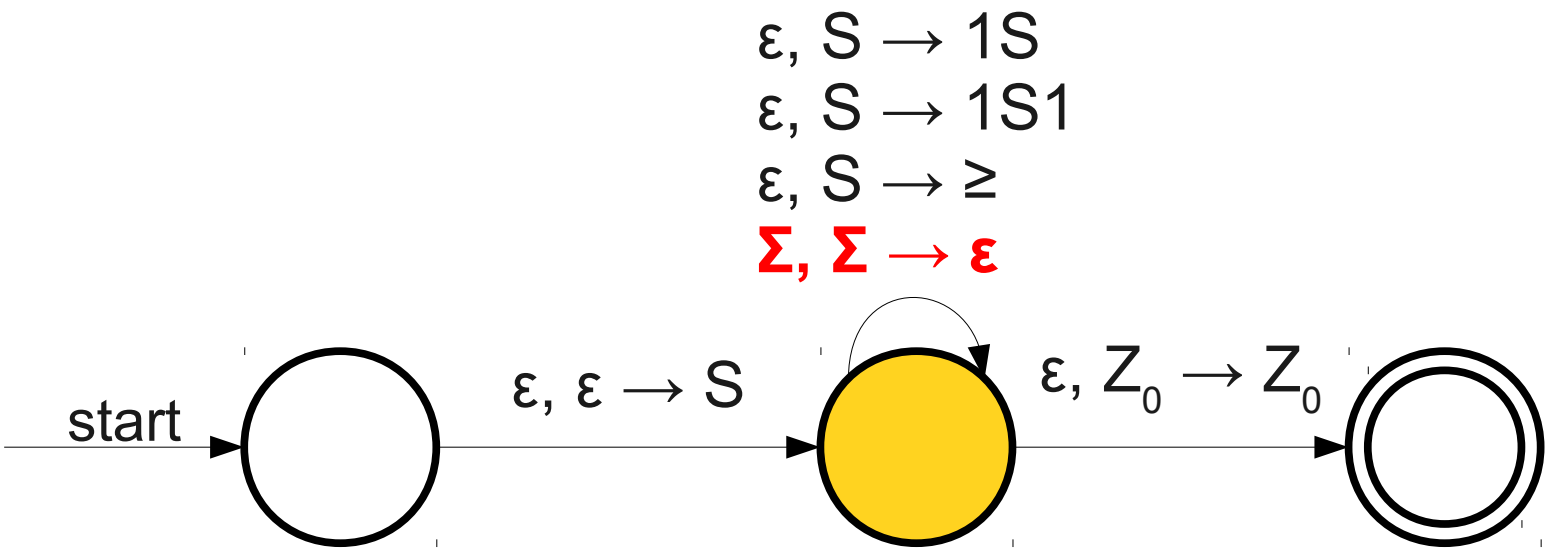
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

