# Mathematical Logic

Part II

# Announcements

- Problem Set 2 due right now.
- Problem Set 3 out, due Friday, October 21 at 2:15PM.
  - Drop by office hours with questions!
  - Email us at **cs103@cs.stanford.edu** with questions!
- Friday Four Square today at 4:15 in front of Gates!
  - Have a ball!  Then hit it to someone else!

# Analyzing Proof Techniques

# Proof by Contrapositive

- Recall that to prove that $p \rightarrow q$, we can also show that $\neg q \rightarrow \neg p$.
- Let's verify that $p \rightarrow q \equiv \neg q \rightarrow \neg p$.

# The Contrapositive

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| F | F | |
| F | T | |
| T | F | |
| T | T | |

# The Contrapositive

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

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| $p$ | $q$ | $\neg q$ | $\rightarrow$ | $\neg p$ |
|---|---|---|---|---|
| F | F | T | | T |
| F | T | F | | T |
| T | F | T | | F |
| T | T | F | | F |

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| T | F | T | F | F |
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$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

# Why All This Matters

- Suppose we want to prove the following statement:

“If $x + y = 16$, then $x \geq 8$ or $y \geq 8$”

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"If x + y = 16, then x ≥ 8 or y ≥ 8"

$$x + y = 16 \rightarrow x \geq 8 \lor y \geq 8$$

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“If $x < 8$ and $y < 8$, then $x + y \neq 16$”

# Why All This Matters

- Suppose we want to prove the following statement:

"If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

$$x < 8 \land y < 8 \rightarrow x + y \neq 16$$

"If $x < 8$ and $y < 8$, then $x + y \neq 16$"

- We can prove this second statement instead.

*Theorem:* If $x + y = 16$, then either $x \geq 8$ or $y \geq 8$.

*Proof:* By contrapositive. We prove instead that if $x < 8$ and $y < 8$, then $x + y \neq 16$. To see this, note that $x + y < 8 + y < 8 + 8 = 16$, so $x + y < 16$, so $x + y \neq 16$ ■

# Why This Matters

- Propositional logic is a tool for reasoning about how various statements affect one another.
- To better understand how to prove a result, it often helps to translate what you're trying to prove into propositional logic first.
- **Note**: To truly reason about proofs, we need the more expressive power of **first-order logic**, which we'll talk about later today.

# Proof by Contradiction

- The general structure of a proof by contradiction is
  - To show p, assume p is false.
  - Show that p being false implies something that cannot be true.
  - Conclude, therefore, that p is true.
- What does this look like in propositional logic?

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$$\neg p$$

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$$\neg p \rightarrow \bot$$

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$$(\neg p \to \bot) \to p$$

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| $p$ | $q$ | $(\neg p \rightarrow \bot) \rightarrow p$ |
|---|---|---|
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| T | F | F | T | F | T | T |
| T | T | F | T | F | T | T |

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| $p$ | $q$ | $(\neg p$ | $\rightarrow$ | $\bot)$ | $\rightarrow$ | $p$ |
|---|---|---|---|---|---|---|
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| $p$ | $q$ | $(\neg p$ | $\to$ | $\bot)$ | $\to$ | $p$ |
|---|---|---|---|---|---|---|
| F | F | T | F | F | T | F |
| F | T | T | F | F | T | F |
| T | F | F | T | F | T | T |
| T | T | F | T | F | T | T |

This statement is always true!

# Tautologies

- A **tautology** is a statement that is always true.
- Examples:
  - $\top$
  - $p \lor \neg p$ (the **Law of the Excluded Middle**)
  - $\bot \to p$ (**vacuous truth**)
- Once a tautology has been proven, we can use that tautology anywhere.

# First-Order Logic

# What is First-Order Logic?

- **First-order logic** is a powerful logical system for reasoning about properties of objects.
- Augments the logical connectives from propositional logic with
  - **predicates** that describe properties of objects, and
  - **functions** that map objects to one another,
  - **quantifiers** that allow us to reason about multiple objects simultaneously.

# The Universe of Propositional Logic

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$$p \land q \rightarrow \neg r \lor \neg s$$

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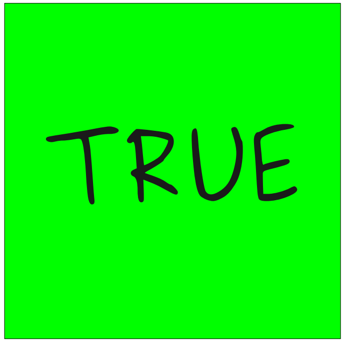


