Section Handout 7

Problem One: Undecidability Reductions

For each of the following languages, show that the language is undecidable by reducing *HALT* to it.

i. Prove that *INFINITE* = { ⟨M⟩ | *L*(M) is finite } is undecidable.

ii. Prove that *JUSTONE* = { ⟨M⟩ | M loops on all but one string } is undecidable.

iii. Prove that *ENTERS* = { ⟨M, *w*, q⟩ | q is a state in M and M enters q when run on *w* } is undecidable.

Problem Two: Unrecognizability Reductions

For each of the following problems, show that the problem is unrecognizable by reducing the indicated problem to it.

i. Prove that $SUBSET_{TM}$ = { ⟨M_1, M_2⟩ | M_1 and M_2 are TMs, and $L(M_1) \subseteq L(M_2)$ } is unrecognizable. *(Hint: Reduce A_{ALL} from the problem set to $SUBSET_{TM}$)*

ii. Prove that *MISSONE* = { ⟨M⟩ | M halts on all but one string } is unrecognizable. *(Hint: Reduce NOHALT to MISSONE)*

iii. Prove that E_{TM} = { ⟨M⟩ | *L*(M) = Ø } is unrecognizable. *(Hint: Reduce NOHALT to E_{TM}).*

Problem Three: Rice's Theorem

For each of the following languages, state whether or not Rice's Theorem applies.

i. L = { ⟨M⟩ | |*L*(M)| = 2011. }

ii. L = { ⟨M, *w*⟩ | M halts on *w*. }

iii. L = { ⟨M⟩ | M accepts all strings of odd length }

iv. L = { ⟨M⟩ | M rejects all strings of odd length }