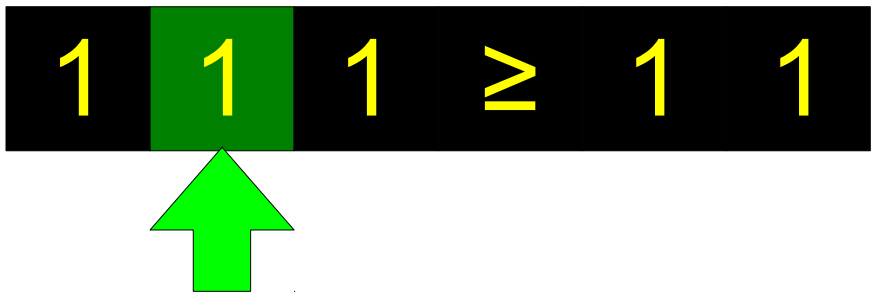


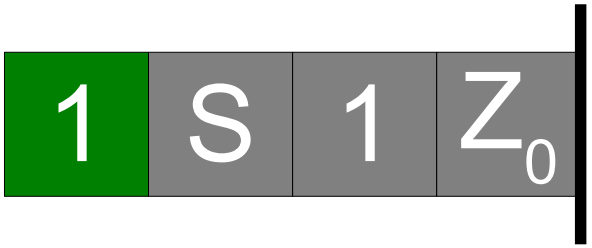


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

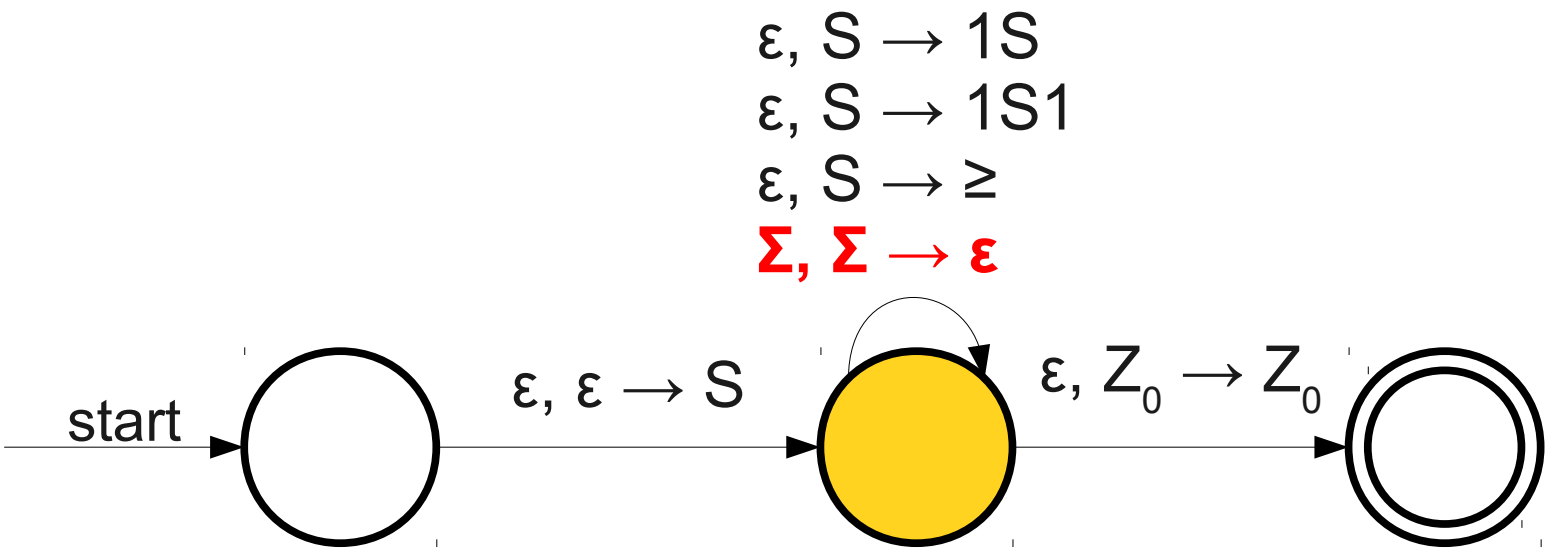


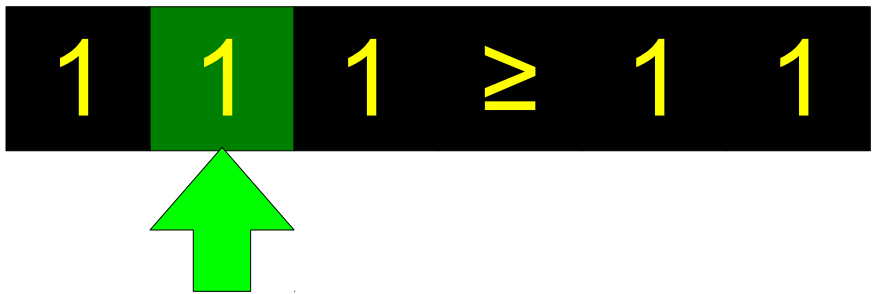


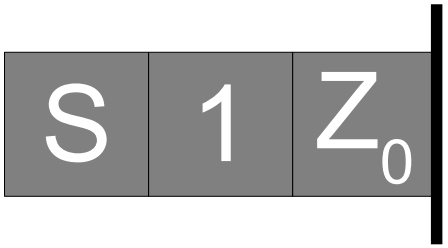


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

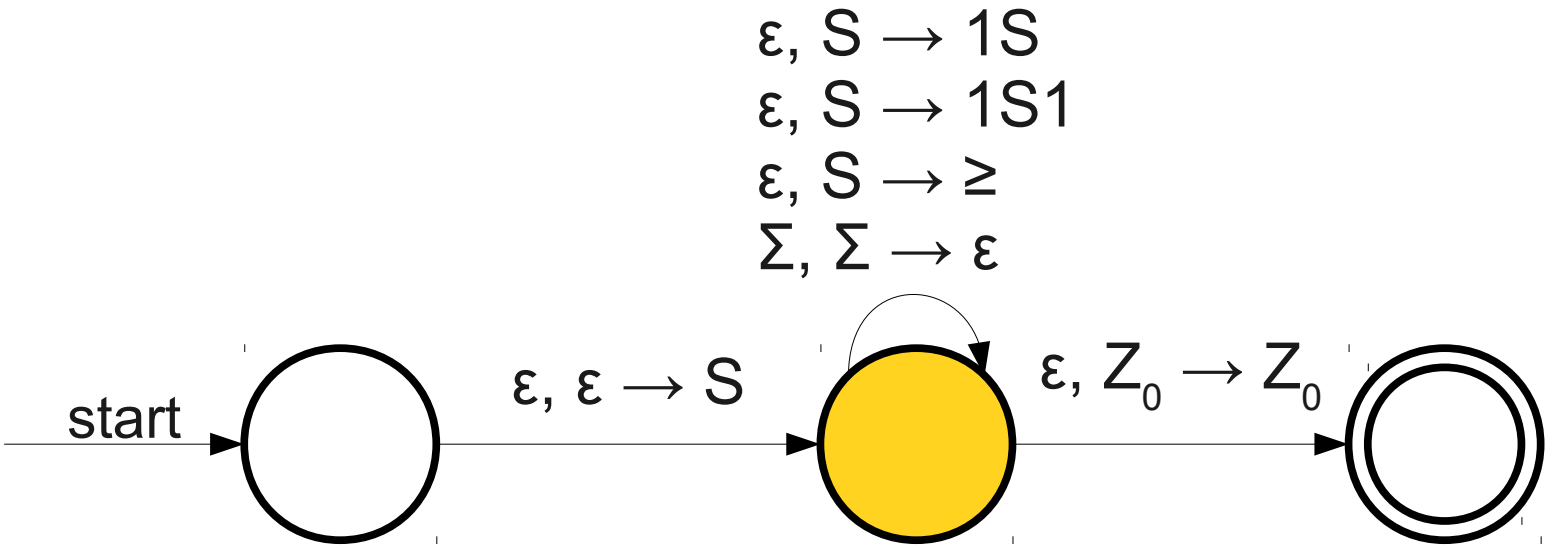


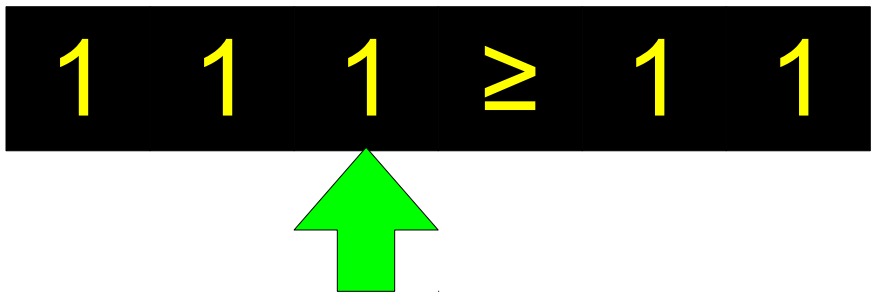


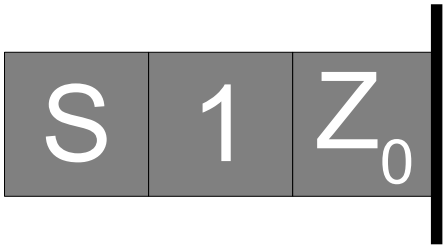


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

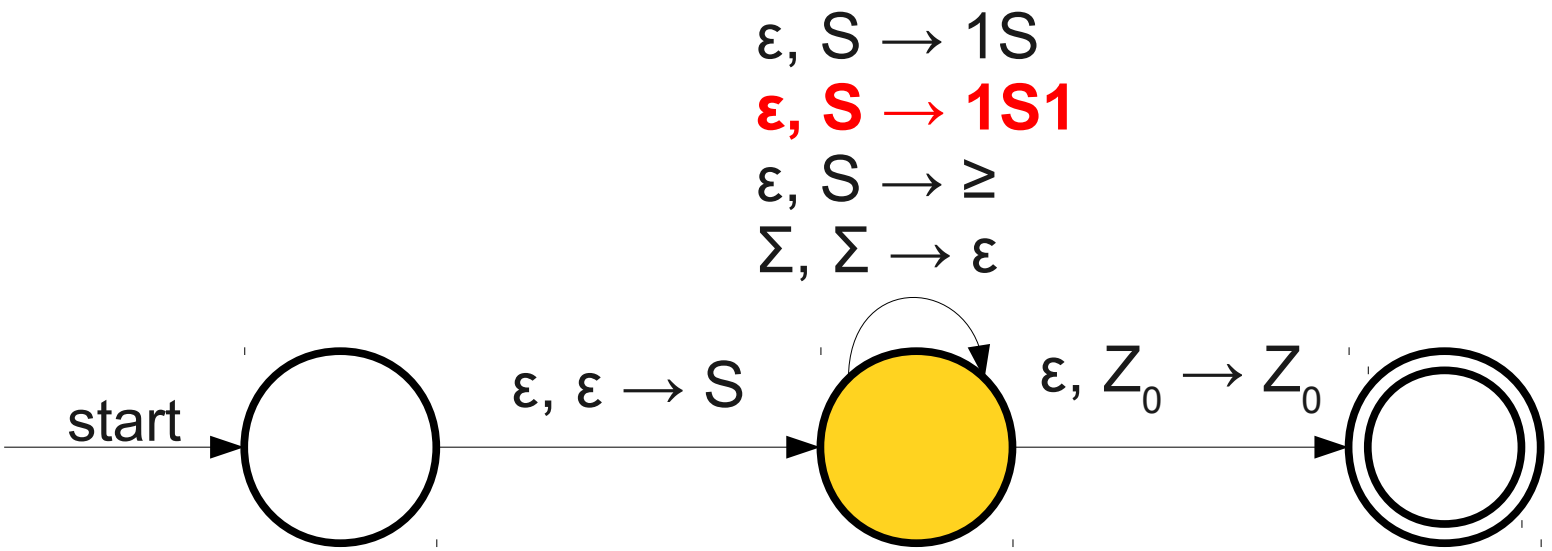


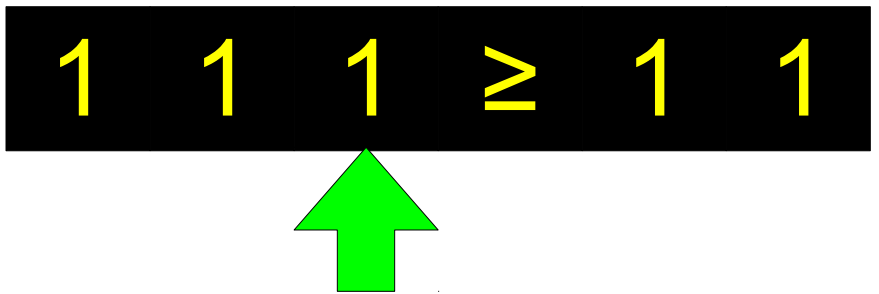


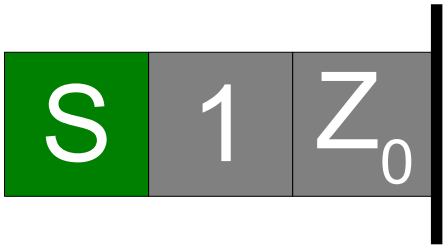


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

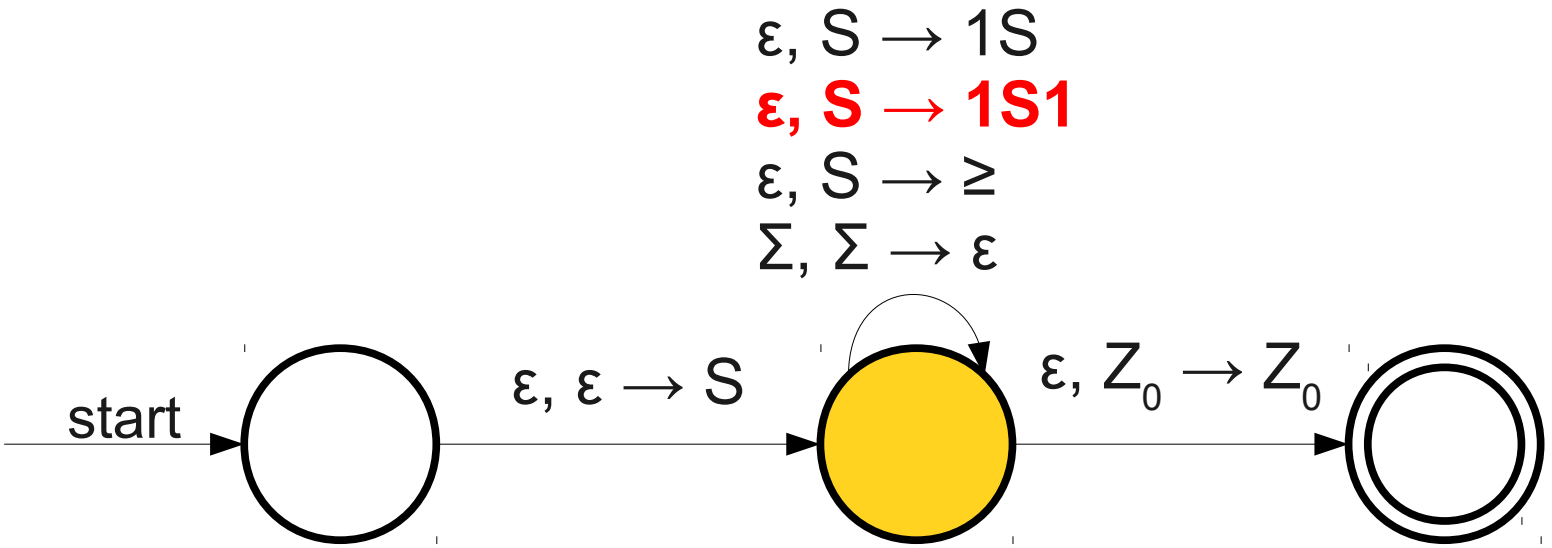


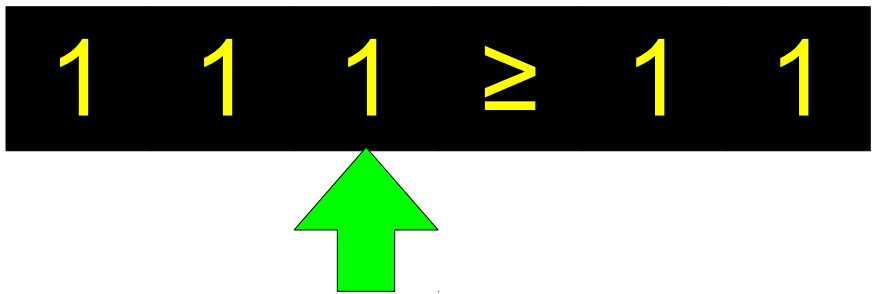


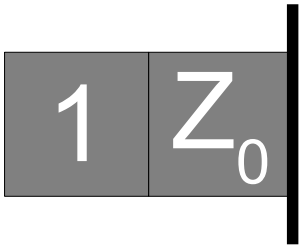


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

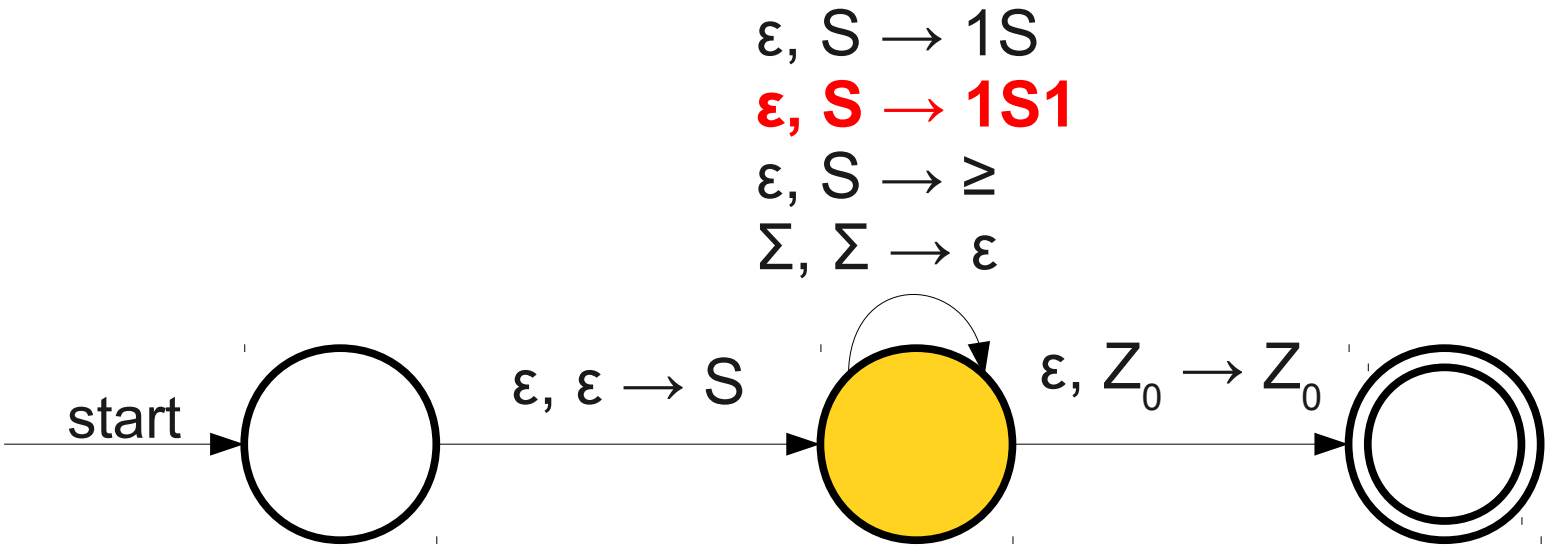


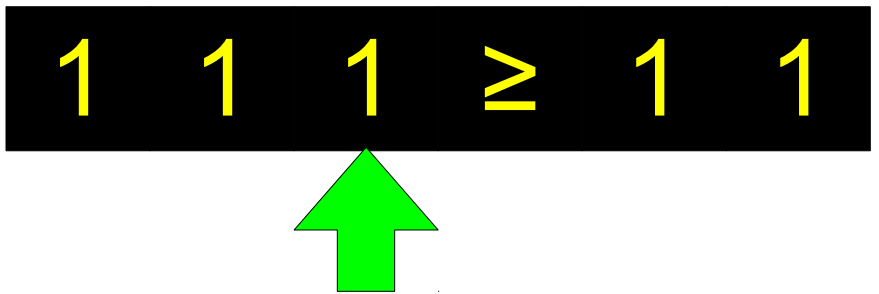


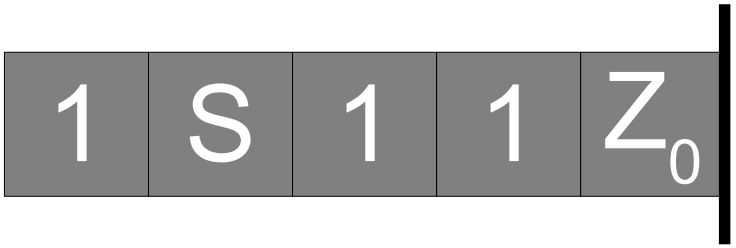


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

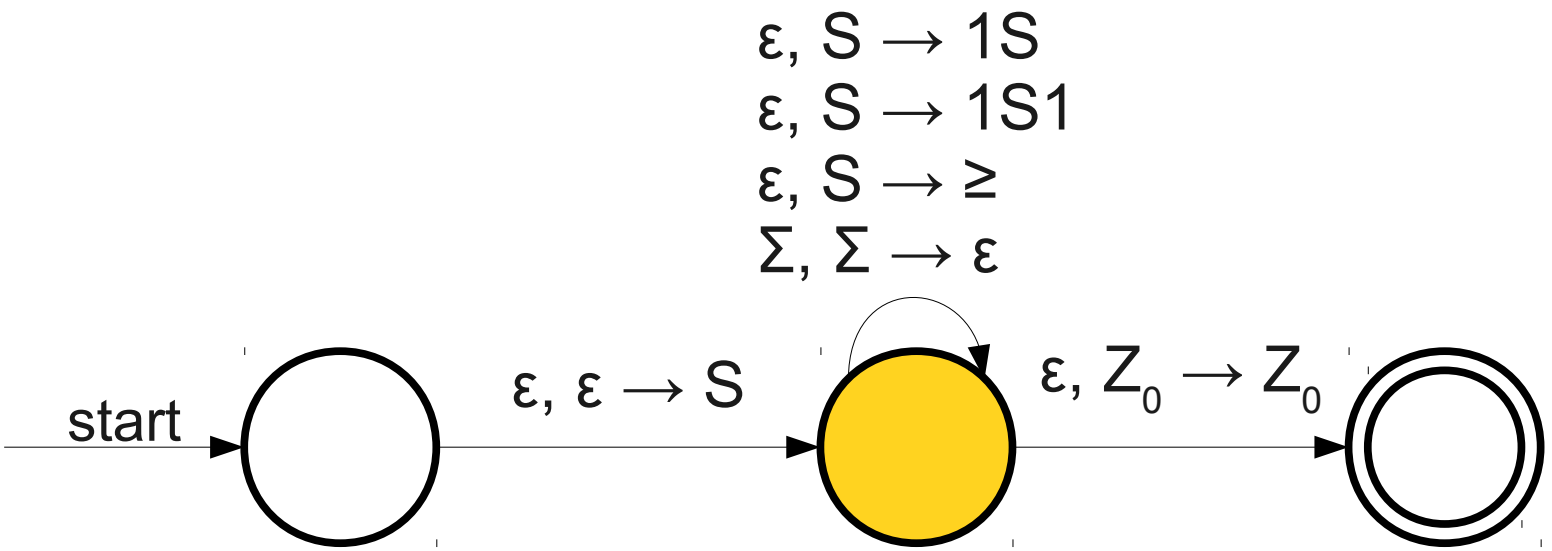


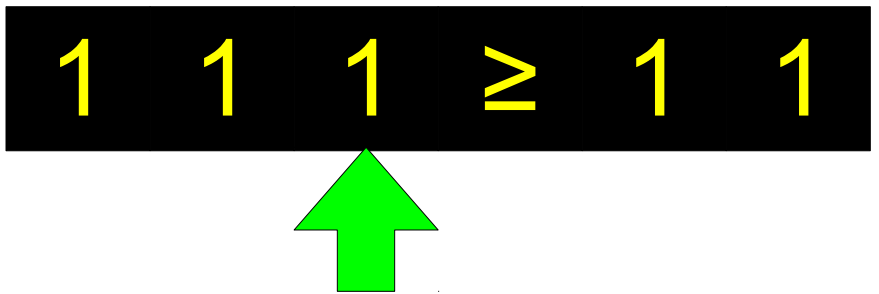


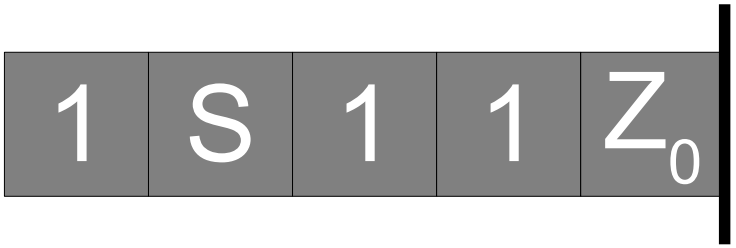


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

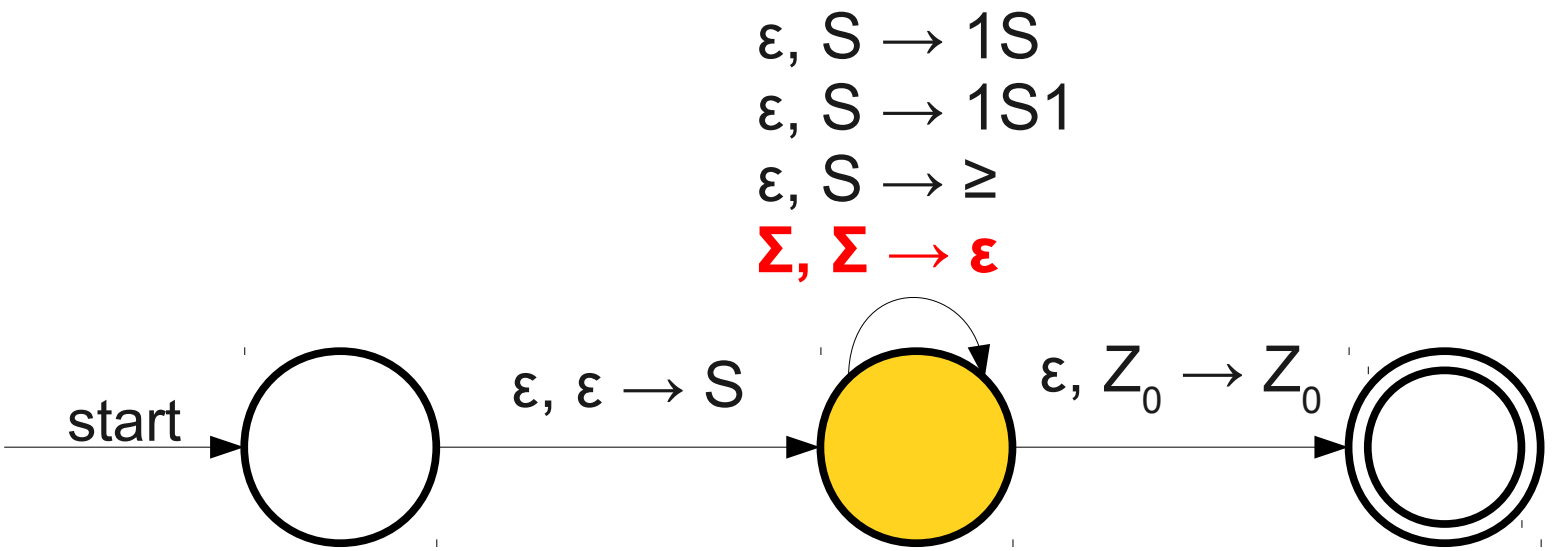


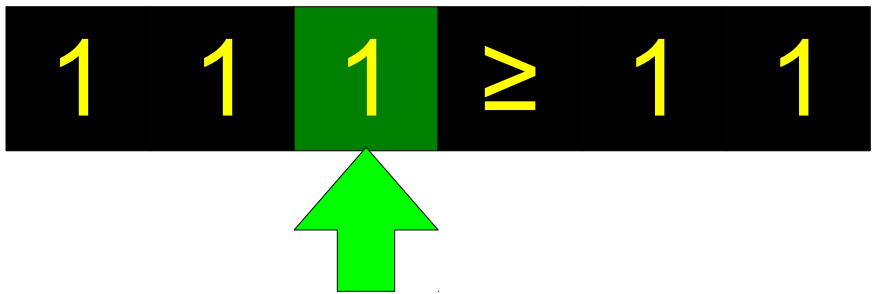


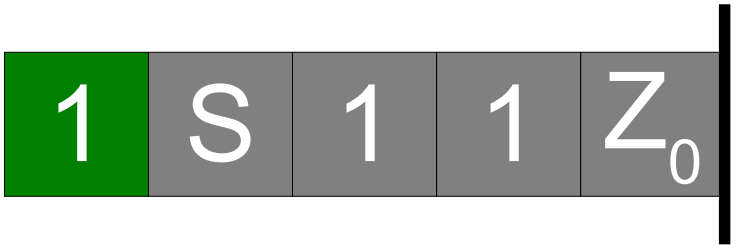


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

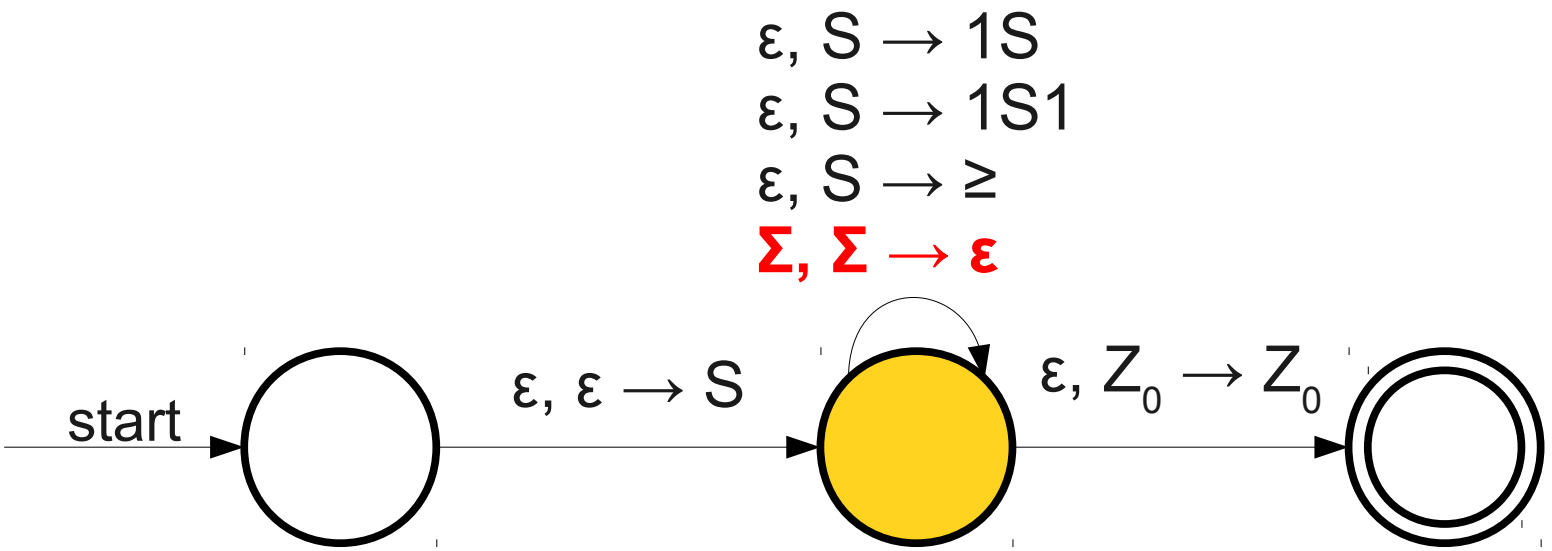


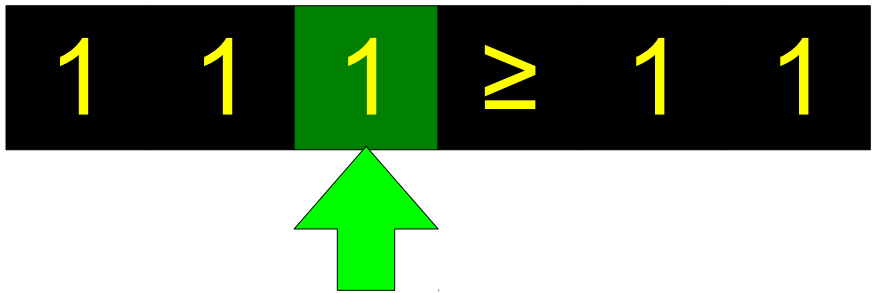


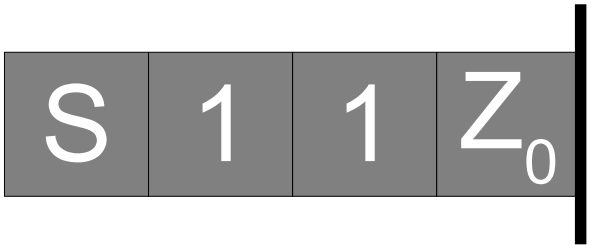


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

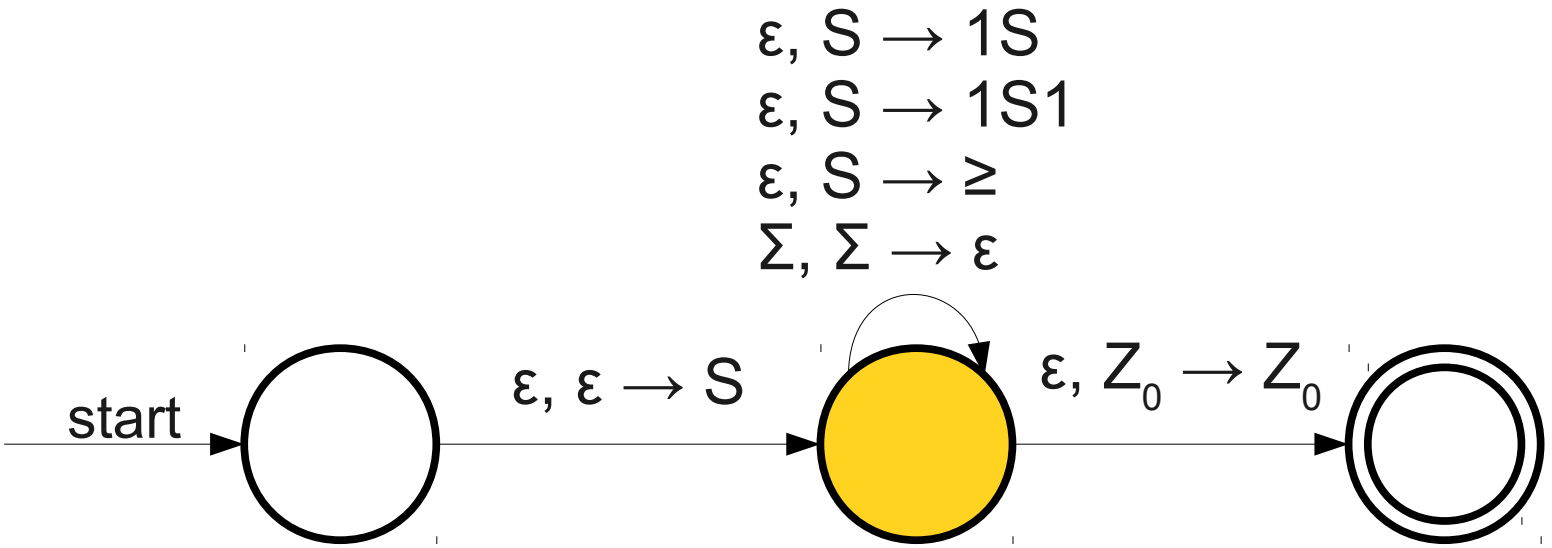


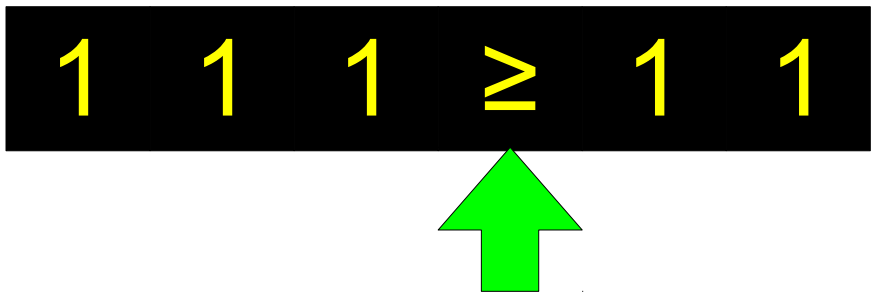