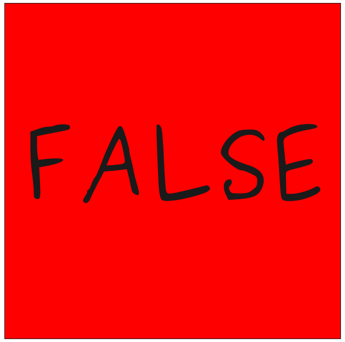

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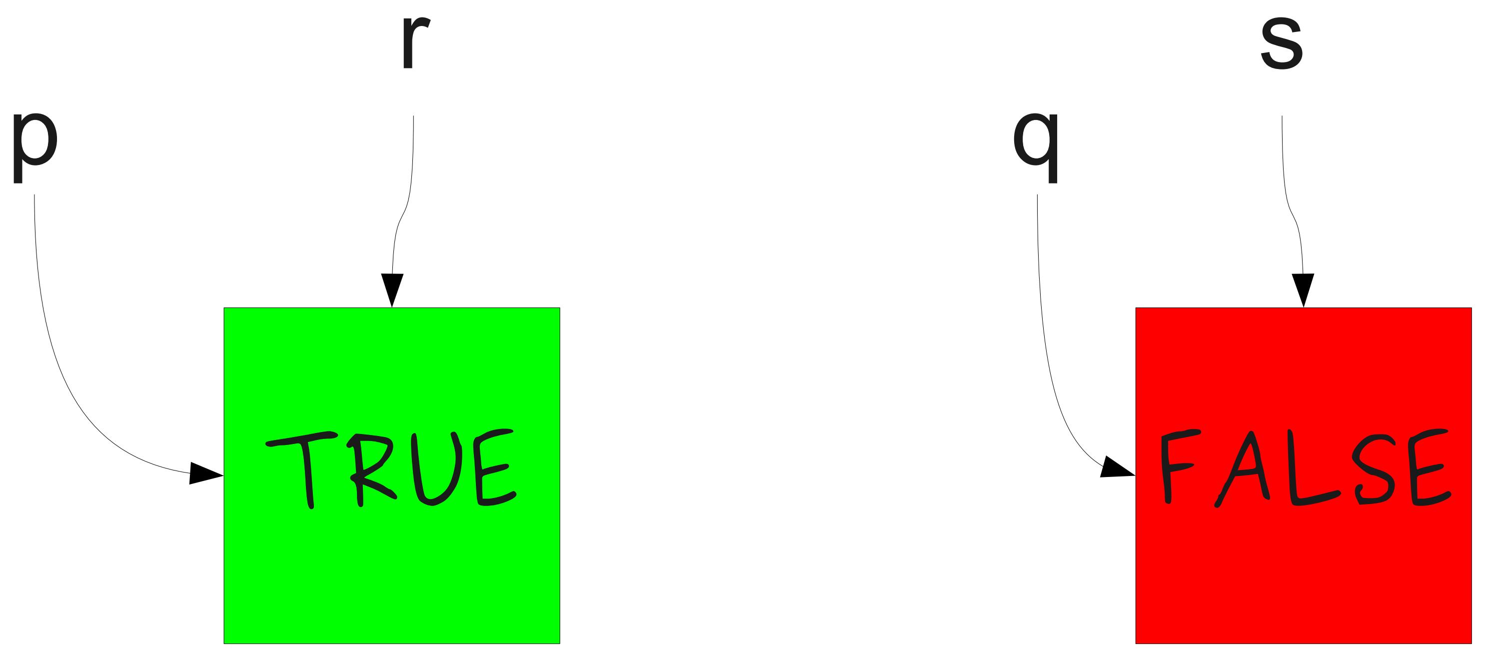

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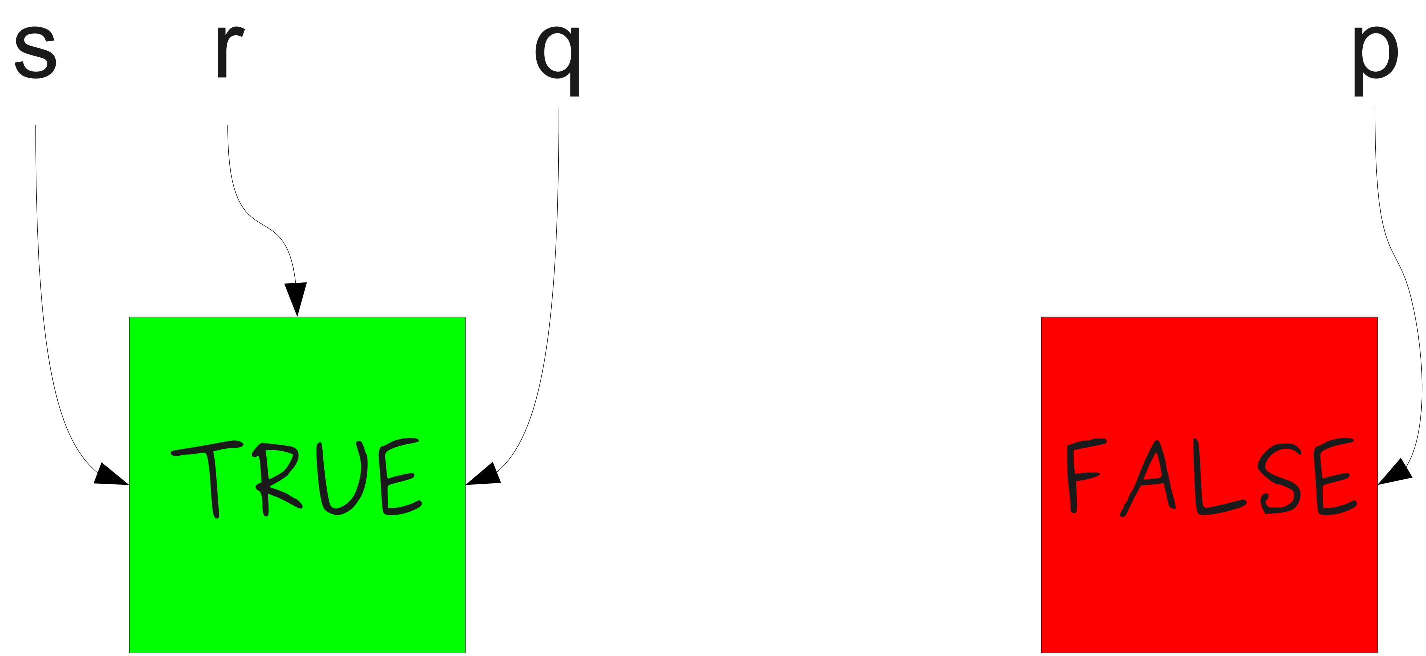

# Propositional Logic

- In propositional logic, each variable represents a **proposition**, which is either true or false.
- Consequently, we can directly apply connectives to propositions:
  - $p \to q$
  - $\neg p \land q$
- The truth or falsity of a statement can be determined by plugging in the truth values for the input propositions and computing the result.
- We can see all possible truth values for a statement by checking all possible truth assignments to its variables.

