Section Handout 6

Problem One: Designing PDAs

Below are a list of alphabets and languages over those alphabets. For each language, design a pushdown automaton that recognizes the given language.

i. Let $\Sigma = \{0, 1, ?\}$ and let $L = \{ x?y \mid x, y \in \{0, 1\}^* \land y \text{ is the reverse of } x \}$. Design a **deterministic** PDA that recognizes L.

ii. Let $\Sigma = \{0, 1\}$ and let $L = \{ xy \mid x, y \in \Sigma^* \land |x| = |y| \land x \neq y \}$. Design a PDA that recognizes L.[*]

iii. Let $\Sigma = \{ 0, 1, 2 \}$ and let $L = \{ 0^m1^n2^p \mid m, n, p \in \mathbb{N} \land (m = n \lor m = p) \}$. Design a PDA that recognizes L.

Problem Two: The Pumping Lemma

For each of the following languages, use the pumping lemma to show that the given language is not context free.

i. Let $\Sigma = \{0, 1, A, B\}$ and let *TWOWAYBALANCE* = { w | w contains the same number of 0s and 1s and the same number of As and Bs }. Prove that *TWOWAYBALANCE* is not context-free.[†]

ii. Let $\Sigma = \{1, \times, =\}$ and let $MULTIPLY = \{1^m \times 1^n = 1^{mn} \mid m, n \in \mathbb{N} \}$. Prove that *MULTIPLY* is not context-free.

iii. Let $\Sigma = \{0, 1, 2\}$ and let L = { w | w contains the same number of 0s, 1s, and 2s. } Prove that L is not context-free.

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[*] This problem adapted from Problem 2.23 from Sipser.

[†] This problem adapted from Problem 2.32 from Sipser.

Problem Three: Designing Turing Machines

Design a Turing machine (by specifying a transition table or state-transition diagram) that accepts the language *BALANCE* = { w | w contains the same number of 0s and 1s }, where Σ = {0, 1}.

Problem Four: Turing Machine Equivalents

For each of the following variants of a Turing machine, prove that they are equivalent to a Turing machine by describing how the modified machine could simulate a Turing machine and how a Turing machine could simulate the modified machine.

i. A modified version of the TM where the tape head is allowed to stay in the same location on a transition.

ii. A modified version of the TM where the tape is infinite in both directions, not just in one direction.

iii. A modified version of the TM where the tape is infinite to the left, but if the machine moves the tape head off of the tape, the TM accepts.