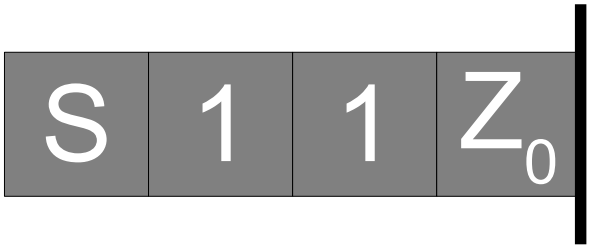


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

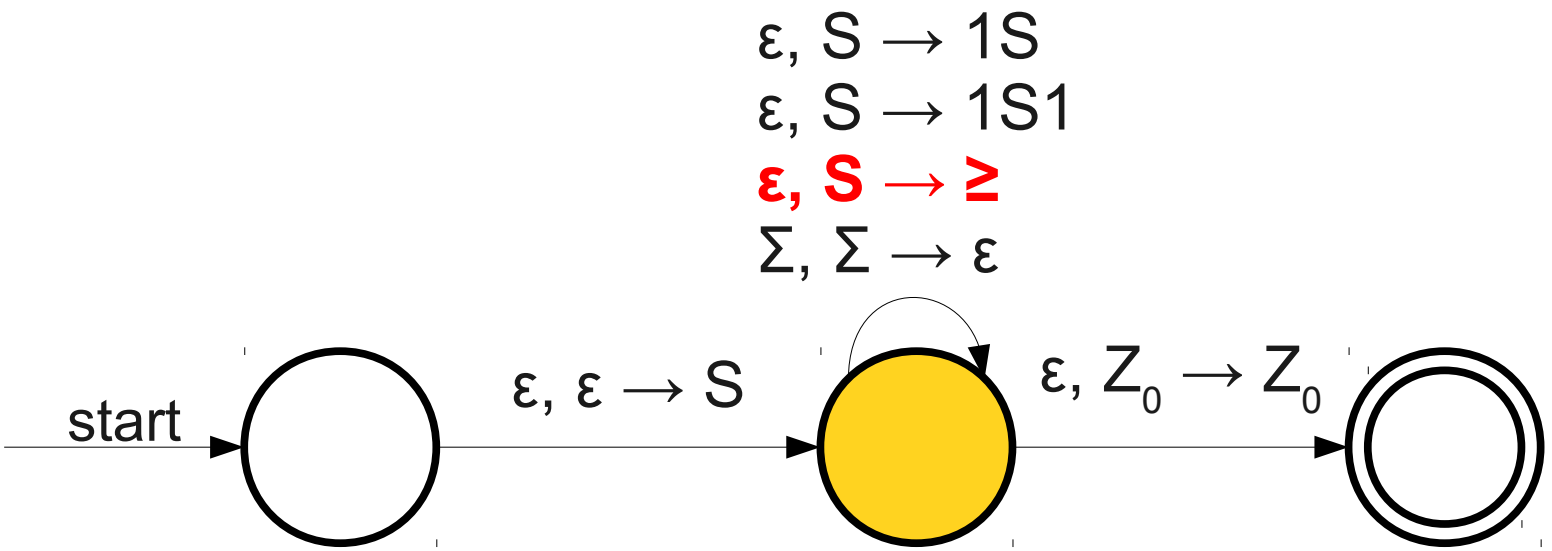


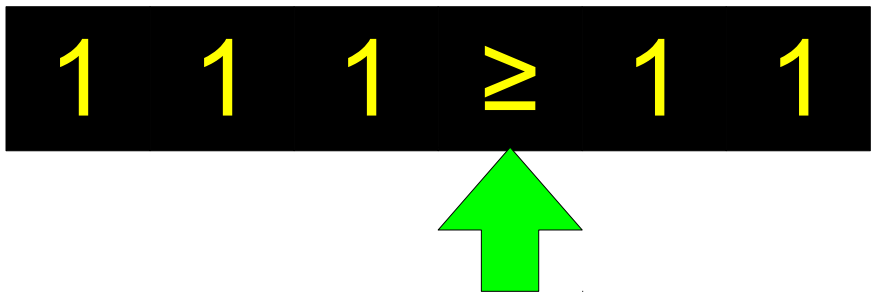


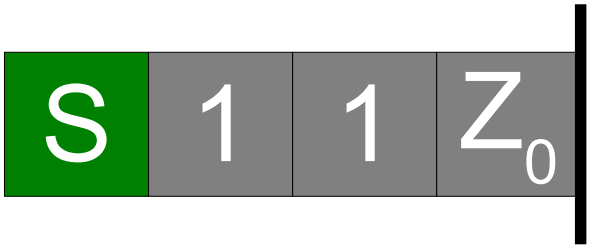


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

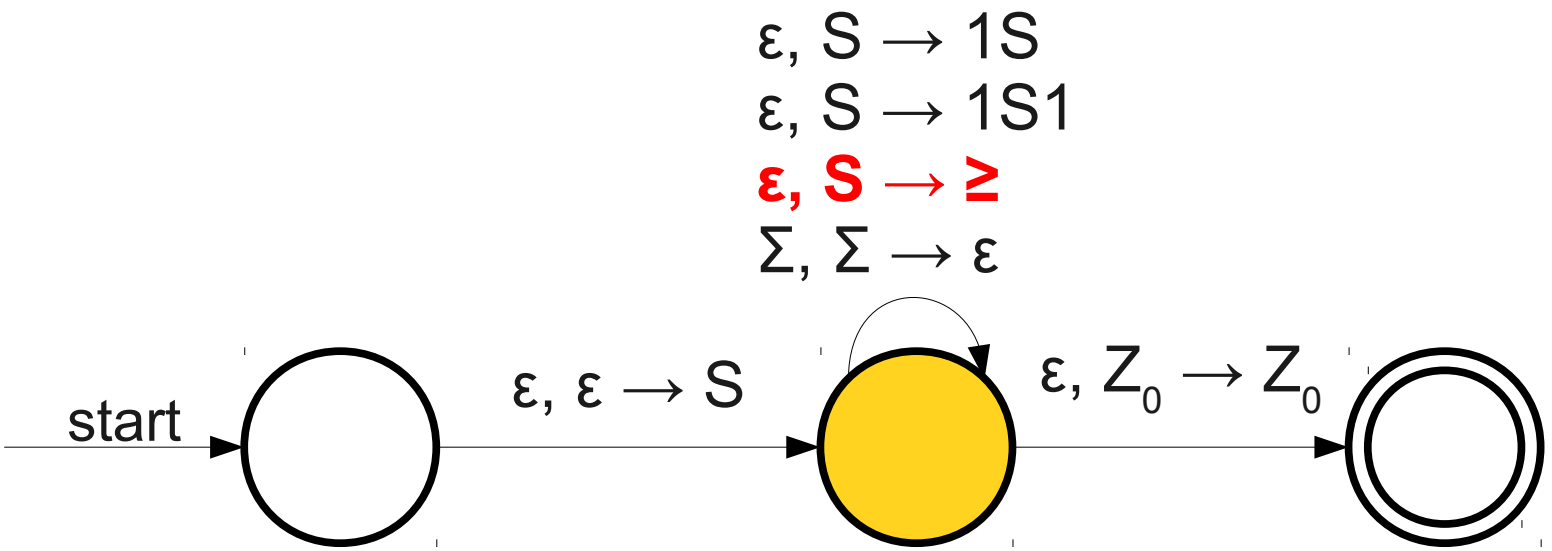


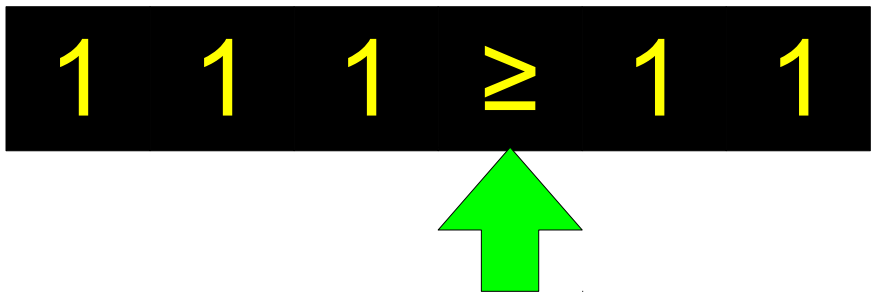


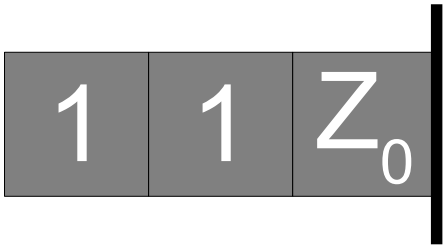


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

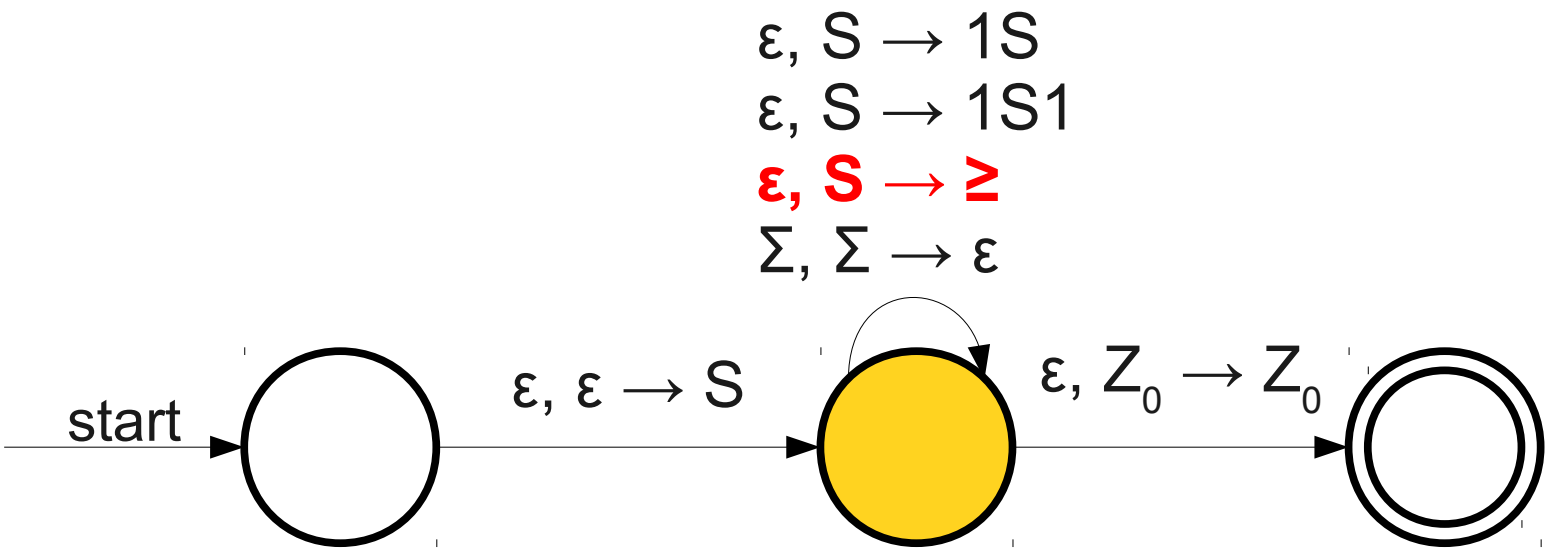


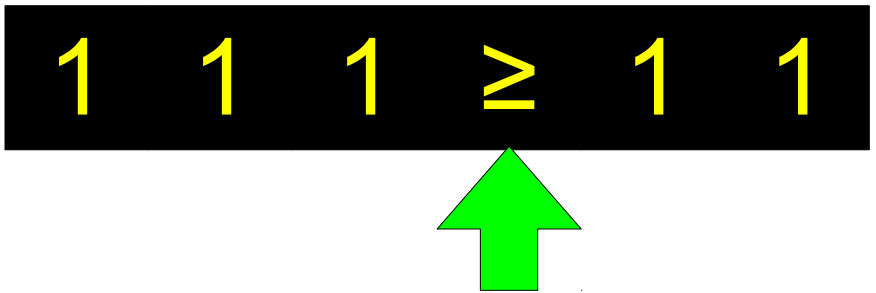


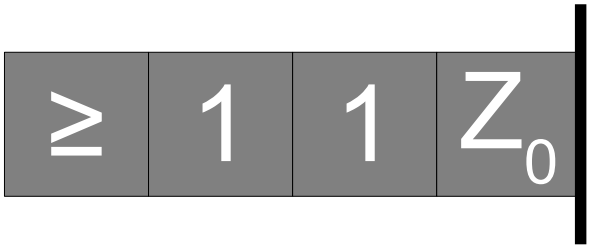


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

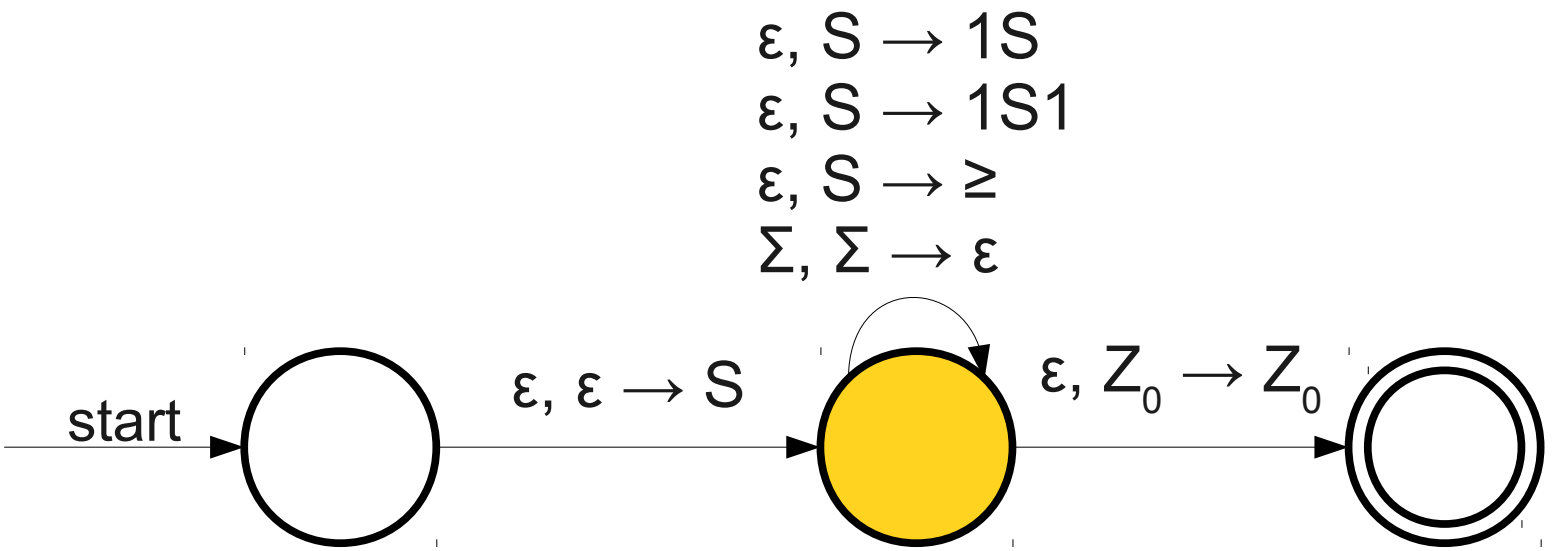


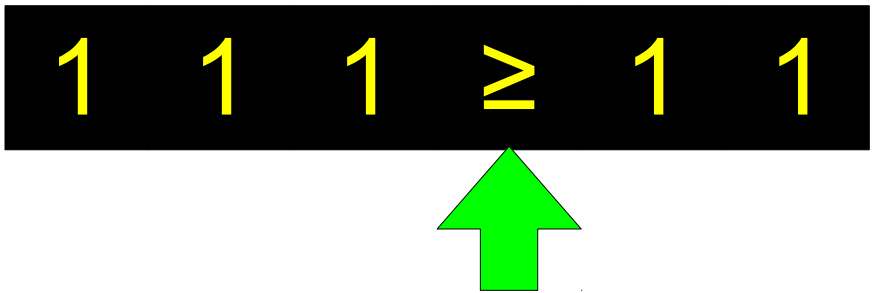


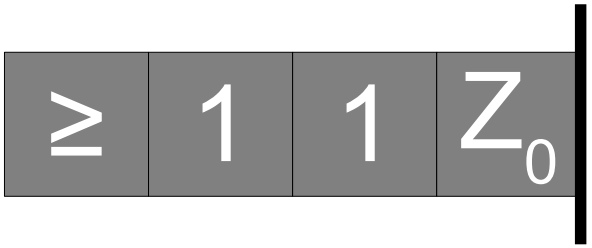


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

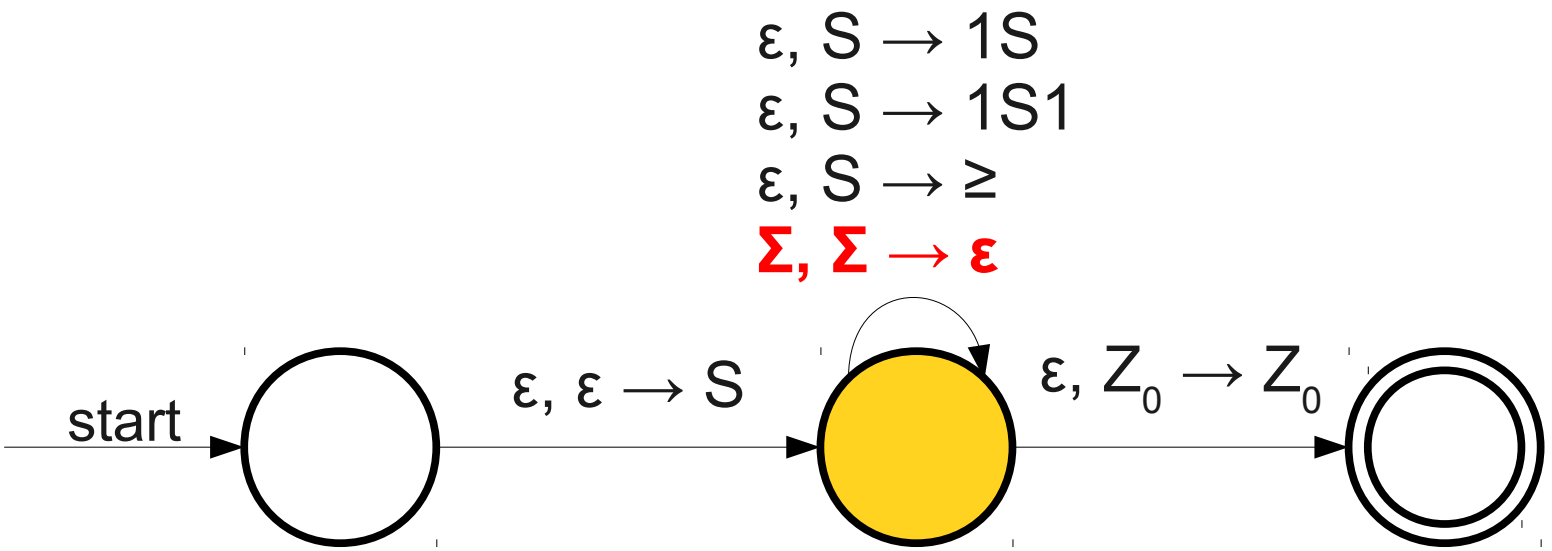


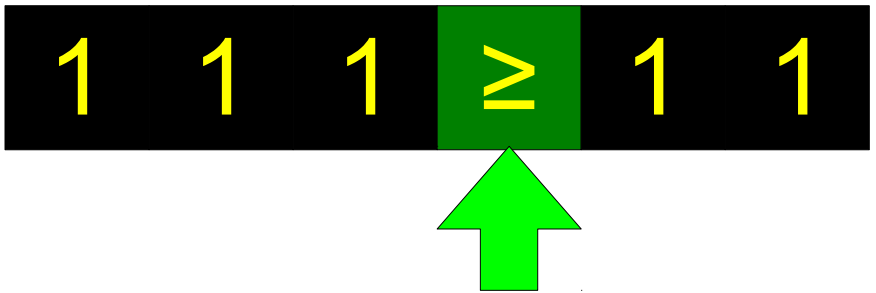


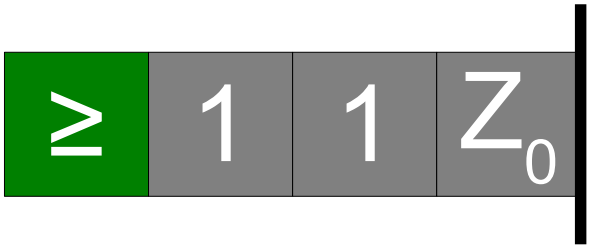


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

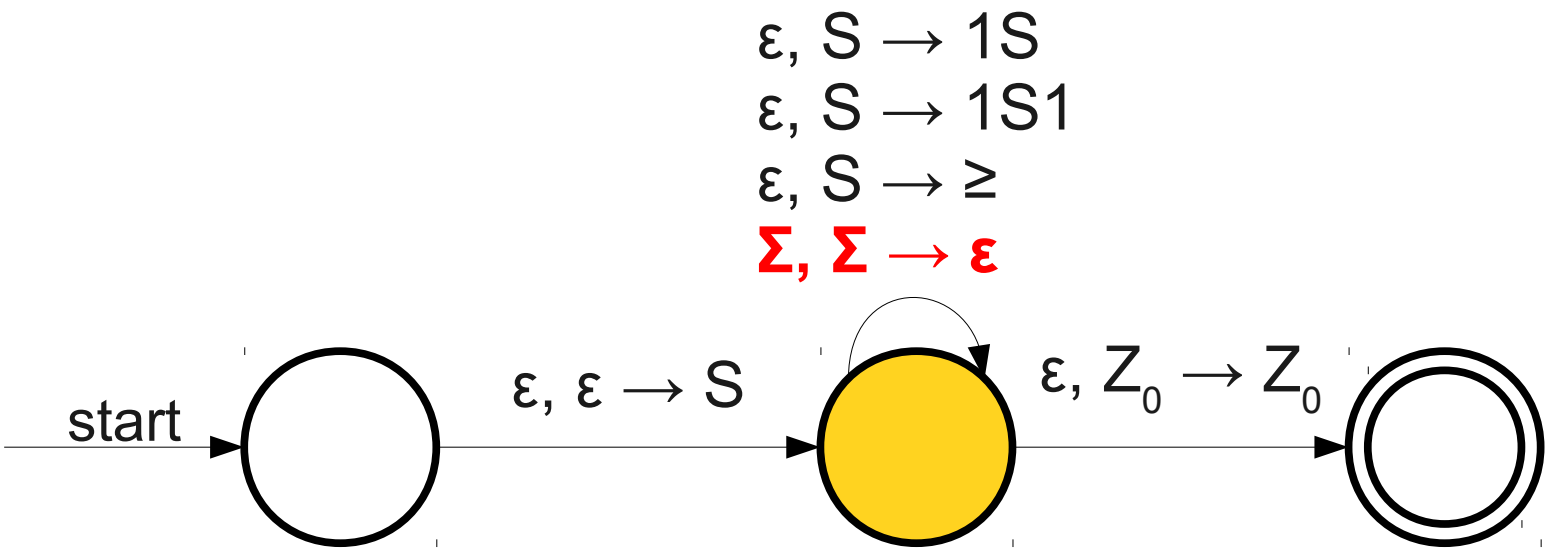


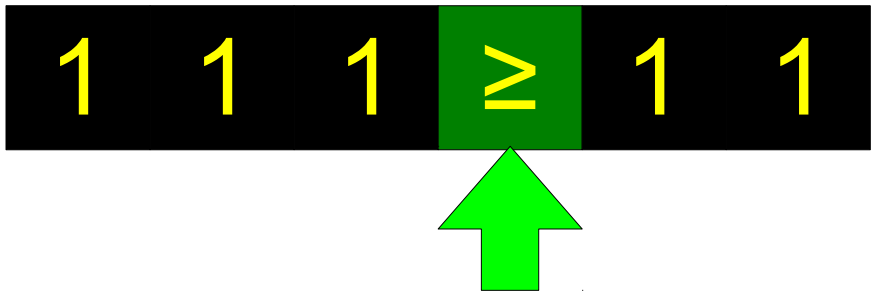


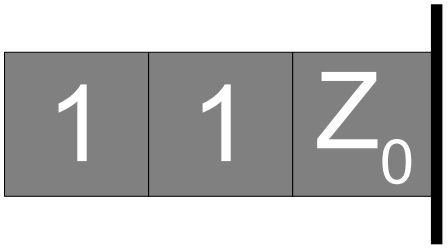


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

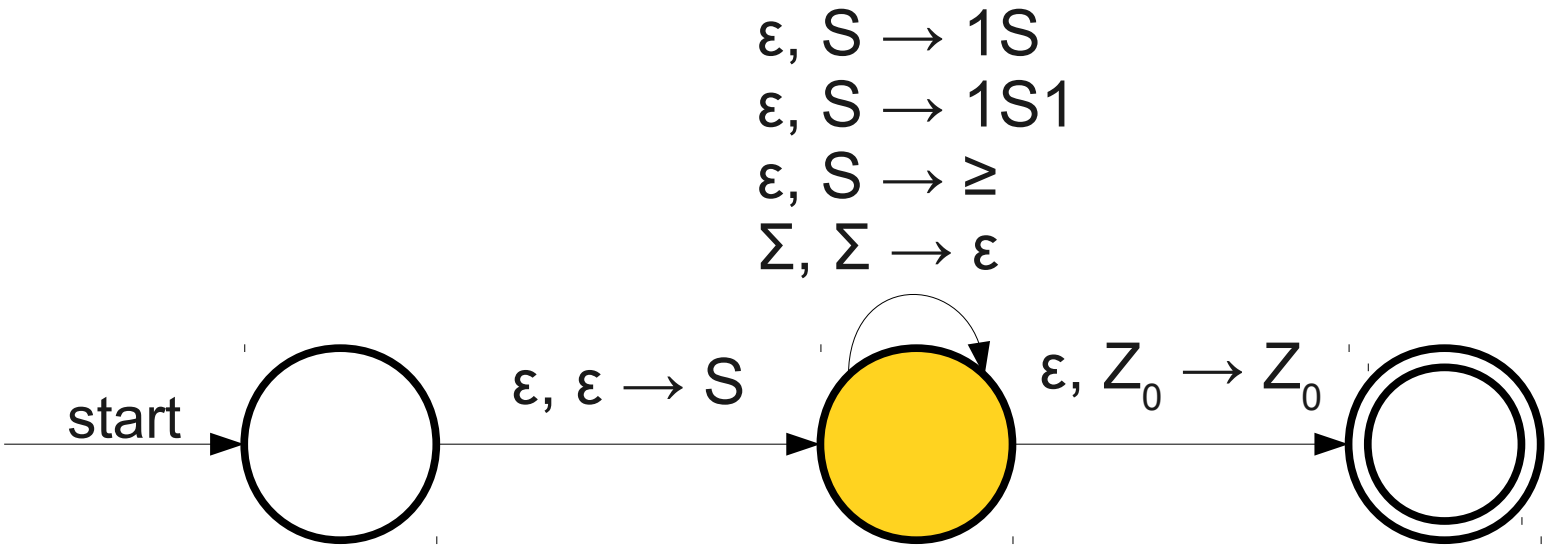


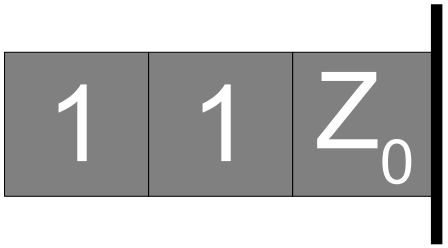




From CFGs to PDAs

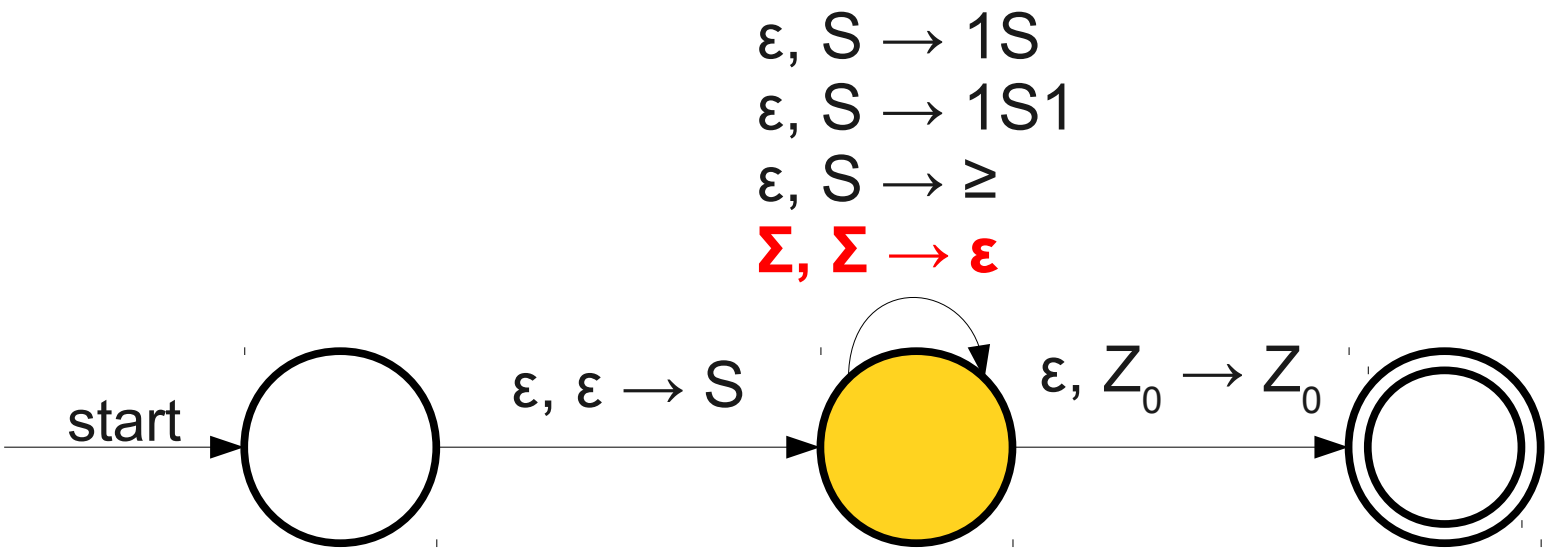
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

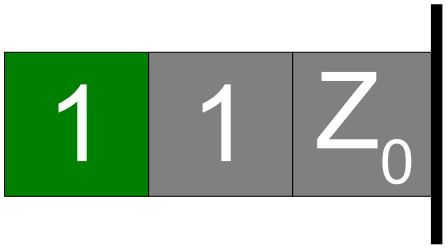




From CFGs to PDAs

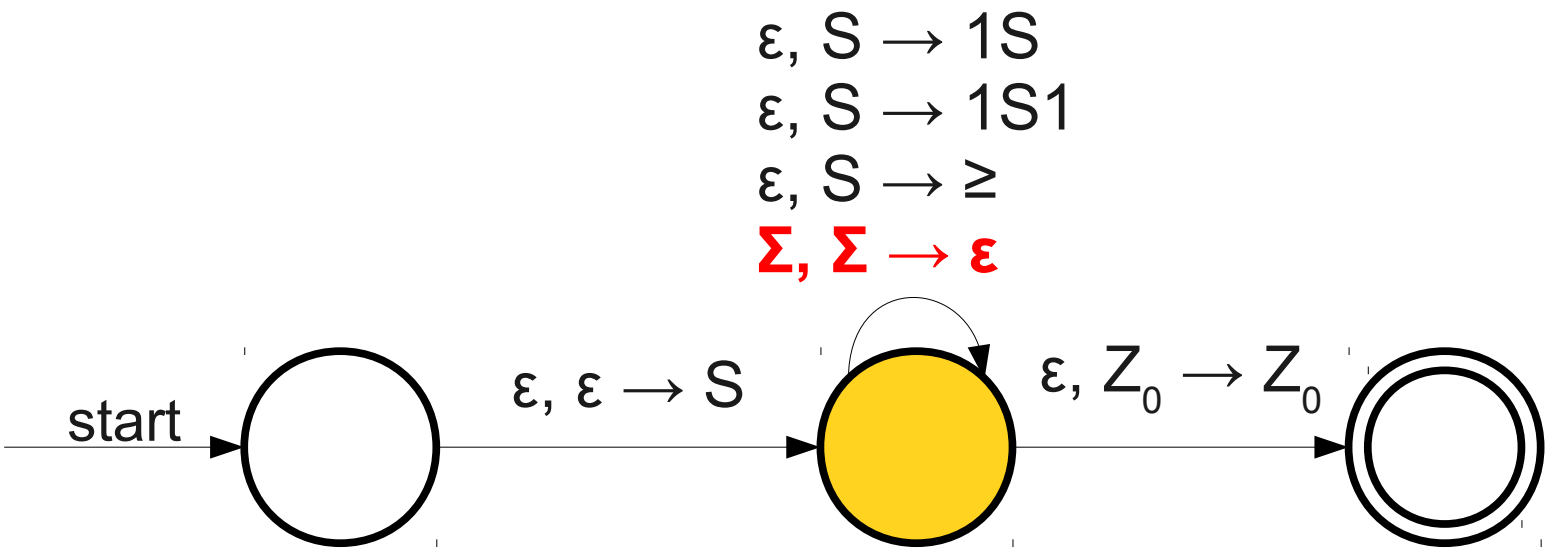
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

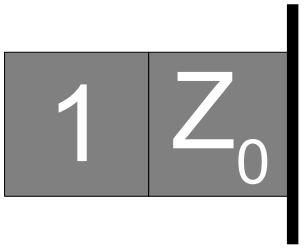




From CFGs to PDAs

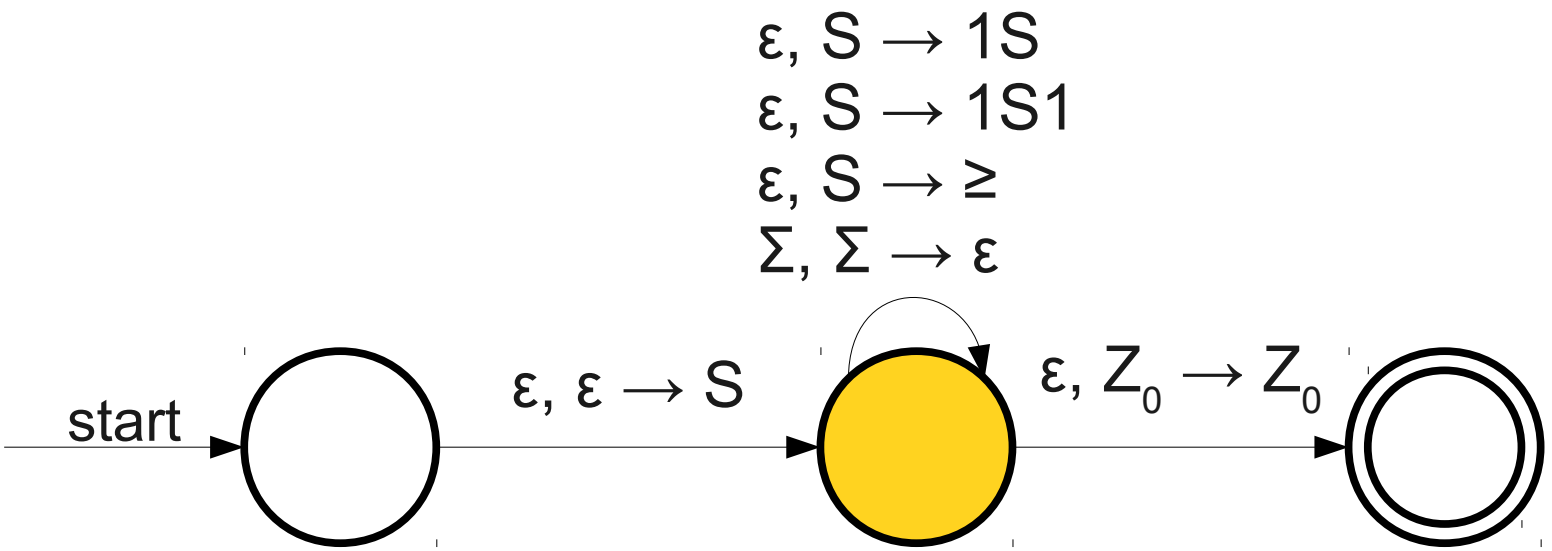
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