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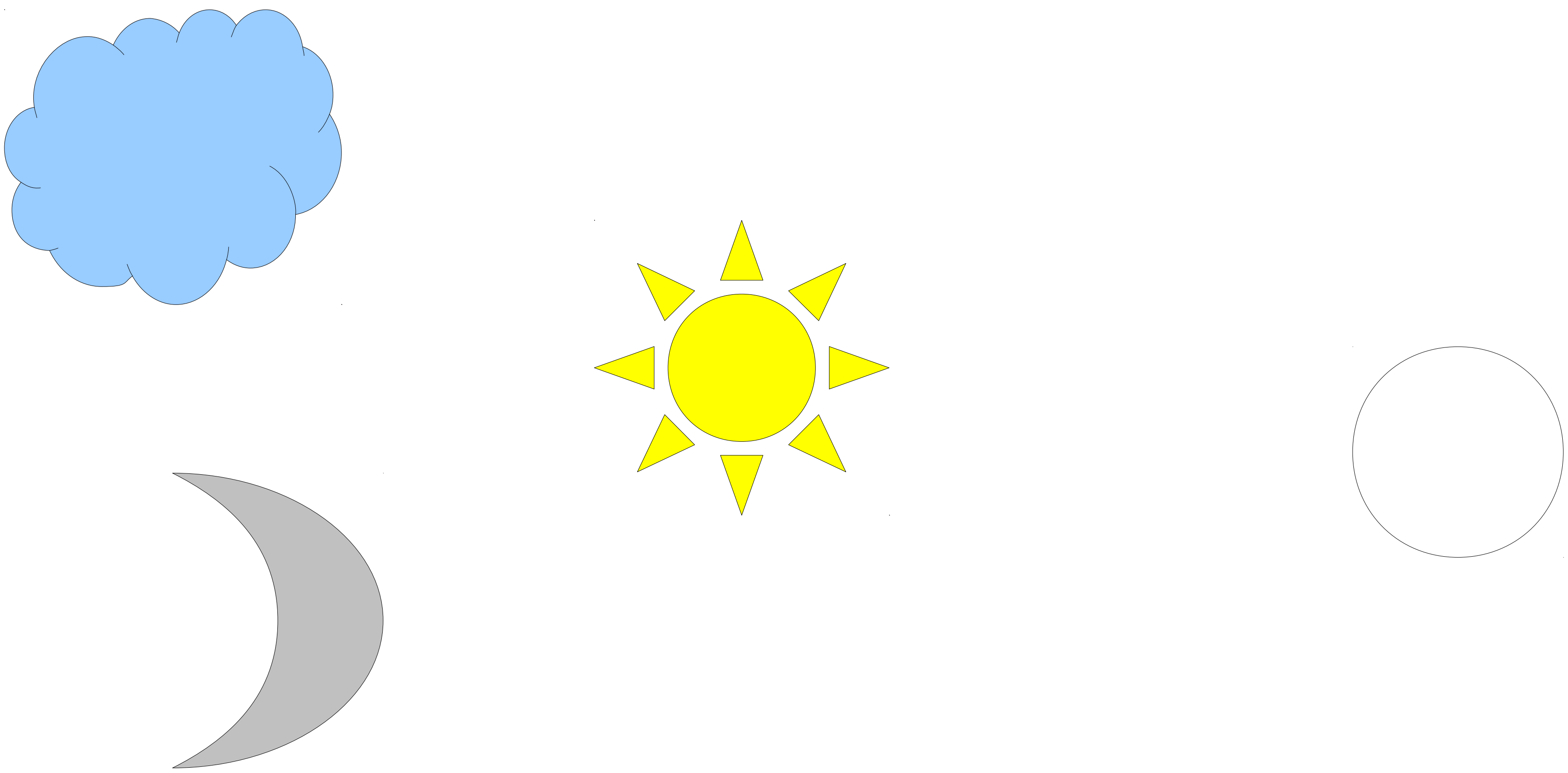

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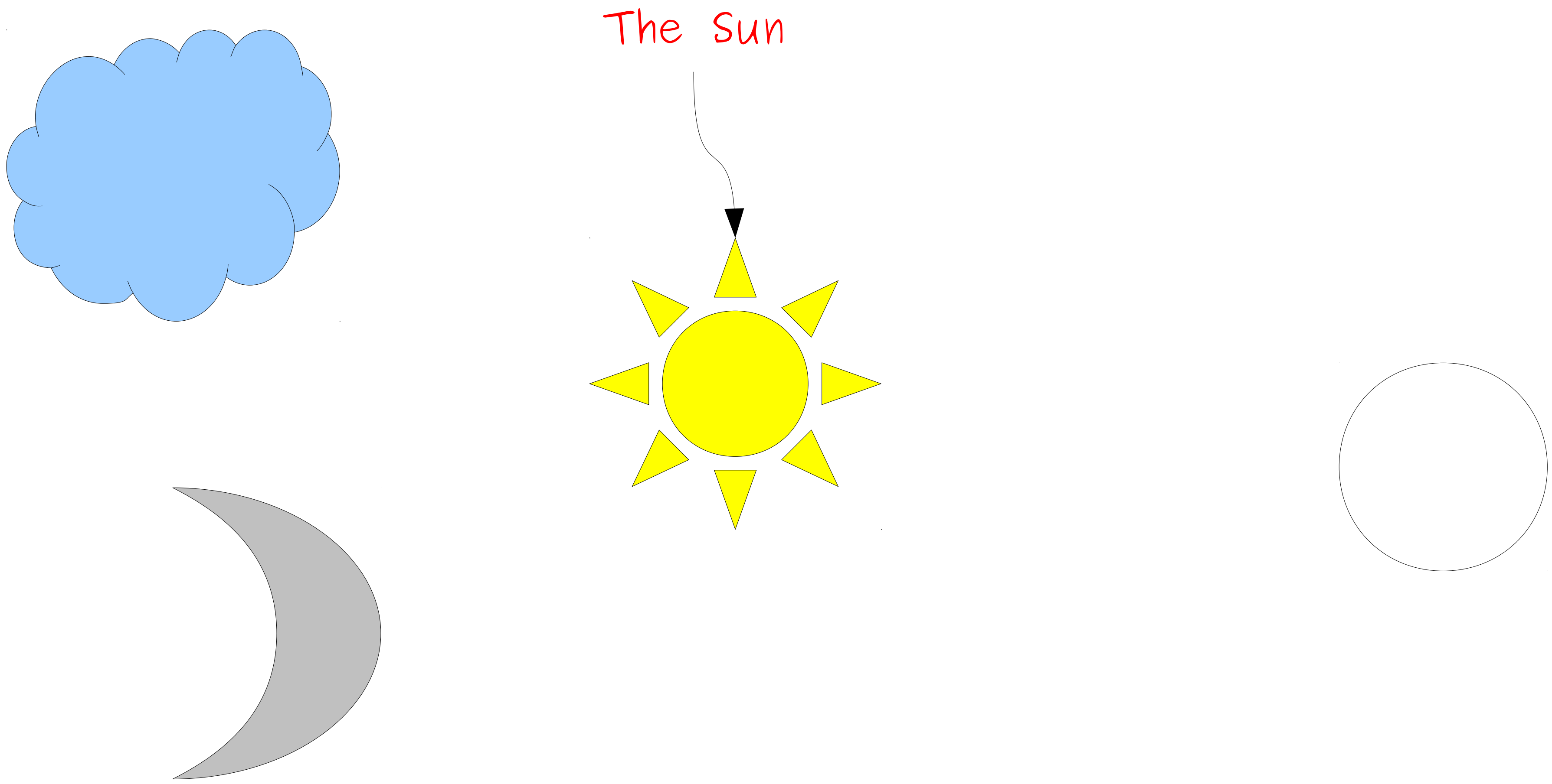

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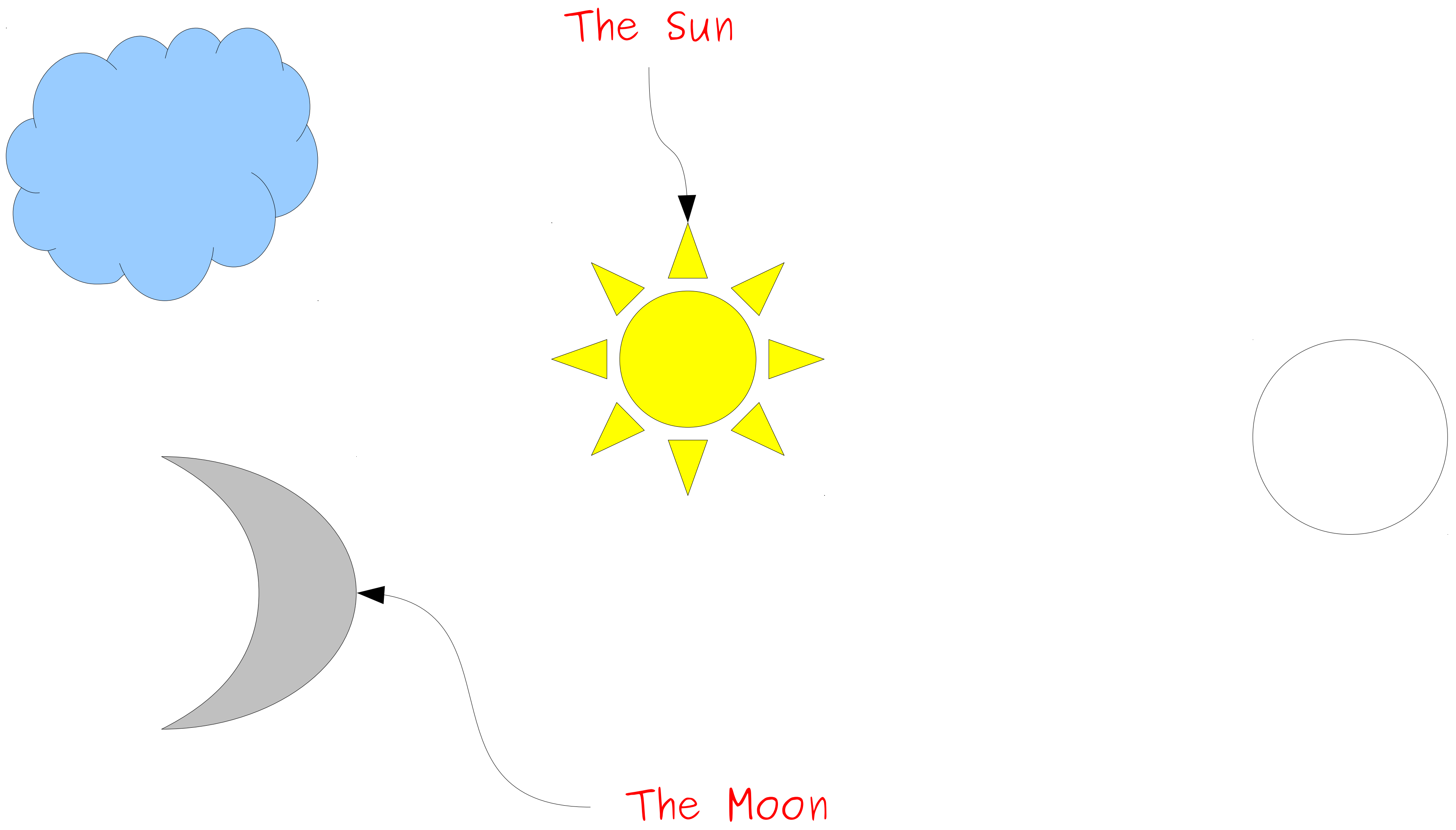