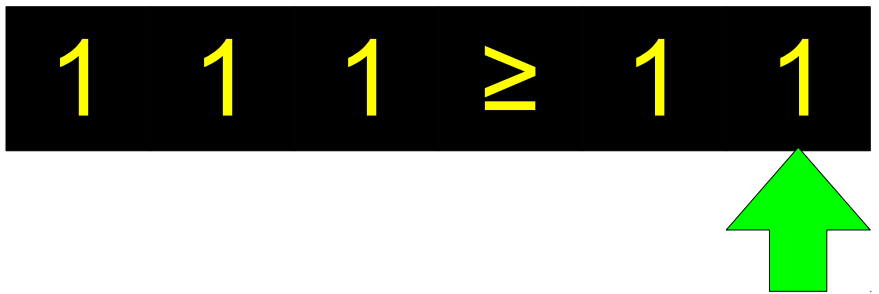


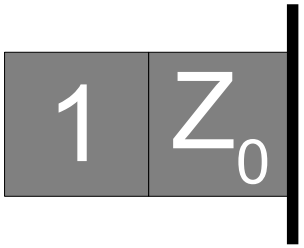


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

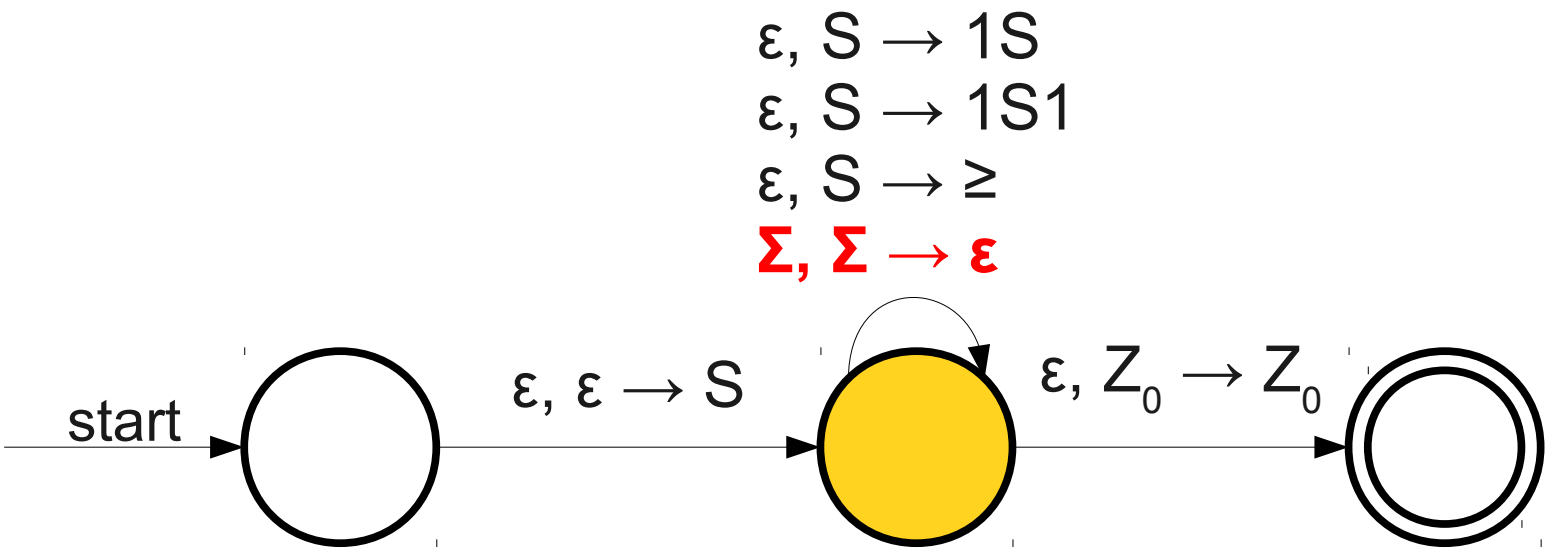


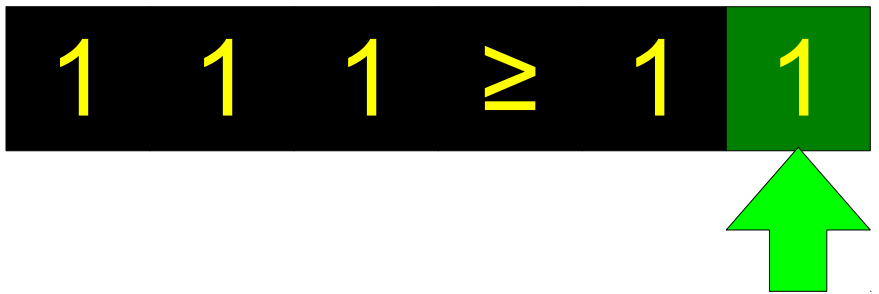


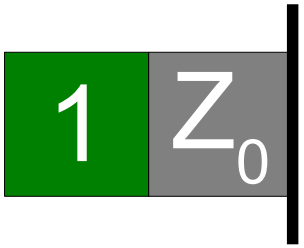


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

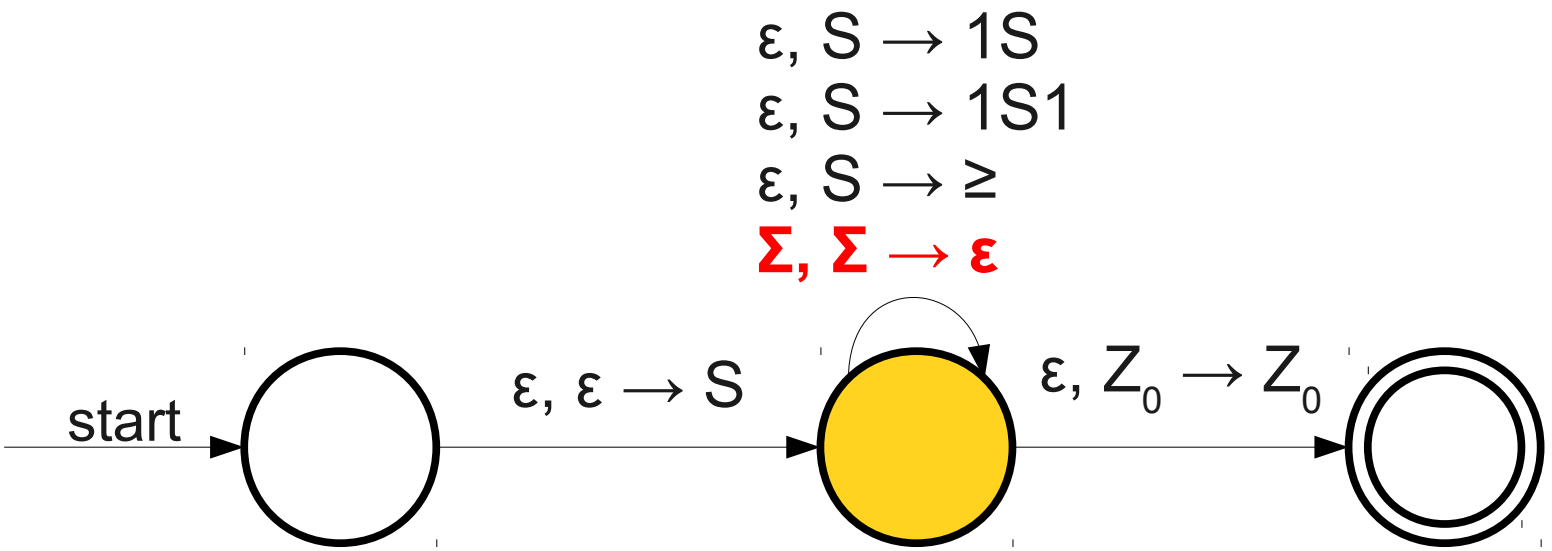


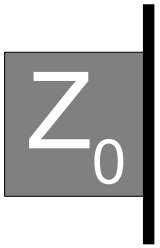




From CFGs to PDAs

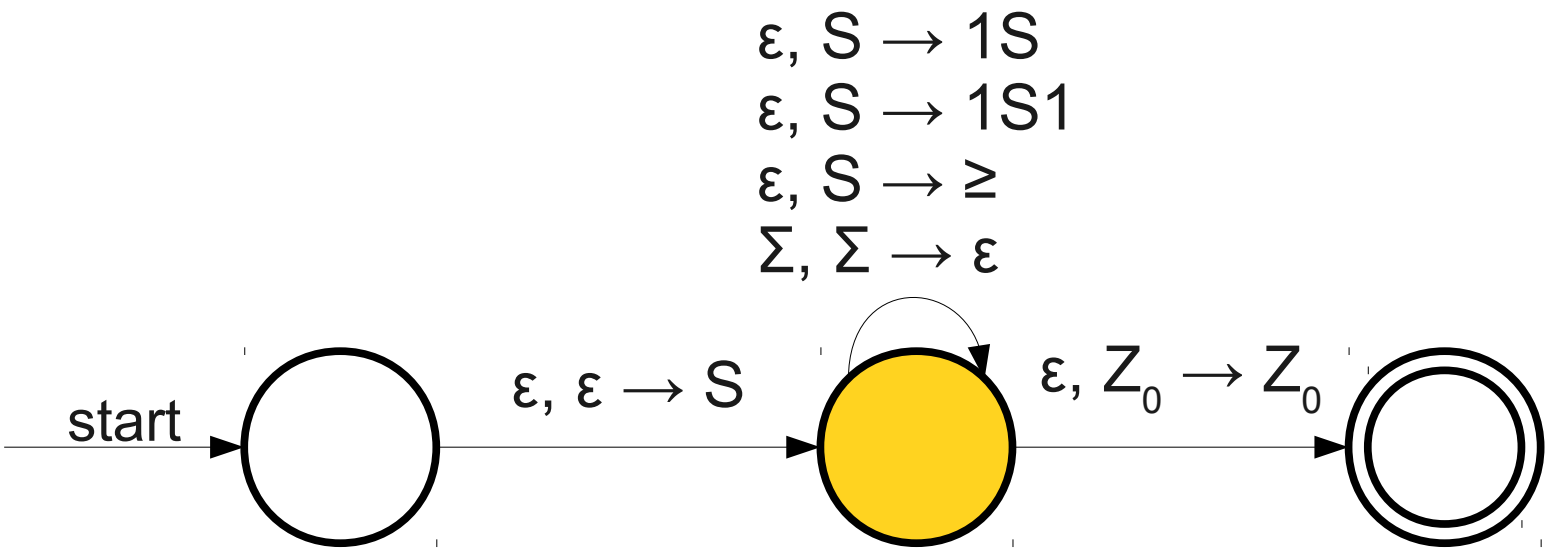
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



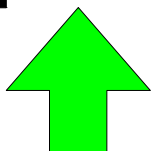


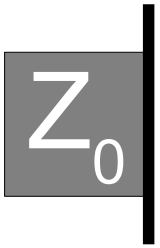
From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



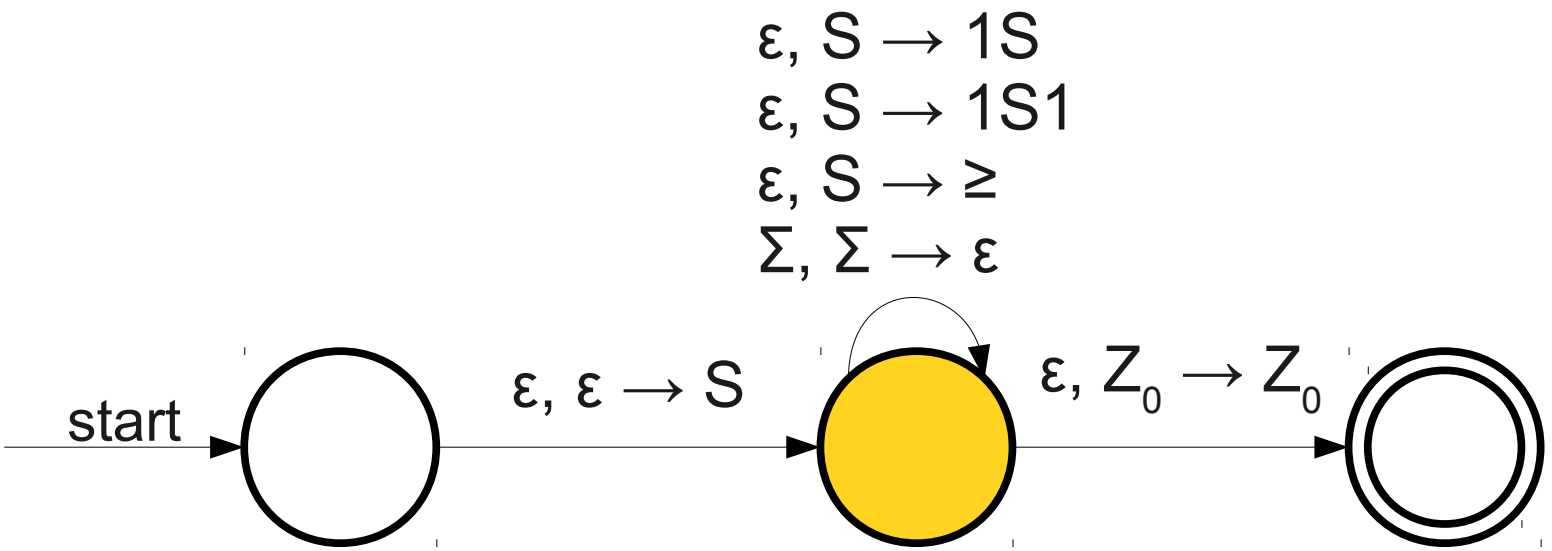
1 1 1 ≥ 1 1



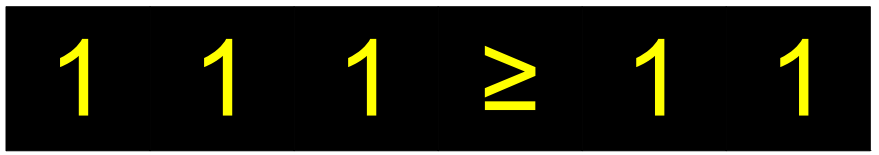


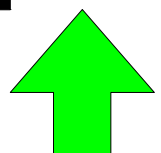
From CFGs to PDAs

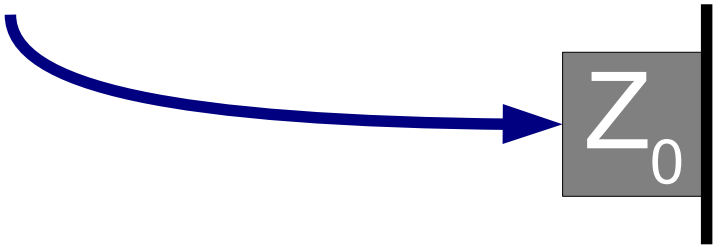
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



At this point we've completely matched the string, so it's time to transition to the accepting state.

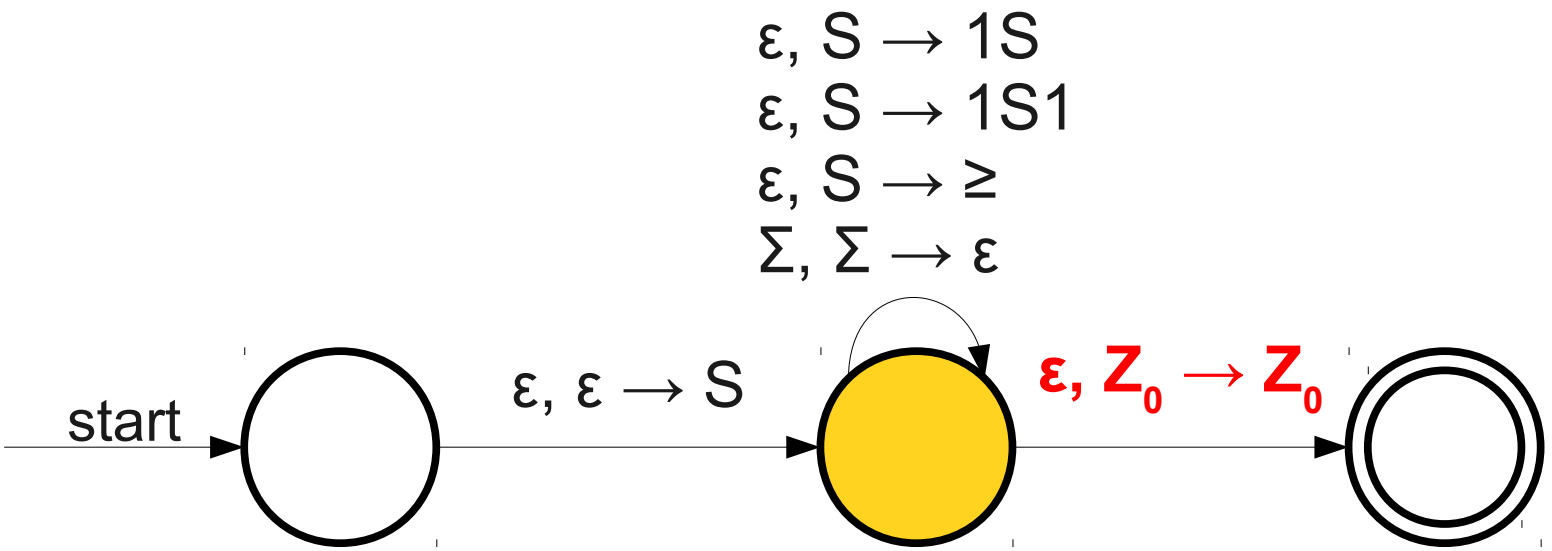




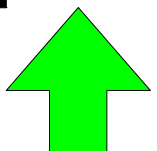


From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$

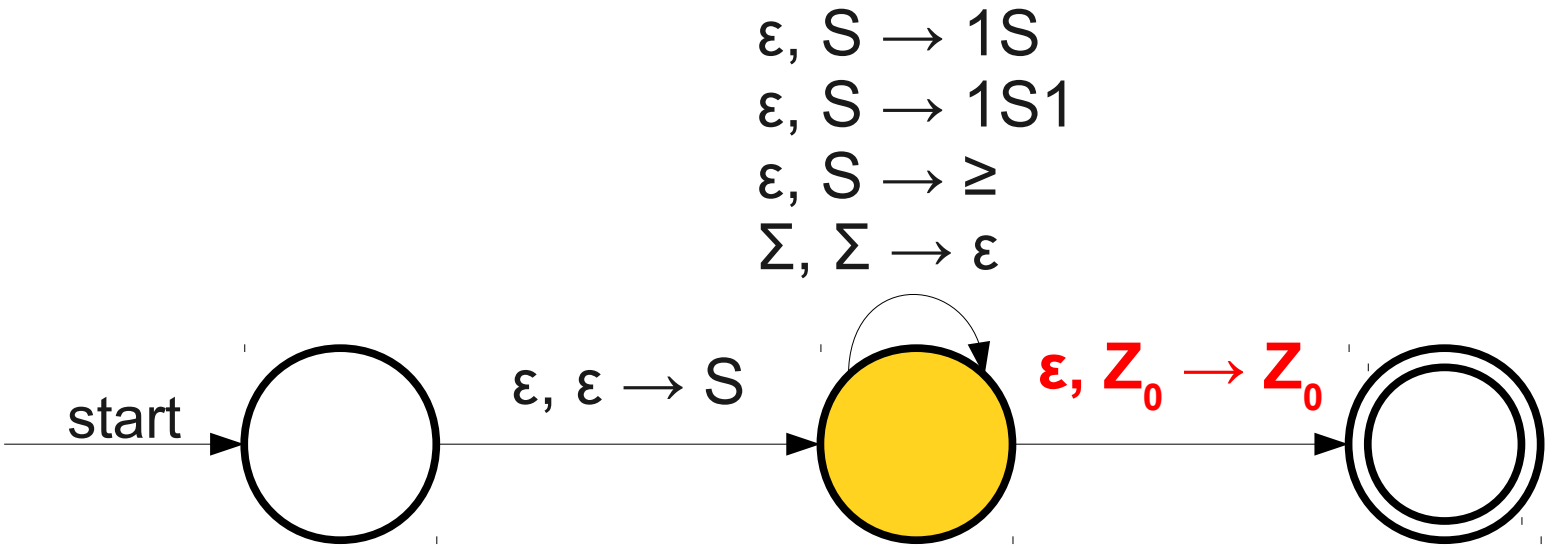


1 1 1 ≥ 1 1



From CFGs to PDAs

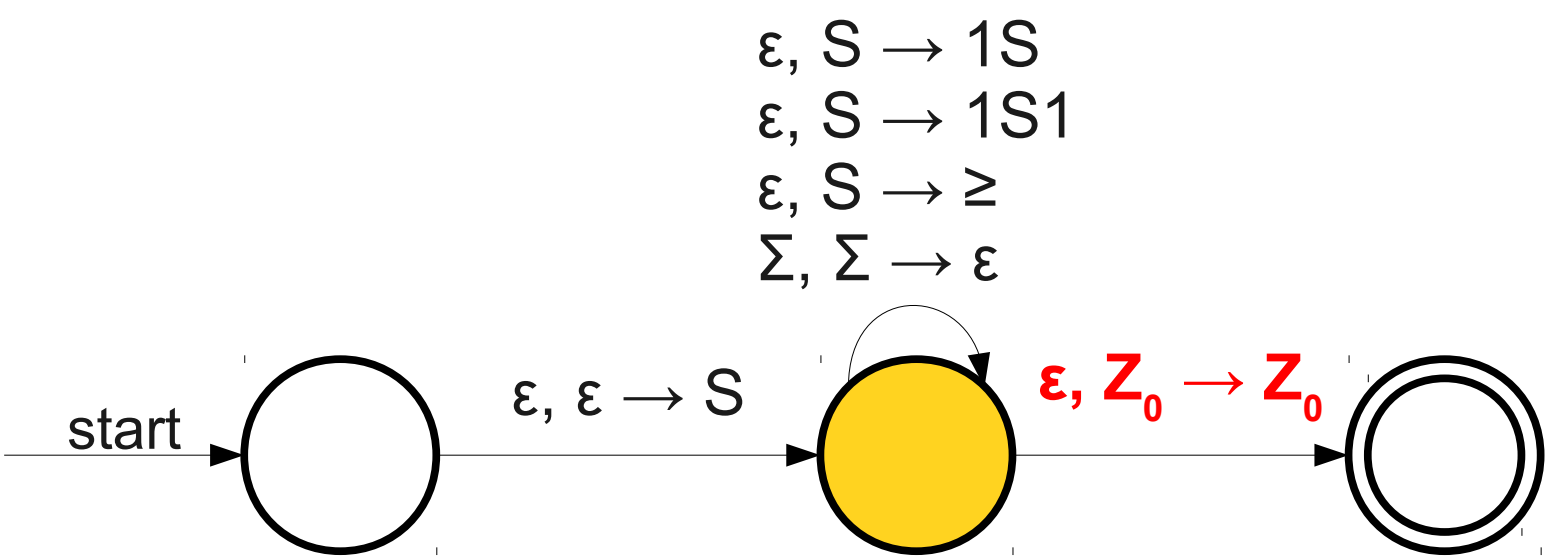
$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



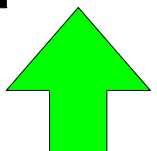
1 1 1 ≥ 1 1

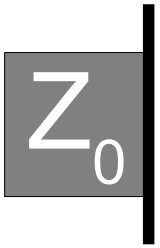
From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