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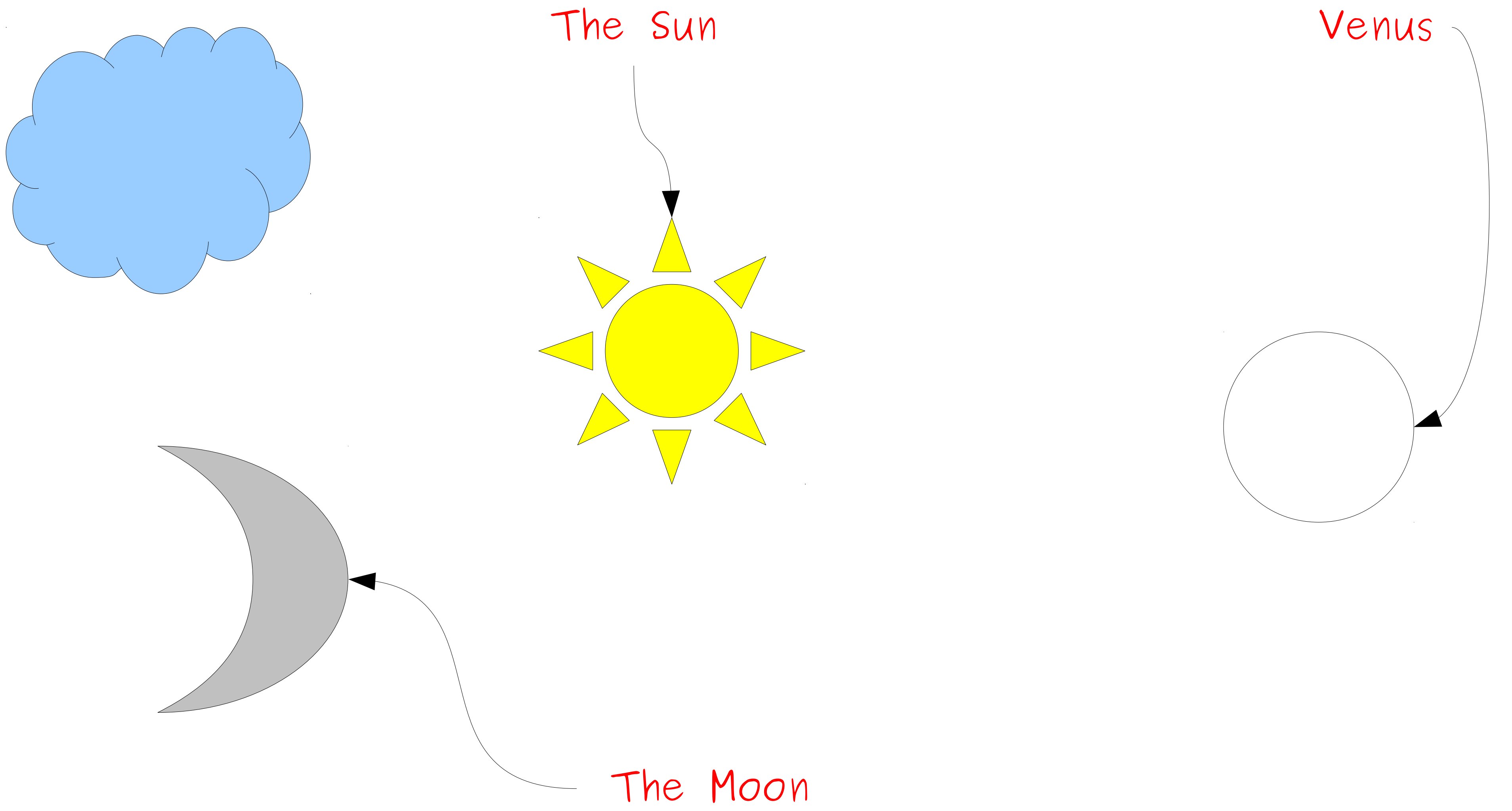