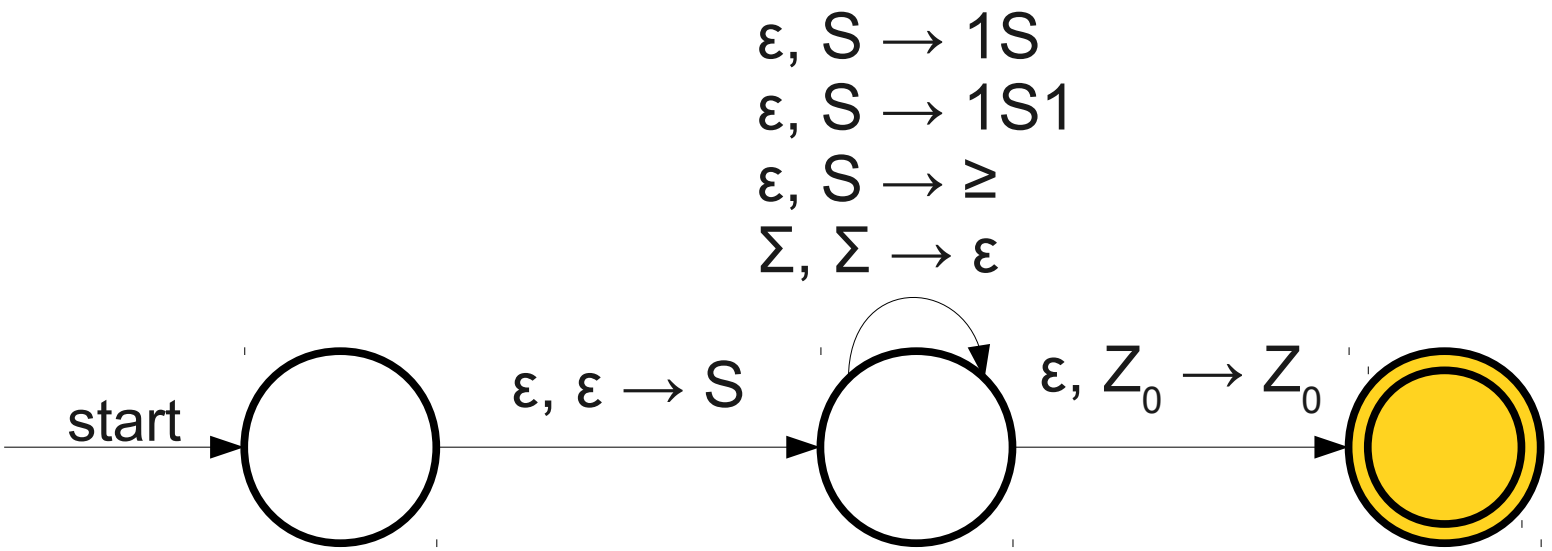
1 1 1 ≥ 1 1



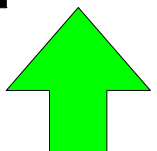


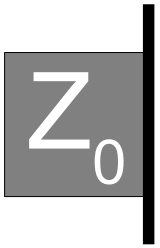
From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



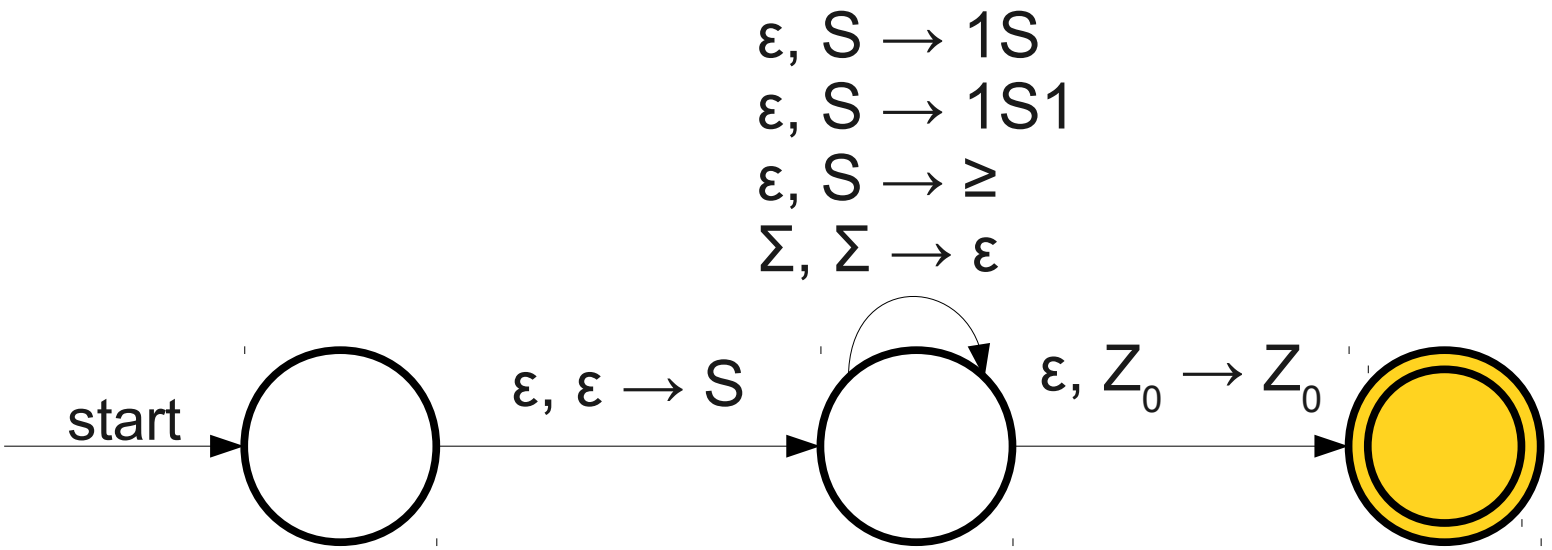
1 1 1 ≥ 1 1





From CFGs to PDAs

$S \rightarrow 1S1$
$S \rightarrow 1S$
$S \rightarrow \geq$



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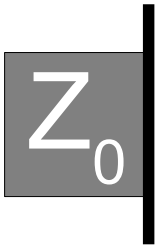

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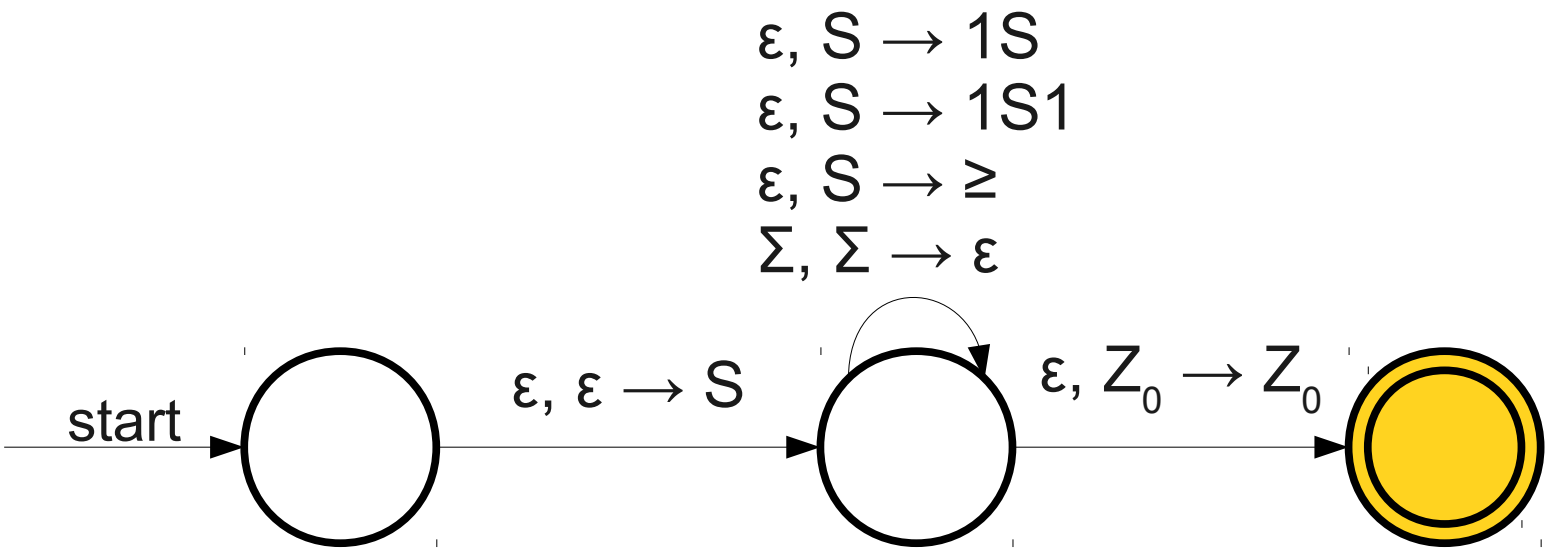

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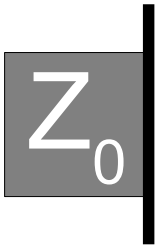

From CFGs to PDAs

- Make three states: **start**, **parsing**, and **accepting**.
- There is a transition ε, ε → S from **start** to **parsing**.
  - Corresponds to starting off with the start symbol S.
- There is a transition ε, A → *w* from **parsing** to itself for each production A → *w*.
  - Corresponds to predicting which production to use.
- There is a transition Σ, Σ → ε from **parsing** to itself.
  - Corresponds to matching a character of the input.
- There is a transition ε, $Z_0 \rightarrow Z_0$ from **parsing** to **accepting**.
  - Corresponds to completely matching the input.

From CFGs to PDAs

- The PDA constructed this way is called a **predict/match parser**.
- Each step either **predicts** which production to use or **matches** some symbol of the input.

From PDAs to CFGs

- The other direction of the proof (converting a PDA to a CFG) is much harder.
- Intuitively, create a CFG representing paths between states in the PDA.
- Lots of tricky details, but a marvelous proof.
  - It's just too large to fit into the margins of this slide.
- Read Sipser for more details.

Regular and Context-Free Languages

Theorem: Any regular language is context-free.

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Proof Sketch: Let L be any regular language and consider a DFA D for L.

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And now for an **awesome** idea not in Sipser...

NPDAs and DPDAs

- With finite automata, we considered both deterministic (DFAs) and nondeterministic (NFAs) automata.
- So far, we've only seen nondeterministic PDAs (or **NPDAs**).
- What about deterministic PDAs (**DPDAs**)?

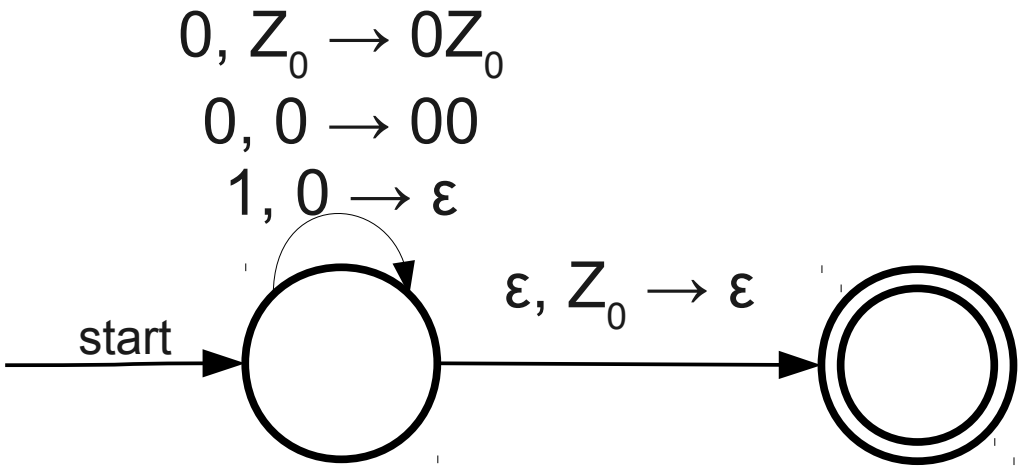
DPDAs

- A **deterministic pushdown automaton** is a PDA with the extra property that

  For each state in the PDA, and for any combination of a current input symbol and a current stack symbol, there is **at most** one transition defined.

- In other words, there is **at most** one legal sequence of transitions that can be followed for any input.
- This does **not** preclude ε-transitions, as long as there is never a conflict between following the ε-transition or some other transition.
- However, there can be **at most** one ε-transition that could be followed at any one time.

Is this a DPDA?



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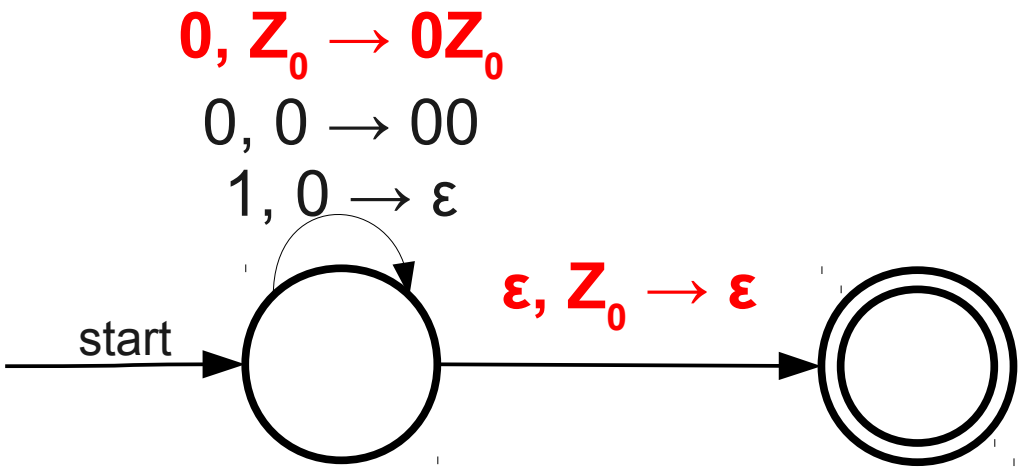

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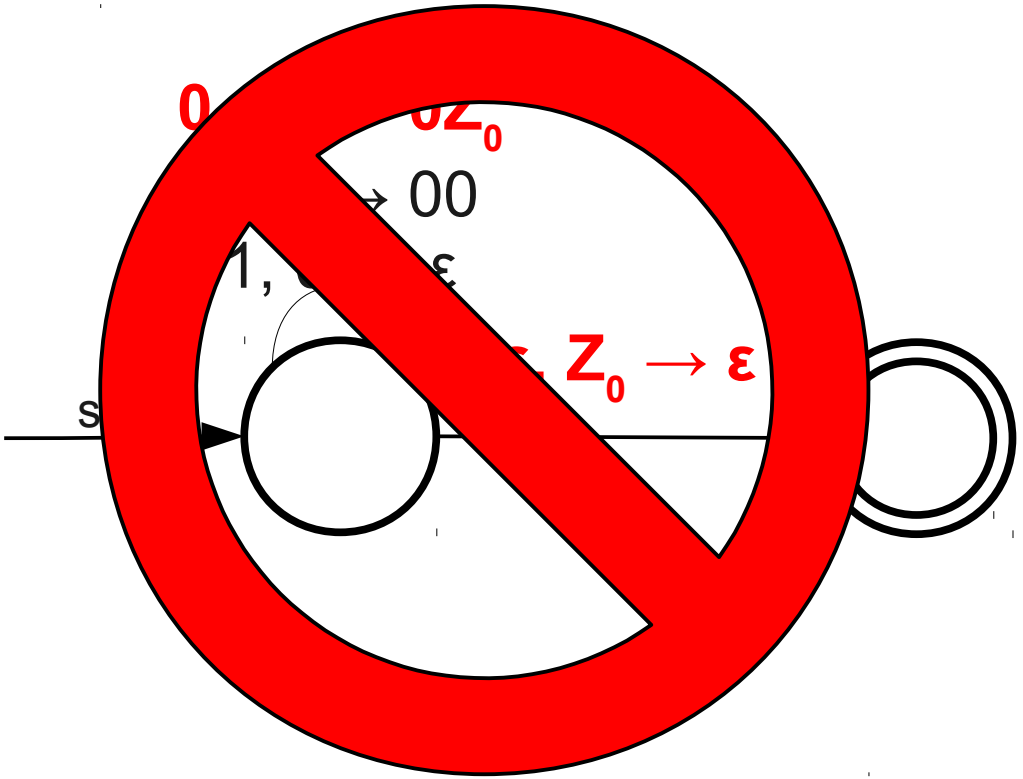

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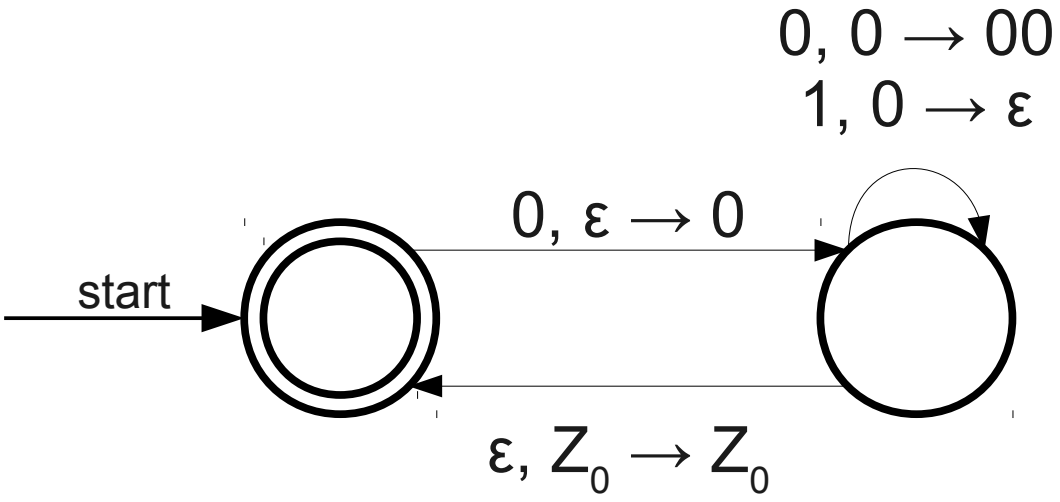
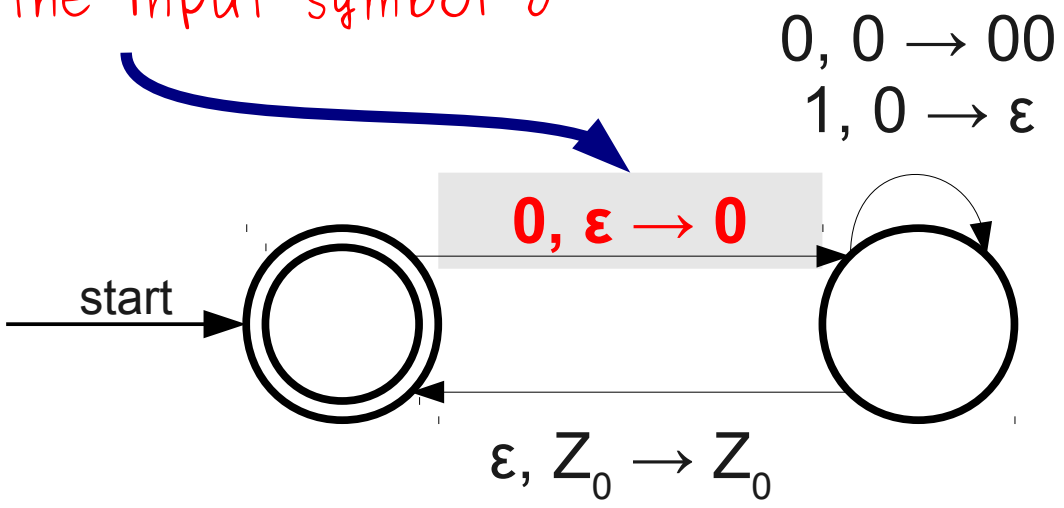


Is this a DPDA?

This ε-transition is allowable because no other transitions in this state use the input symbol 0



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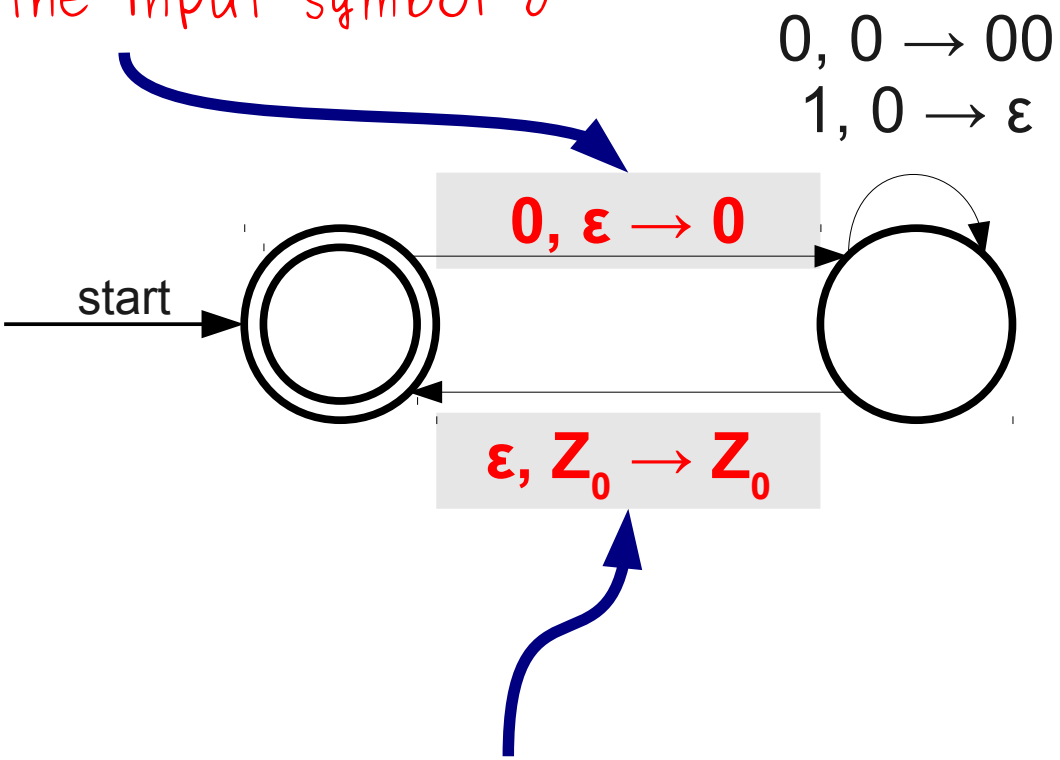
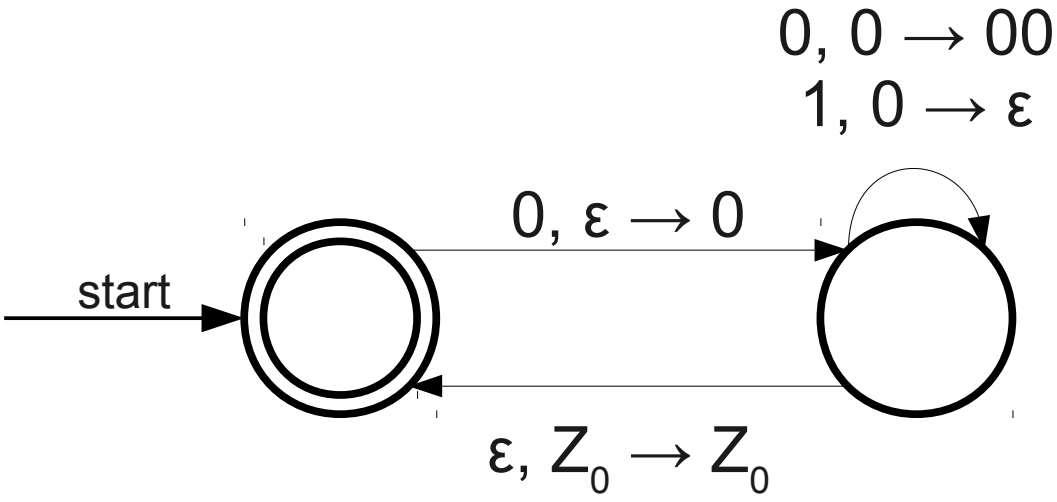


This ε–transition is allowable because no other transitions in this state use the stack symbol Z_0.

Is this a DPDA?



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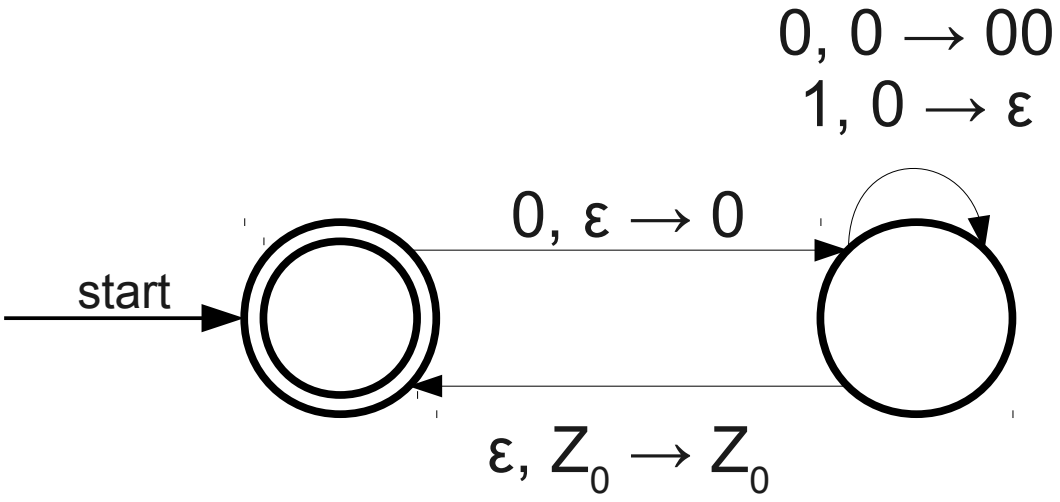

0 1 0 0 1 1

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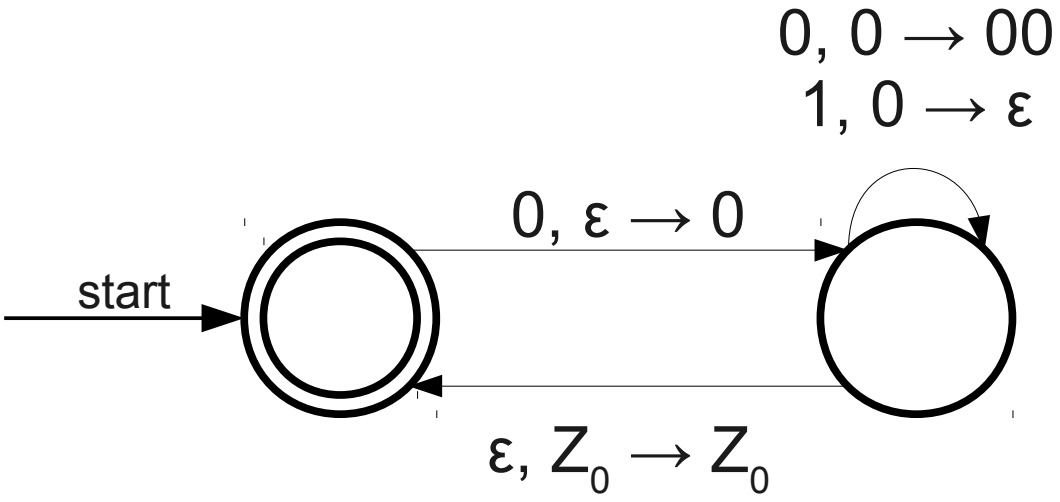

0 1 0 0 1 1

Z_0

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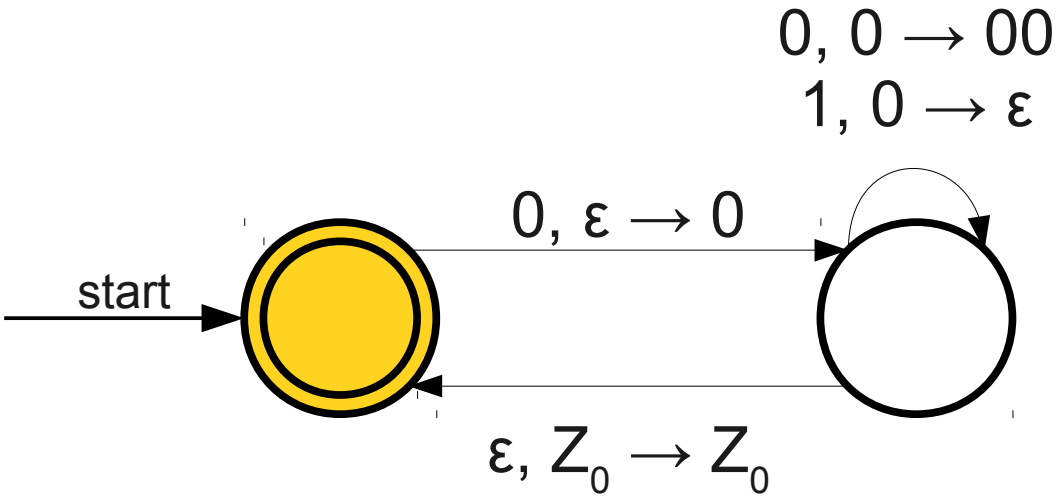

0 1 0 0 1 1

Z_0

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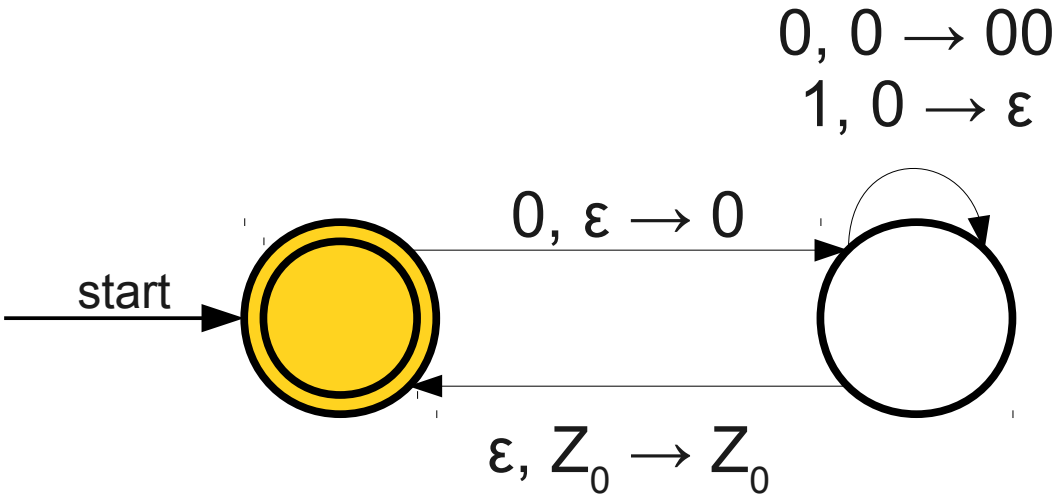

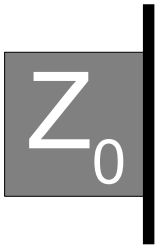

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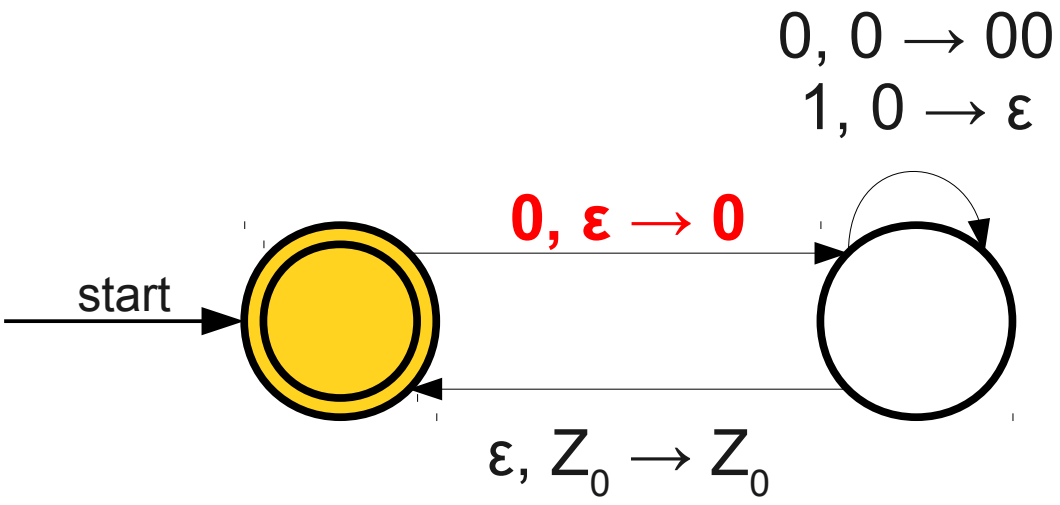

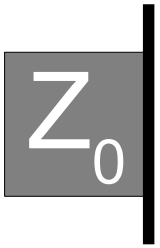

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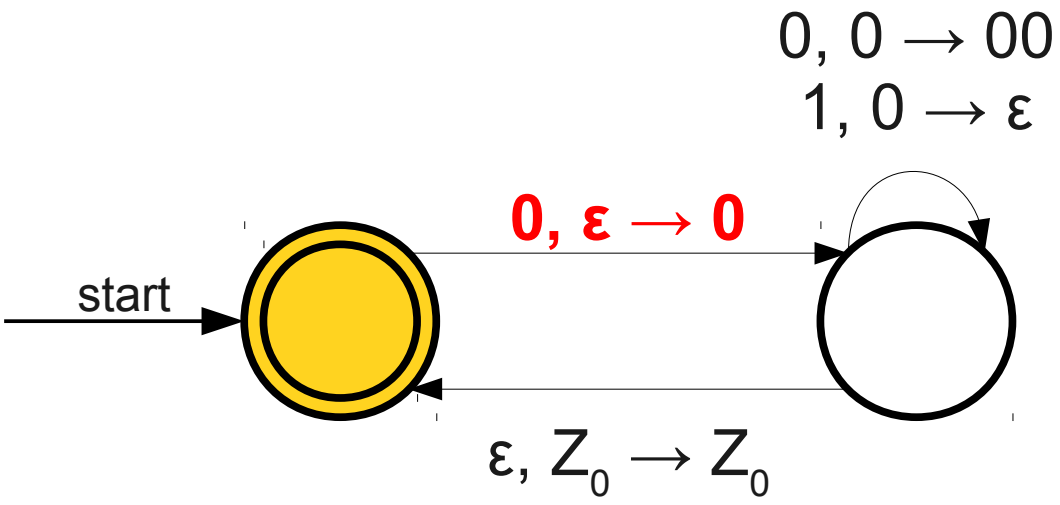

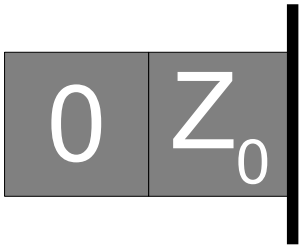

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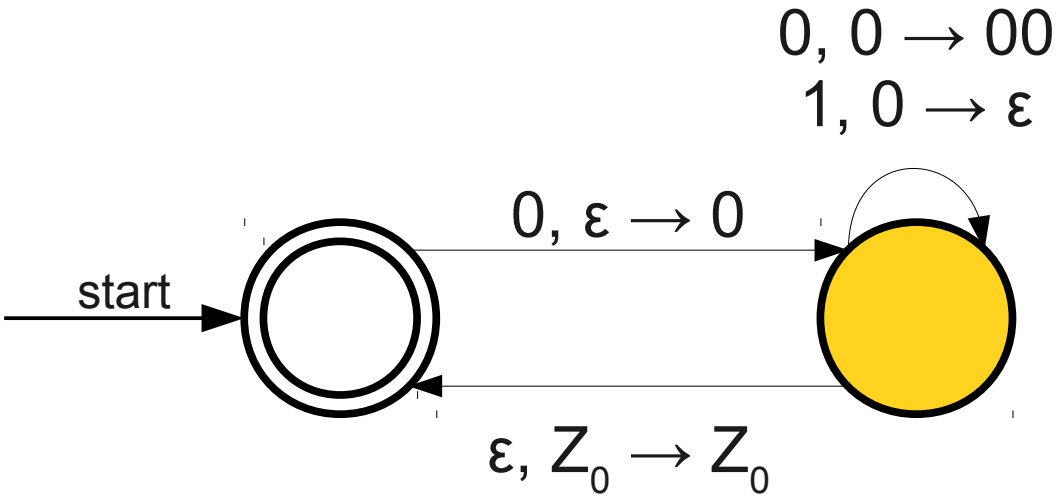

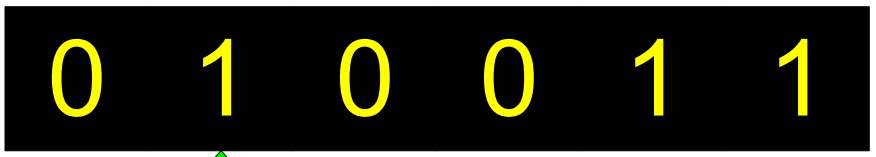

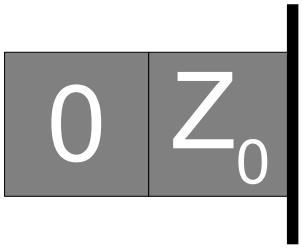

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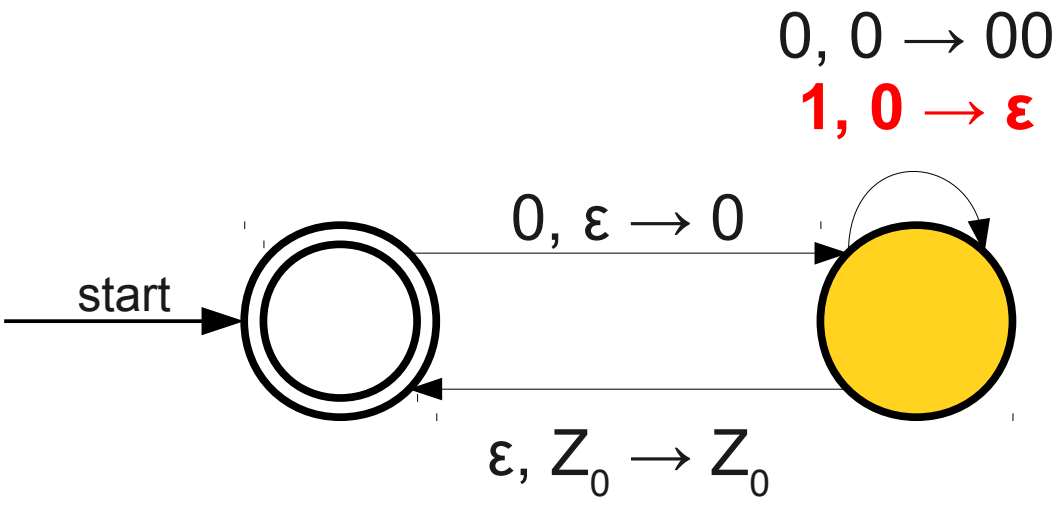

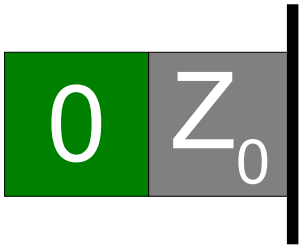

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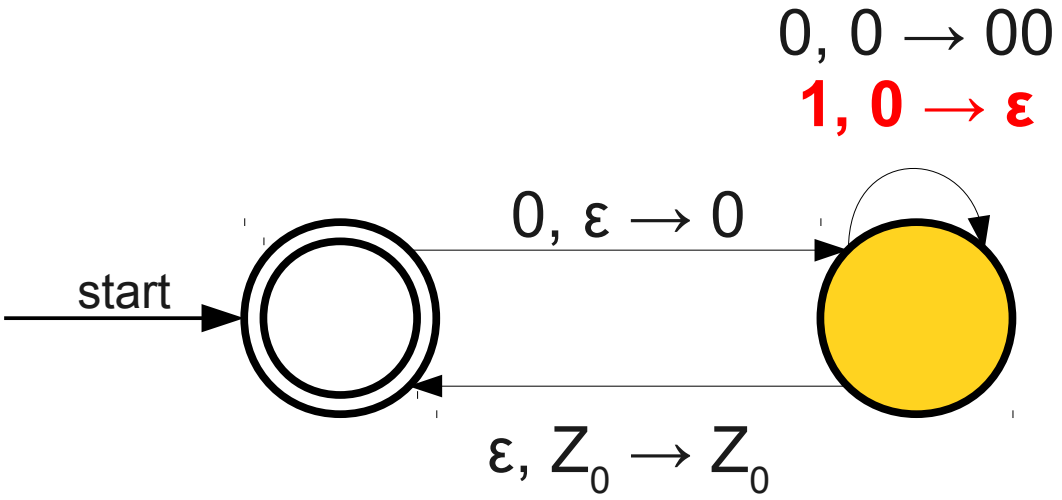

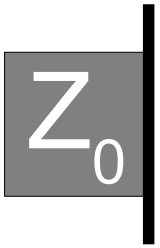

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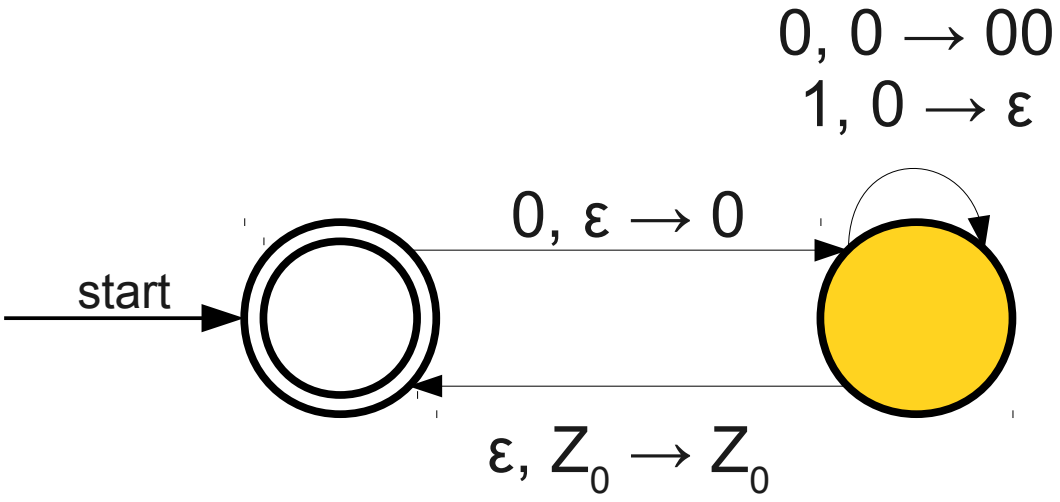

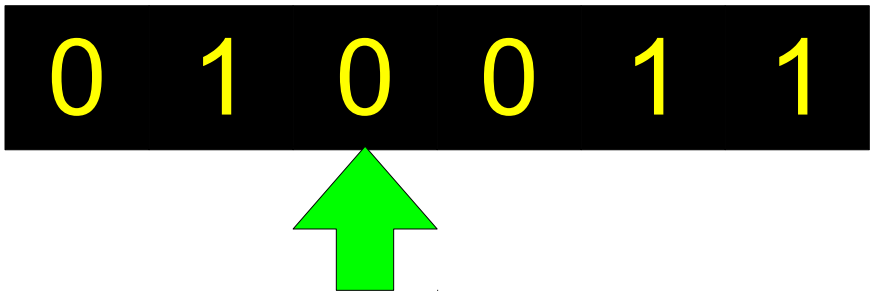

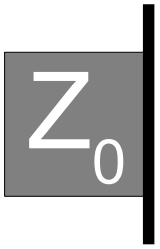

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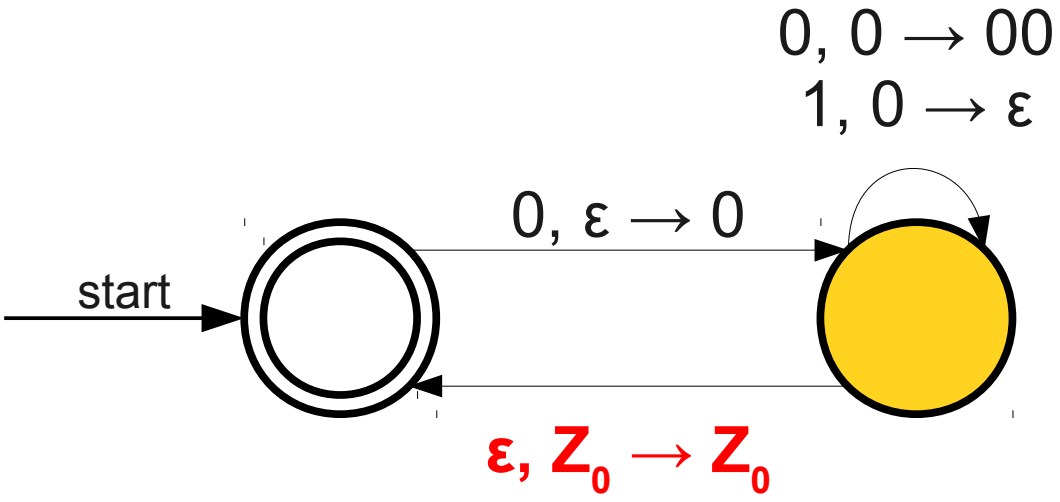

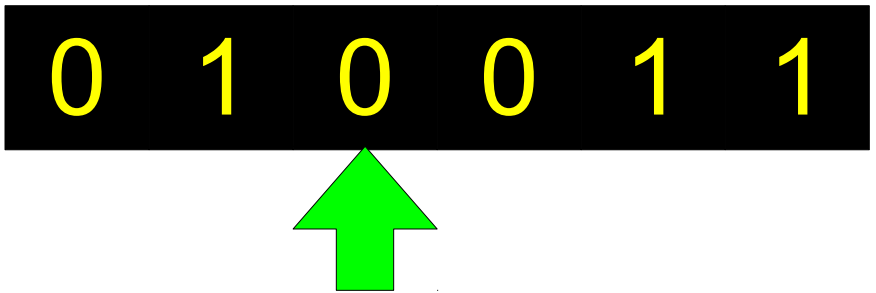

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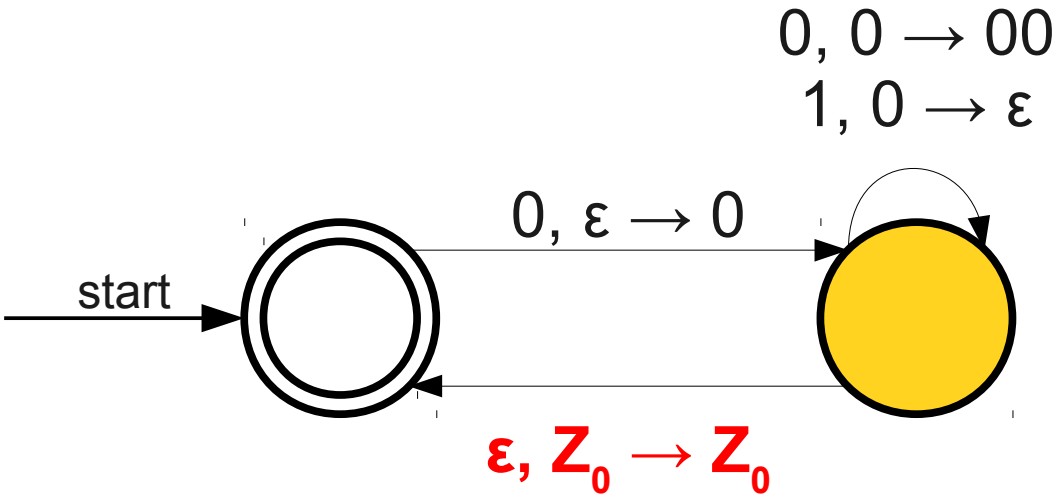

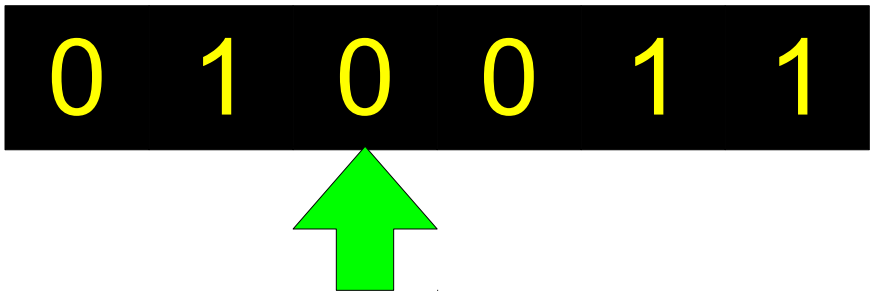

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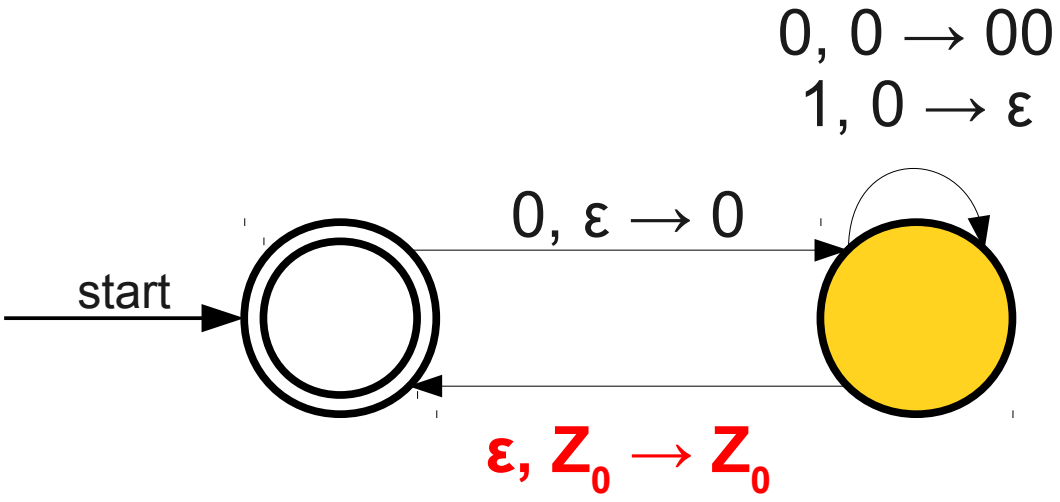