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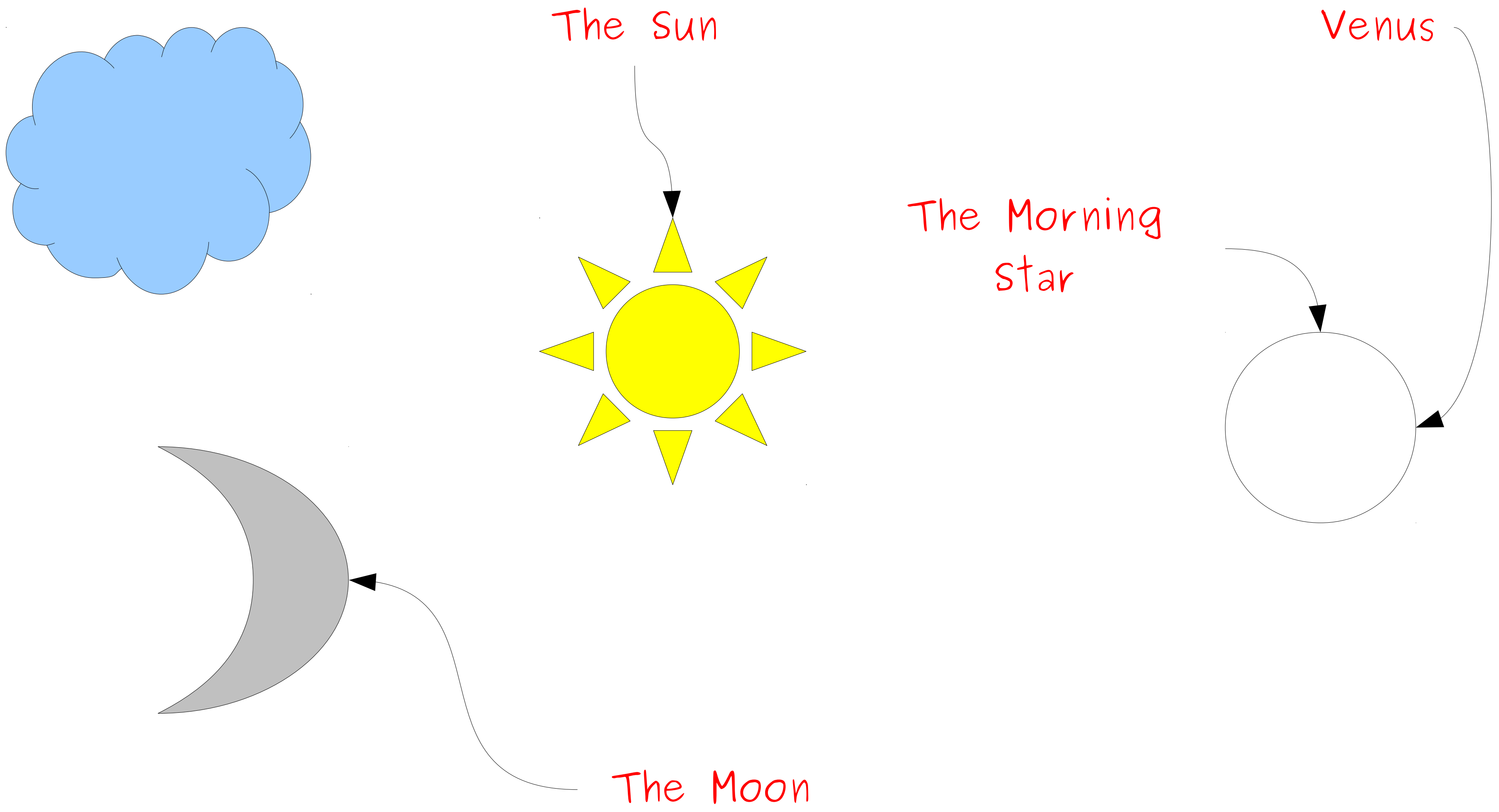

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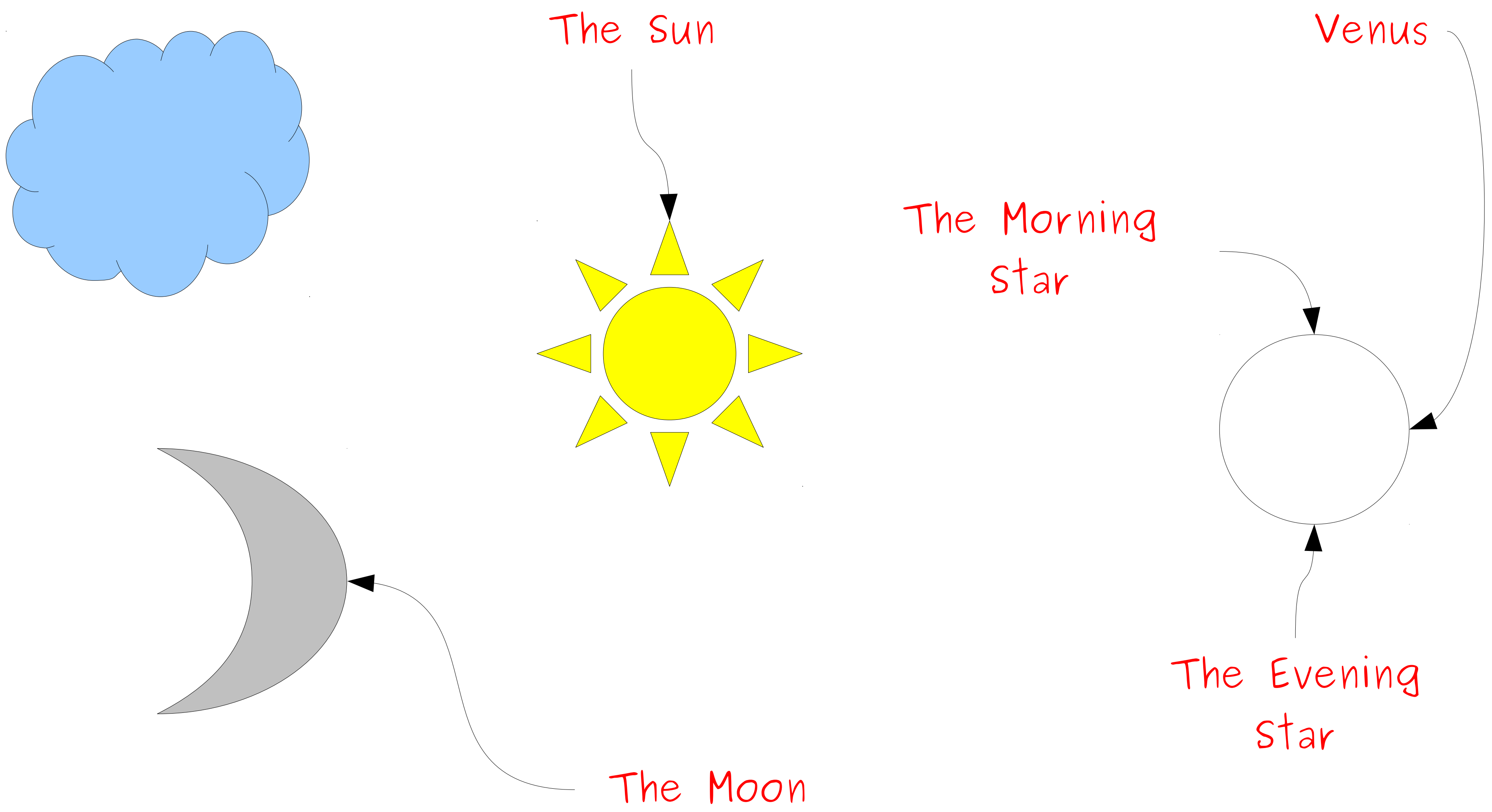
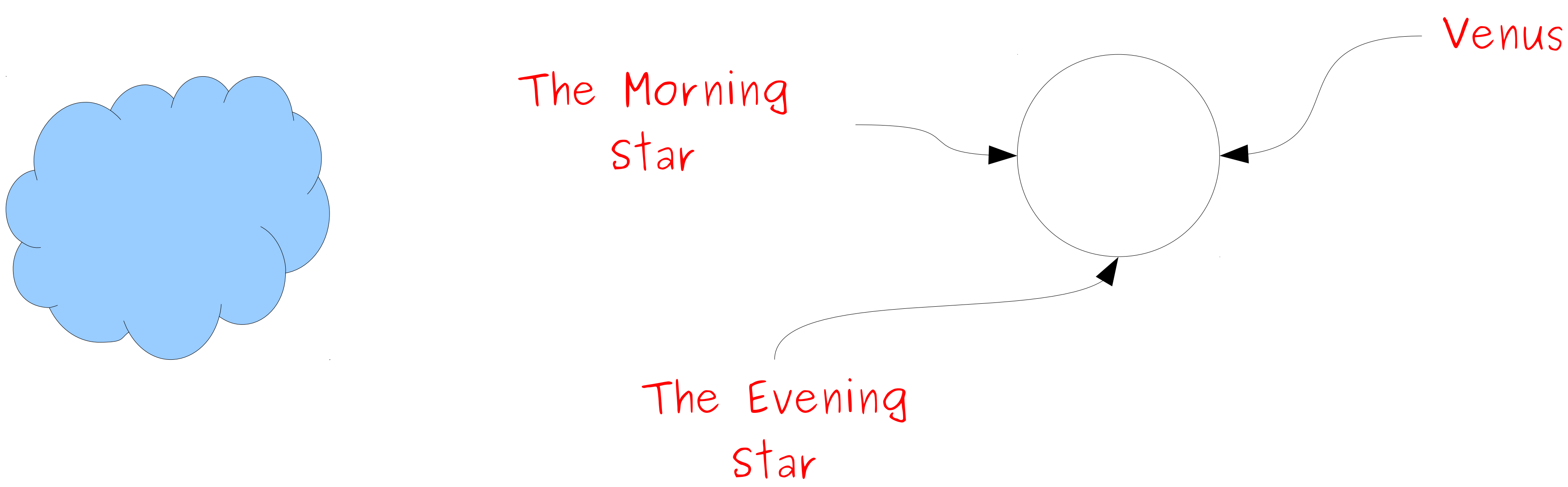