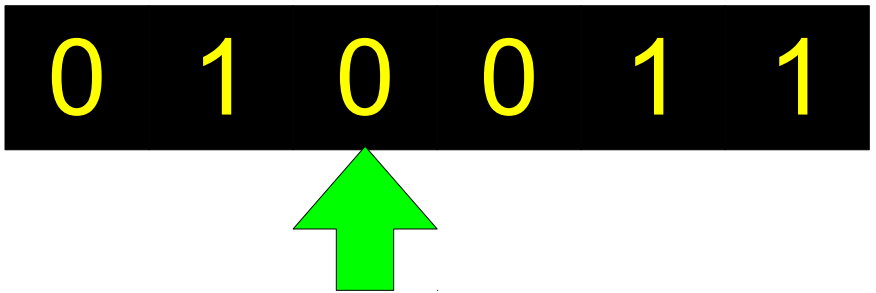

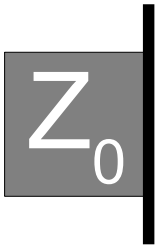

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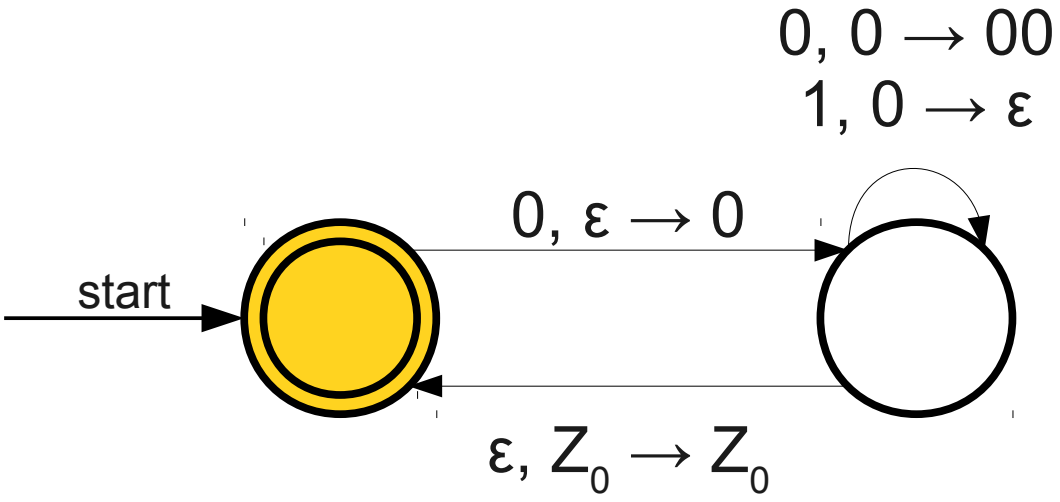

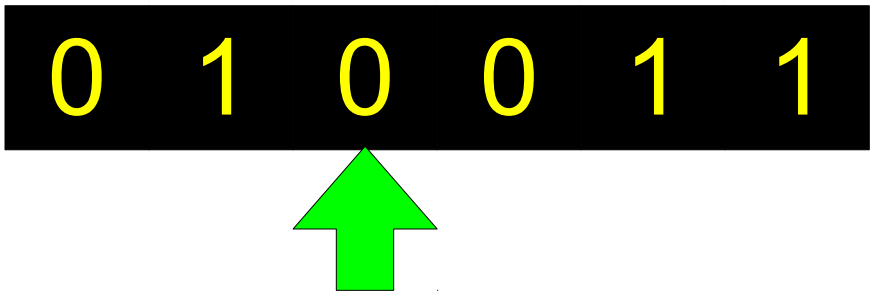

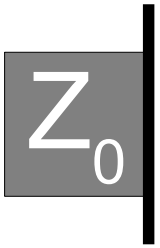

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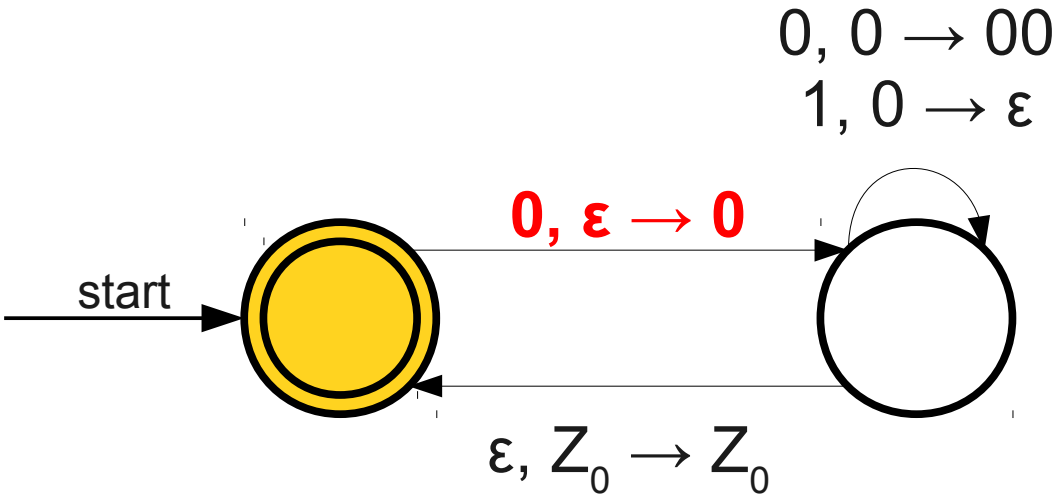

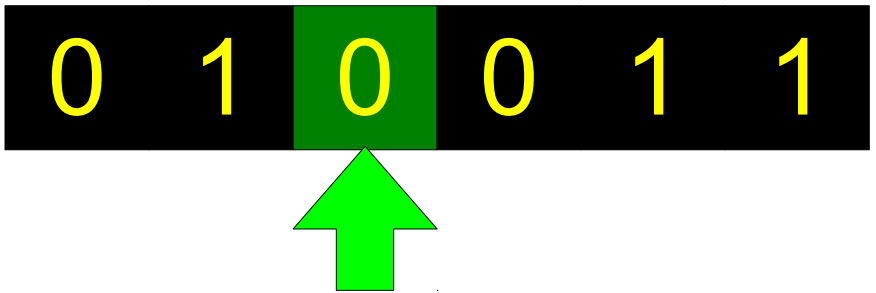

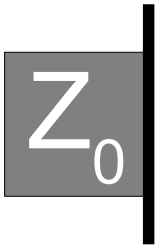

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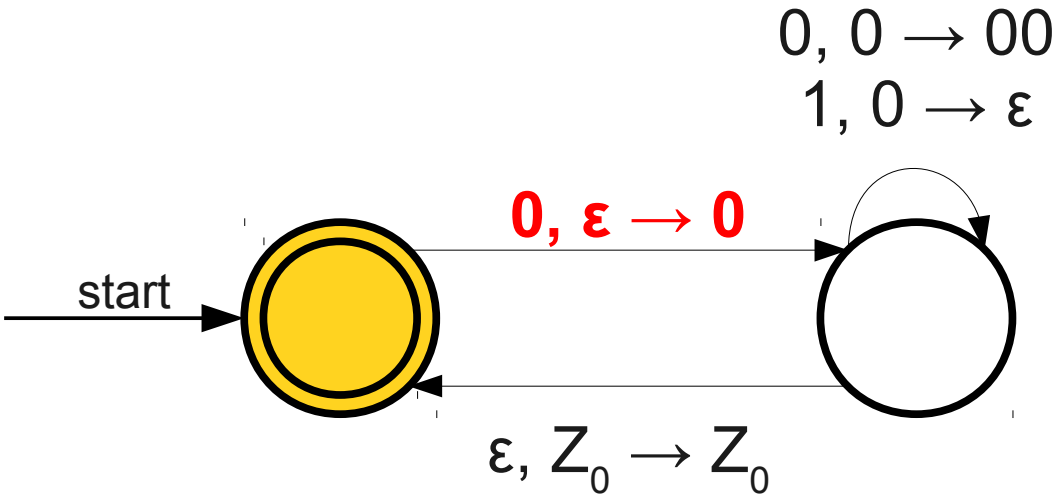

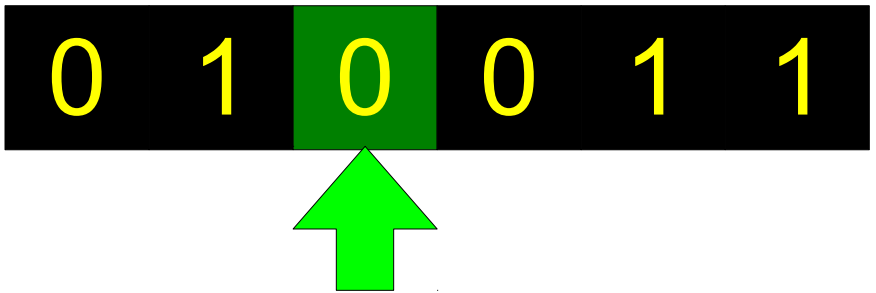

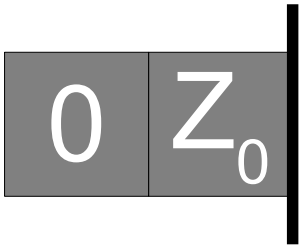

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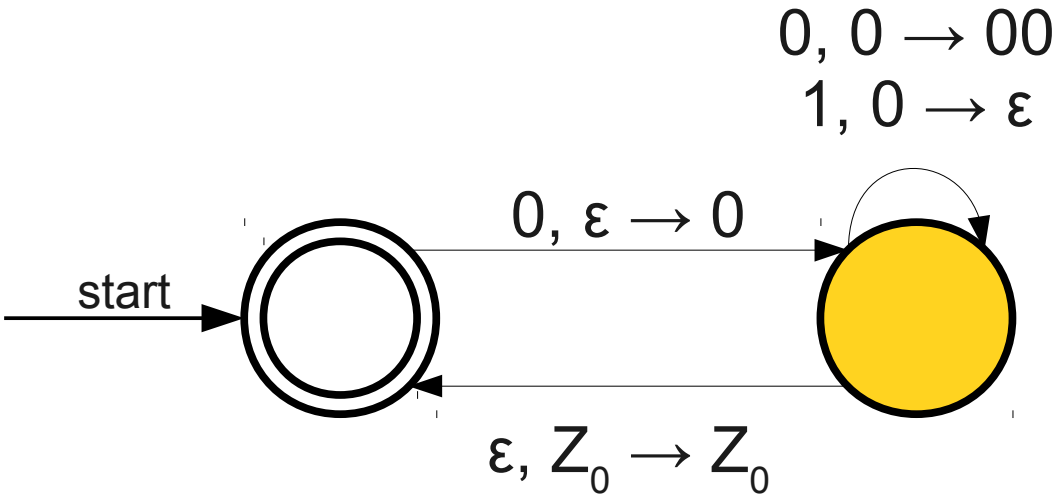

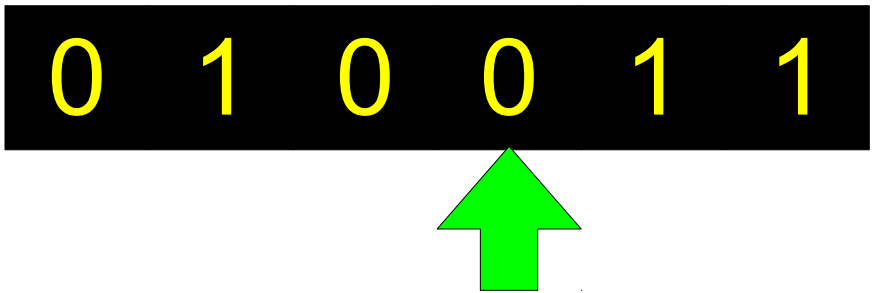

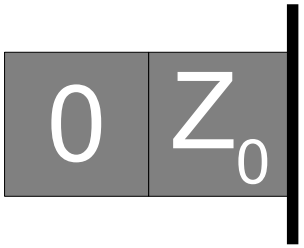

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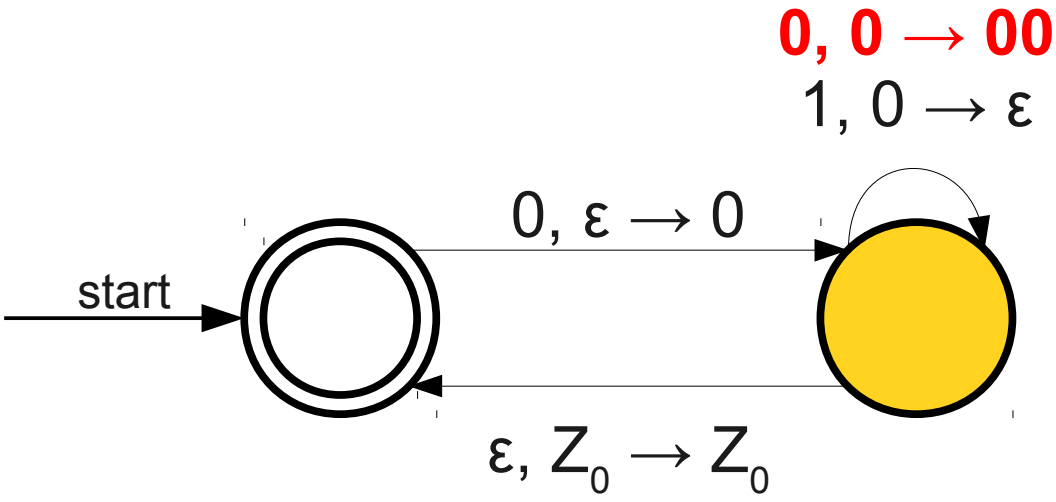

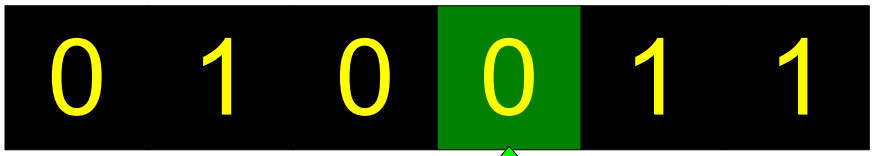

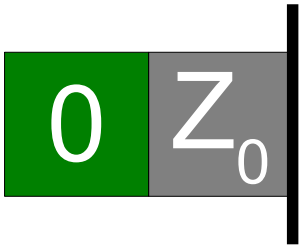

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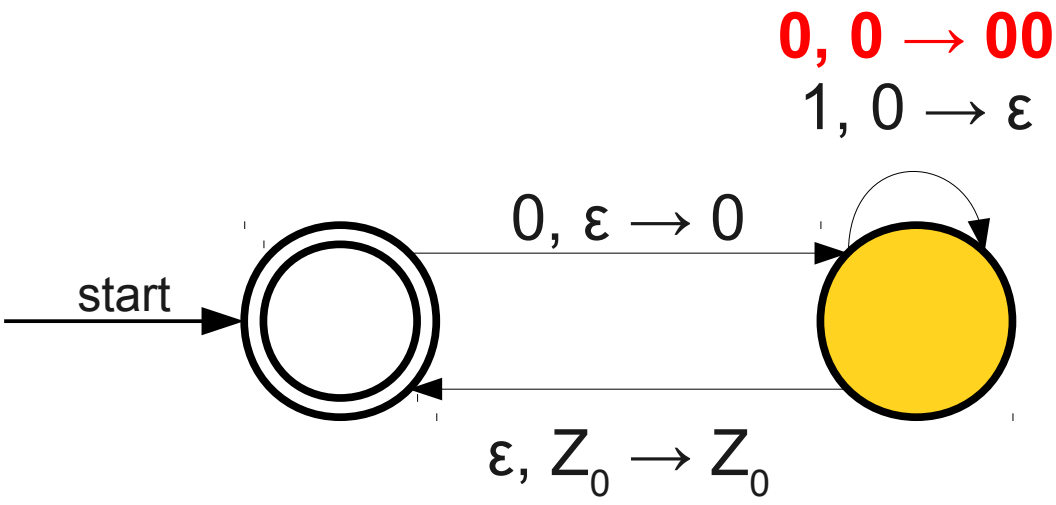

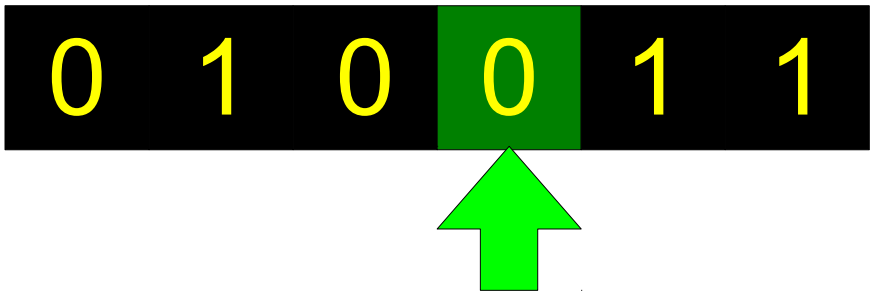

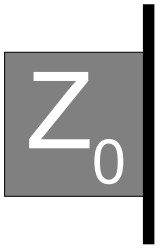

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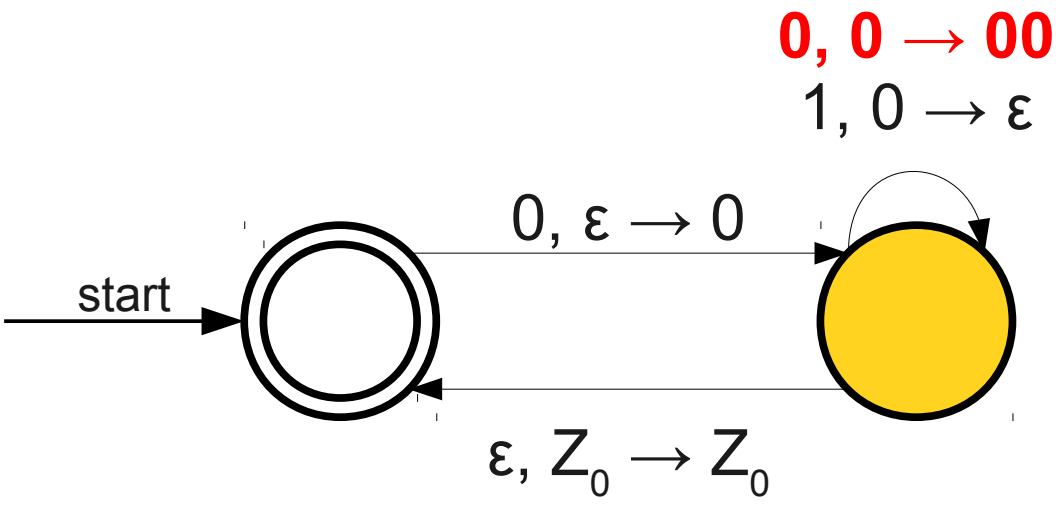

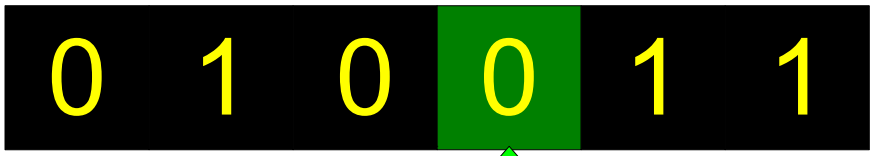

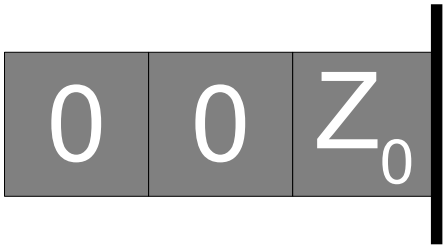

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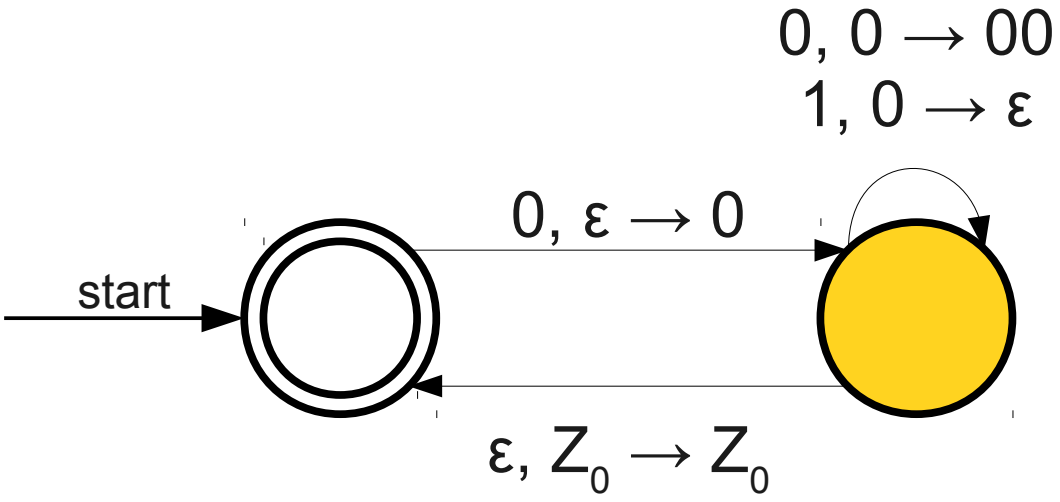

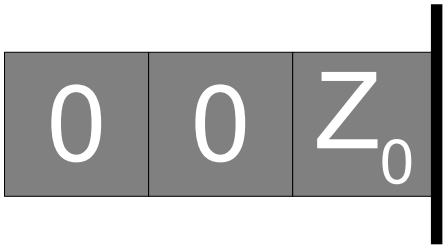

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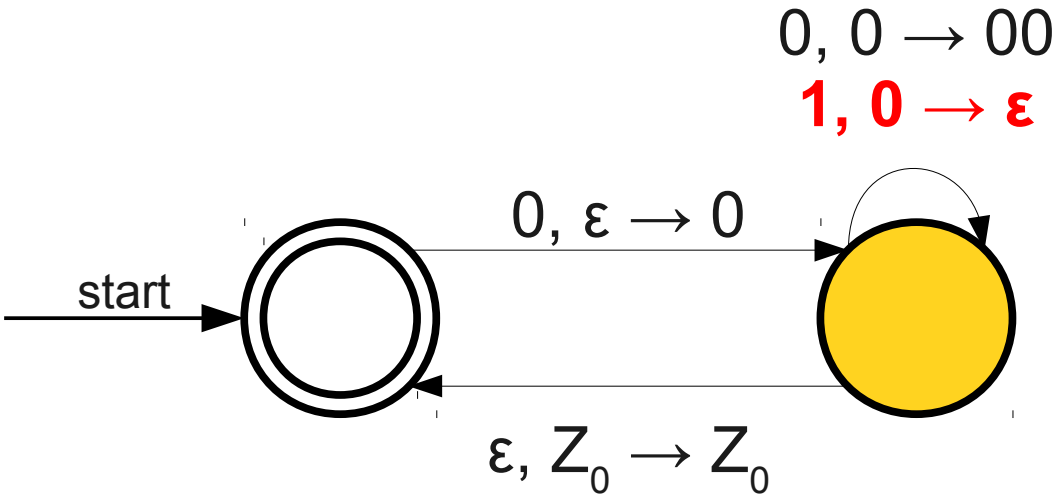

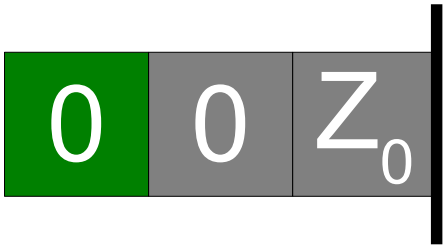

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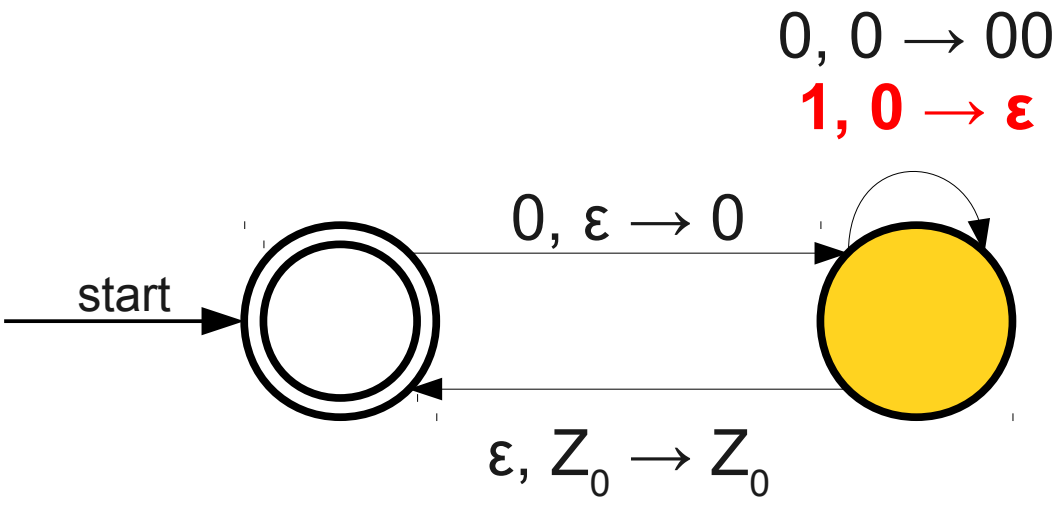

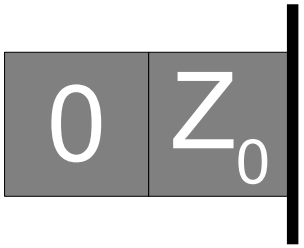

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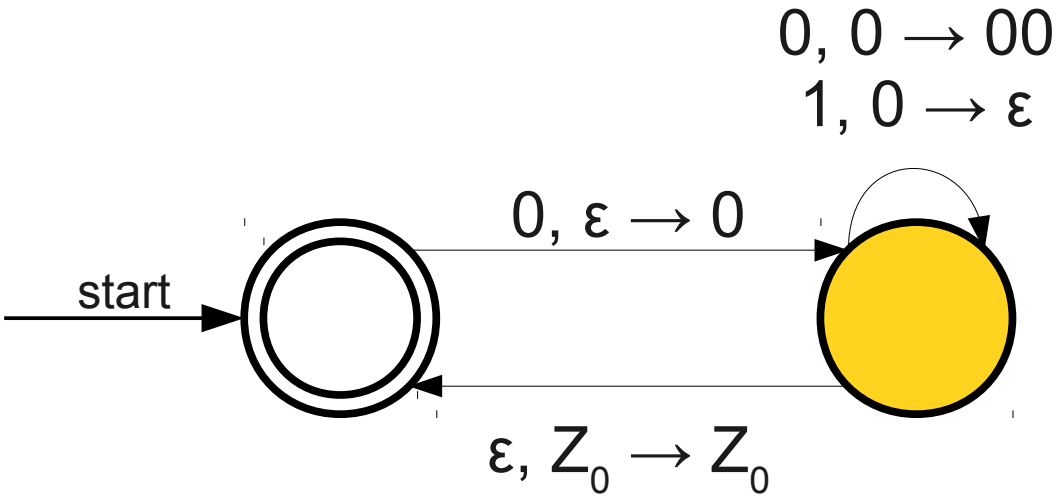

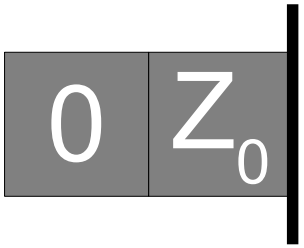

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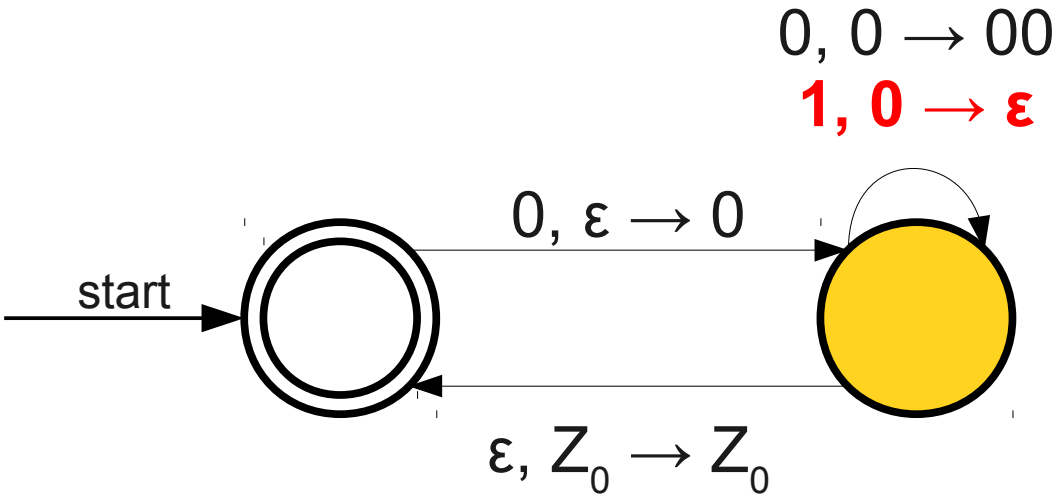

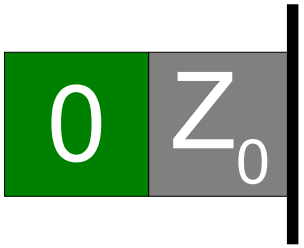

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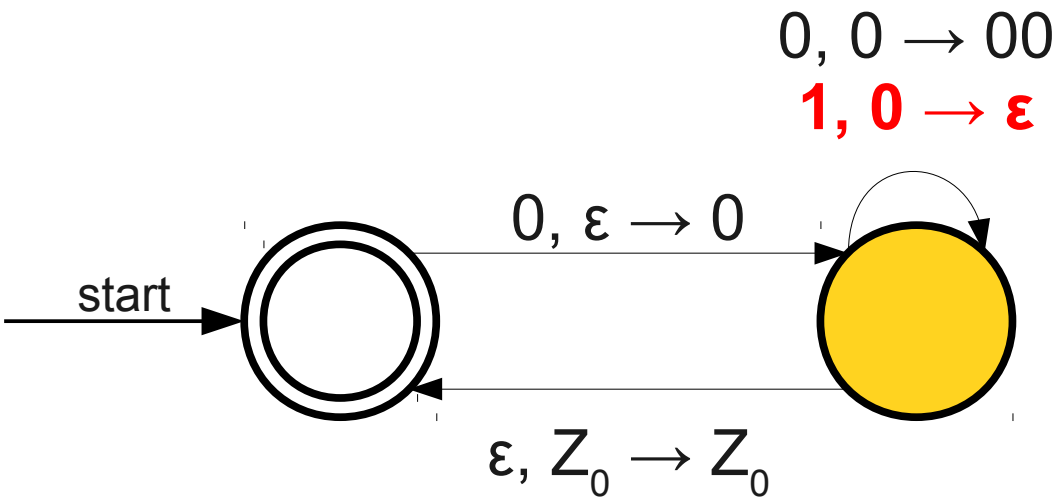

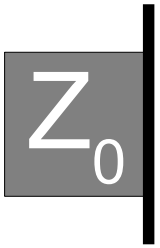

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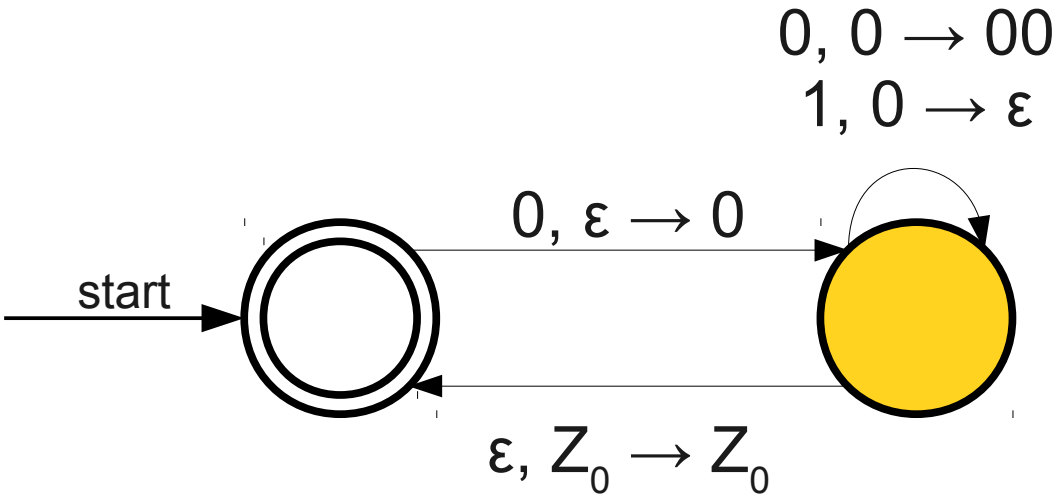

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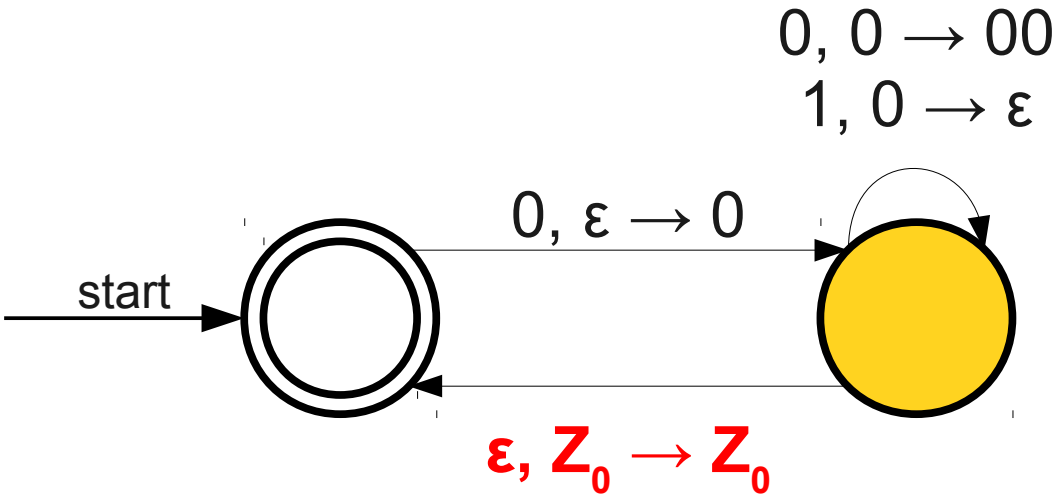

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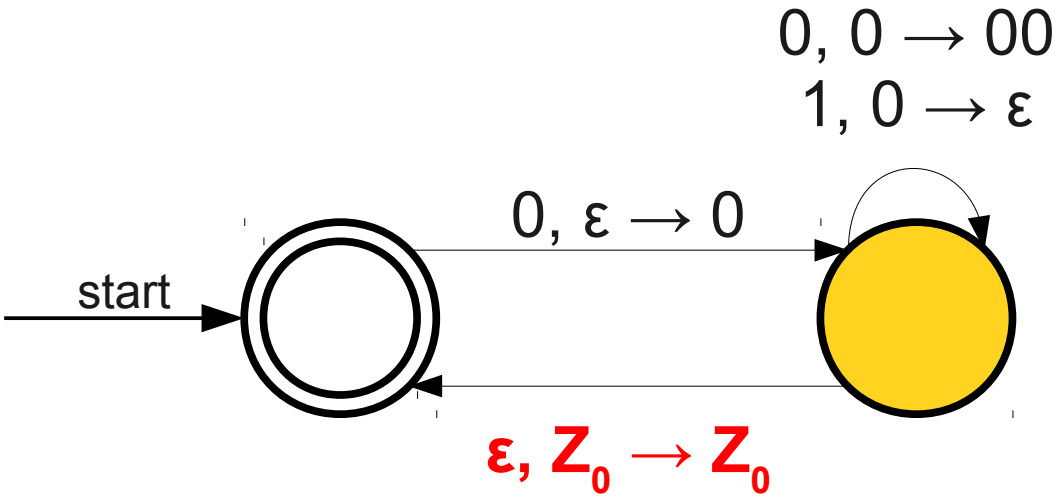

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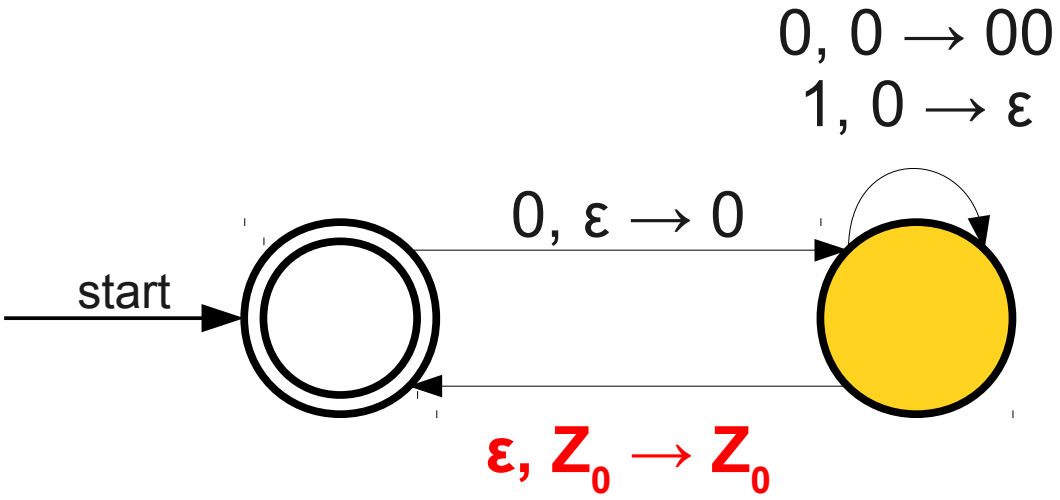

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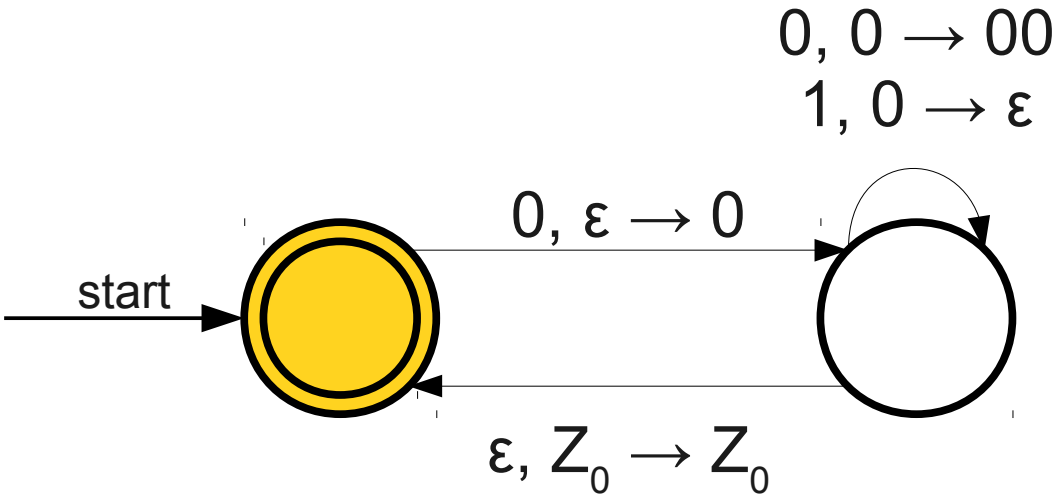

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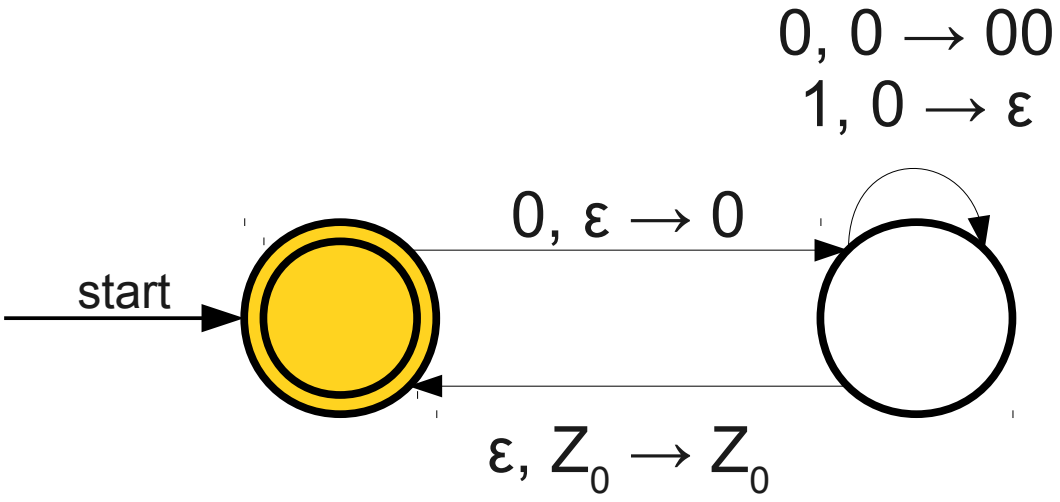

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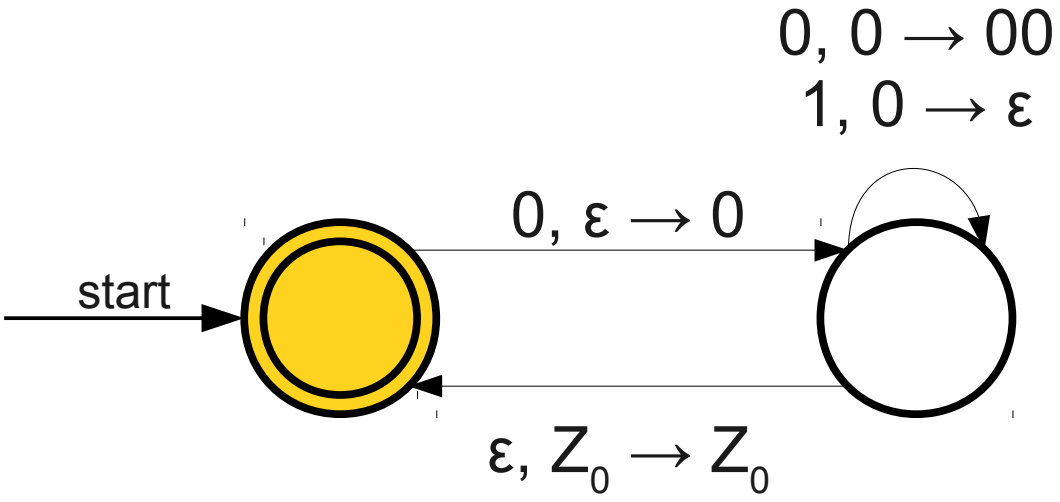

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Why DPDAs Matter

- Because DPDAs are deterministic, they can be simulated efficiently:
  - Keep track of the top of the stack.
  - Store an **action/goto table** that says what operations to perform on the stack and what state to enter on each input/stack pair.
  - Loop over the input, processing input/stack pairs until the automaton rejects or ends in an accepting state with all input consumed.
- If we can find a DPDA for a CFL, then we can recognize strings in that language efficiently.

If we can find a DPDA for a CFL, then we can recognize strings in that language efficiently.

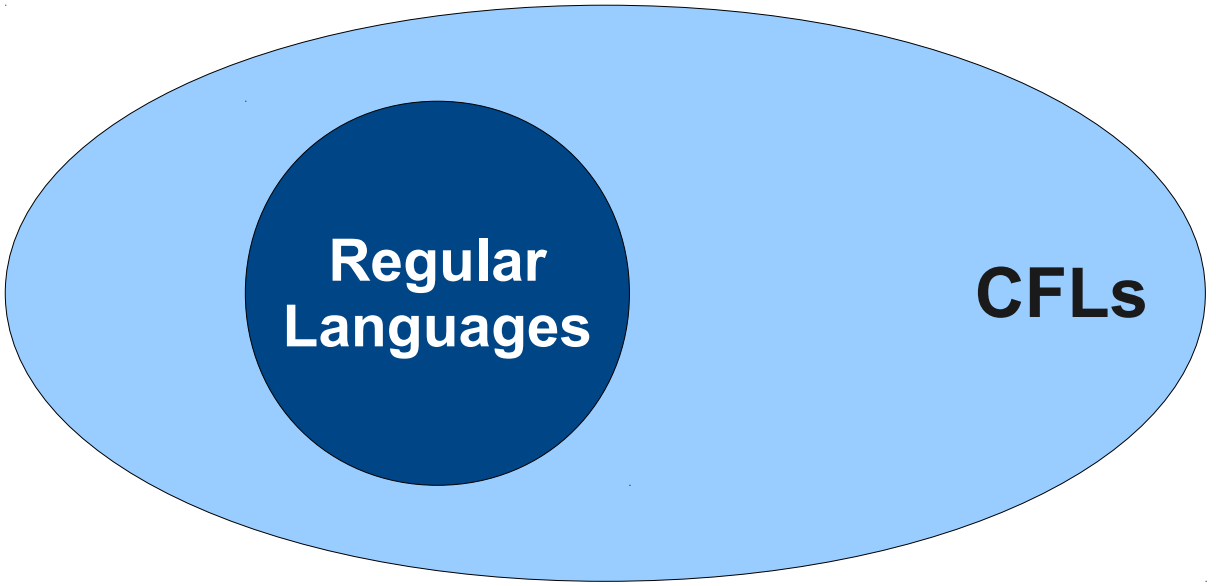
Can we guarantee that we can always find a DPDA for a CFL?

The Power of Nondeterminism

- When dealing with finite automata, there is no difference in the power of NFAs and DFAs.
- However, when dealing with PDAs, there are CFLs that can be recognized by NPDAs that **cannot** be recognized by DPDAs.
- Simple example: The language of palindromes.
  - How do you know when you've read half the string?
- NPDAs are **more powerful** than DPDAs.

Deterministic CFLs

- A context-free language L is called a **deterministic context-free language** (DCFL) if there is some DPDA that recognizes L.
- Not all CFLs and DCFLs, though many important ones are.
  - Balanced parentheses, most programming languages, etc.



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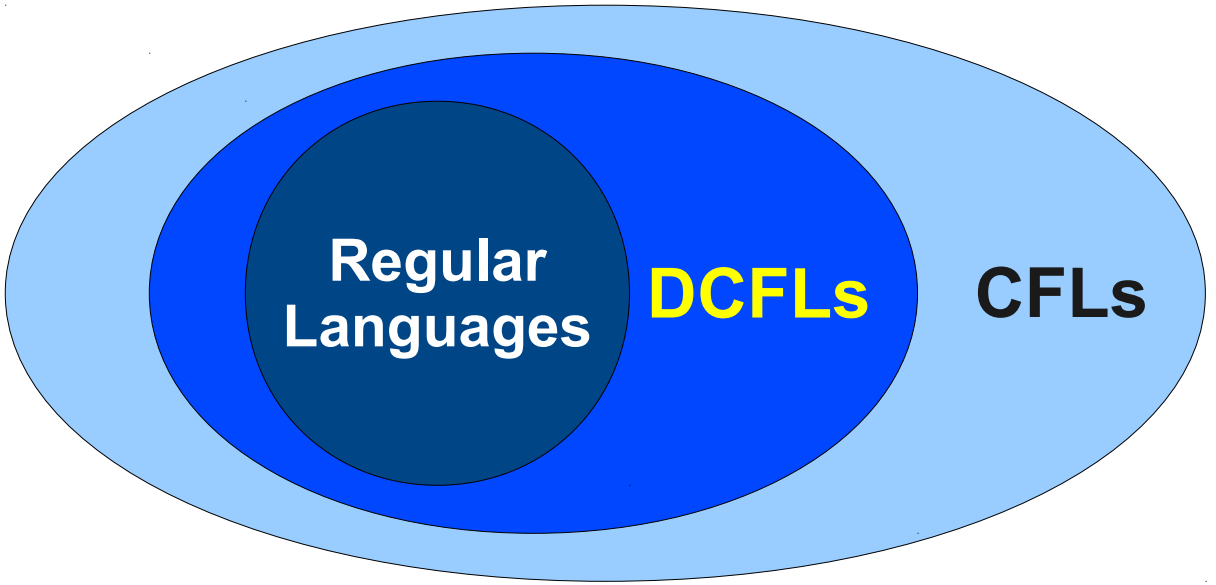

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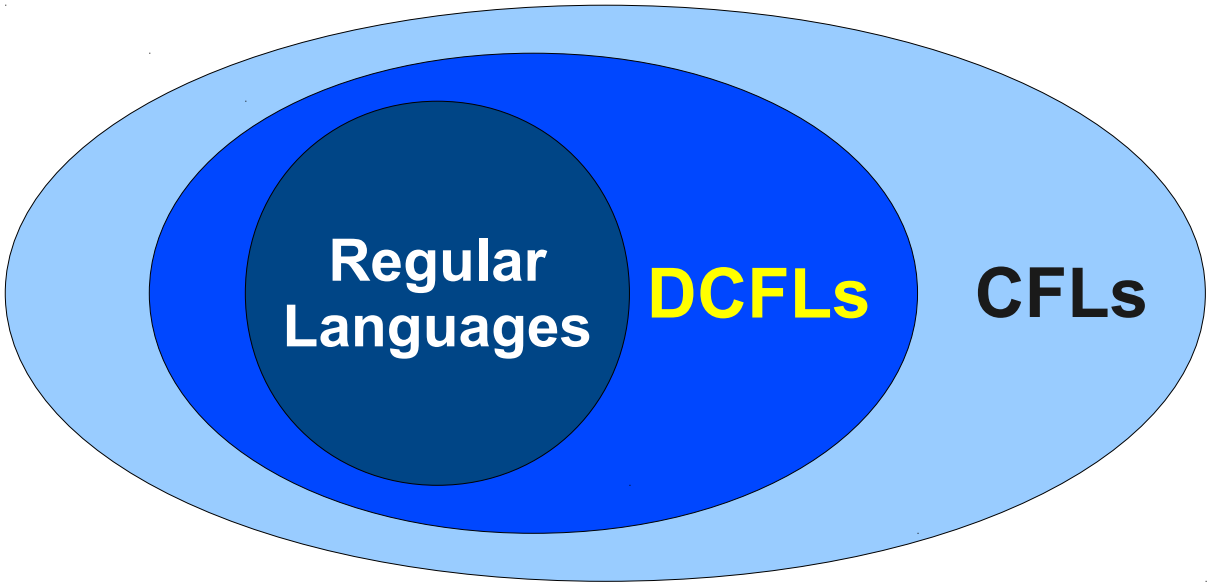
Why are all regular languages DCFLs?



Summary

- Automata can be augmented with a memory storage to increase their power.
- PDAs are finite automata equipped with a stack.
- PDAs accept precisely the context-free languages:
  - Any CFG can be converted to a PDA.
  - Any PDA can be converted to a CFG.
- Deterministic PDAs are strictly weaker than nondeterministic PDAs.

Next Time

- The Limits of CFLs
- A surprisingly powerful automaton: **The Turing Machine**.