# First-Order Logic

- In first-order logic, each variable refers to some **object** in a set called the **domain of discourse**.
- Some objects may have multiple names.
- Some objects may have no name at all.



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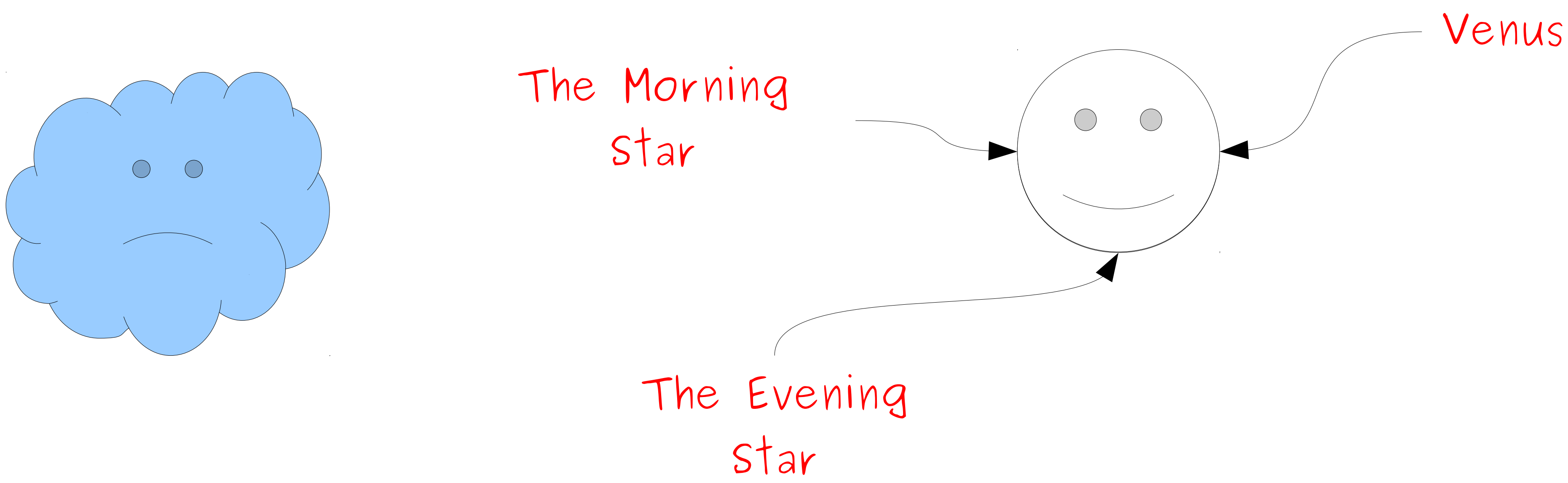

# Propositional vs. First-Order Logic

- Because propositional variables are either true or false, we can directly apply connectives to them.
  - $p \to q$
  - $\neg p \leftrightarrow q \land r$
- Because first-order variables refer to arbitrary objects, it does not make sense to apply connectives to them.
  - Venus → Sun
  - 137 ↔ ¬42

# Reasoning about Objects

- To reason about objects, first-order logic uses **predicates**.
- Examples:
  - *GottaGetDownOn*(Friday)
  - *LookingForwardTo*(Weekend)
  - *ComesAfterwards*(Sunday, Saturday)
- Predicates can take any number of arguments, but each predicate has a fixed number of arguments (called its **arity**)
- Applying a predicate to arguments produces a proposition, which is either true or false.

# First-Order Sentences

- Sentences in first-order logic can be constructed from predicates applied to objects:

$$LikesToEat(V, M) \land Near(V, M) \rightarrow WillEat(V, M)$$

$$\neg GetsCake(\text{Chell}) \rightarrow Destroys(\text{Chell}, \text{GlaDOS})$$

$$x < 8 \rightarrow x < 137$$

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$$x < 8 \rightarrow x < 137$$

The notation $x < 8$ is just a shorthand for something like LessThan(x, 8). Binary predicates in math are often written like this, but they are not a part of first-order logic.

# Equality

- First-order logic is equipped with a special predicate = that says whether two objects are equal to one another.
- Equality is a part of first-order logic, just as → and ¬ are.
- Examples:

MorningStar = EveningStar

Glenda = GoodWitchOfTheNorth

- Equality can only be applied to **objects**; to see if **propositions** are equal, use ↔.

# Expanding First-Order Logic

$$x < 8 \land y < 8 \rightarrow x + y < 16$$

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$$x < 8 \land y < 8 \rightarrow x + y < 16$$

Why is this allowed?

# Functions

- First-order logic allows **functions** that return objects associated with other objects.
- Examples:

$$x + y$$

*LengthOf*(path)

*MedianOf*(x, y, z)

- As with predicates, functions can take in any number of arguments, but each function has a fixed arity.
- Functions evaluate to **objects**, not **propositions**.

How would we translate the statement

"For any natural number n, n is even iff $n^2$ is even"

into first-order logic?

# Quantifiers

- The biggest change from propositional logic to first-order logic is the use of **quantifiers**.
- A **quantifier** is a statement that expresses that some property is true for some or all choices that could be made.
- Both of the following require quantifiers:
  - All velociraptors want to eat me.
  - Some test subject will receive cake.

“For any natural number n, n is even iff $n^2$ is even”

“For any natural number n, n is even iff $n^2$ is even”

$$\forall n.\ (Even(n) \leftrightarrow Even(n^2))$$

"For any natural number n, n is even iff $n^2$ is even"

$$\forall n.\ (\text{Even}(n) \leftrightarrow \text{Even}(n^2))$$

∀ is the universal quantifier and says "for any choice of n, the following is true."

# The Universal Quantifier

- A statement of the form $\forall x.\ \psi$ asserts that for every choice of x in our domain, $\psi$ is true.
- Examples:

$$\forall v.\ (\text{Velociraptor}(v) \rightarrow \text{WillEat}(v, \text{me}))$$

$$\forall n.\ (\text{Even}(n) \leftrightarrow \neg\text{Odd}(n))$$

$$\text{Tallest}(x) \rightarrow \forall y.\ (x \neq y \rightarrow \text{IsShorterThan}(y, x))$$

Some velociraptor can open windows.

Some velociraptor can open windows.

$\exists n. (\text{Velociraptor}(n) \land \text{OpensWindows}(n))$

Some velociraptor can open windows.

$\exists n.$ (Velociraptor(n) ∧ OpensWindows(n))

∃ is the existential quantifier and says "for some choice of n, the following is true."

# The Existential Quantifier

- A statement of the form $\exists x. \psi$ asserts that for **some** choice of x in our domain, ψ is true.
- Examples:

  $\exists x. (Even(x) \land Prime(x))$

  $\exists x. (TallerThan(x, me) \land LighterThan(x, me))$

  $(\exists x. Appreciates(x, me)) \rightarrow Happy(me)$

# Operator Precedence (Again)

- When writing out a formula in first-order logic, the quantifiers $\forall$ and $\exists$ have precedence just below $\neg$.
- Thus

$$\forall x.\ P(x) \lor R(x) \to Q(x)$$

is interpreted as

$$((\forall x.\ P(x)) \lor R(x)) \to Q(x)$$

rather than

$$\forall x.\ ((P(x) \lor R(x)) \to Q(x))$$

# A Bad Translation

Everyone who can outrun velociraptors won't get eaten.

$$\forall x. \, (\text{FasterThanVelociraptors}(x) \land \neg\text{WillBeEaten}(x))$$

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What about Invisible Fred, who can't outrun the velociraptors but won't get eaten either?

**"Any time P(x), then Q(x)"**

translates as

$$\forall x.\ (P(x) \rightarrow Q(x))$$

# Another Bad Translation

There is some velociraptor that can open windows and eat me.

$$\exists x. (Velociraptor(x) \land OpensWindows(x) \rightarrow EatsMe(x))$$

# Another Bad Translation

There is some velociraptor that can open windows and eat me.

$\exists x. (Velociraptor(x) \land OpensWindows(x) \to EatsMe(x))$

I'm not a velociraptor!

# Another Bad Translation

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$\exists x. (\mathbf{Velociraptor(x)} \land \mathrm{OpensWindows}(x) \to \mathrm{EatsMe}(x))$

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There is some velociraptor that can open windows and eat me.

$\exists x.$ (**Velociraptor(x) ∧ OpensWindows(x)** → EatsMe(x))

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$\exists x. (Velociraptor(x) \land OpensWindows(x) \land EatsMe(x))$

I'm not a velociraptor!

**“There is some P(x) where Q(x)”**

translates as

$$\exists x.\ (P(x) \land Q(x))$$